The Last Lesson

(i) Given $x = \tan^2 4y$, $0 < y < \frac{\pi}{8}$, find $\frac{dy}{dx}$ as a function of x.

Write your answer in the form $\frac{1}{A\left(x^p + x^q\right)}$, where A, p and q are constants to be found.

(5)

(ii) The volume V of a cube is increasing at a constant rate of 2 $cm^3\ s^{-1}$. Find the rate at which the length of the edge of the cube is increasing when the volume of the cube is 64 cm^3.

(5)

6(i)	$x=\tan^2 4y \Rightarrow \dfrac{dx}{dy}=8\tan 4y\sec^2 4y$ oe	M1A1
	$\dfrac{dy}{dx}=\dfrac{1}{8\tan 4y\sec^2 4y}=\dfrac{1}{8\tan 4y(1+\tan^2 4y)}=\dfrac{1}{8\sqrt{x}(1+x)}=\dfrac{1}{8(x^{0.5}+x^{1.5})}$	M1,M1A1
		(5)
(ii)	$\dfrac{dV}{dt}=2,\quad V=x^3 \Rightarrow \dfrac{dV}{dx}=3x^2$	B1,B1
	Uses $\dfrac{dV}{dt}=\dfrac{dV}{dx}\times\dfrac{dx}{dt}$	M1
	$\left.\dfrac{dx}{dt}\right\|_{x=4}=\dfrac{2}{3x^2}=\dfrac{1}{24}\left(\text{cm s}^{-1}\right)$	M1A1
		(5)
		(10 marks)

Name The School

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2

3

4

5

Name The School

1 = Dorothy Stringer,
2 = Blatchington Mill,
3 = Downlands
4 = Priory
5 = Patcham High

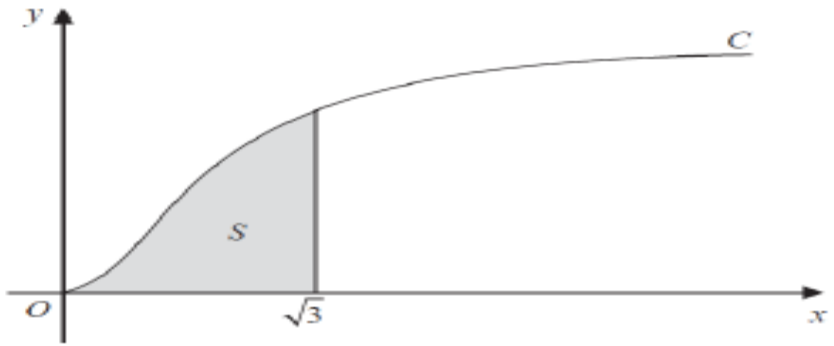

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 4, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the lines with equations $x = 1$ and $x = 3$.

(*a*) Complete the table below with the value of y corresponding to $x = 2$. Give your answer to 4 decimal places.

x	1	1.5	2	2.5	3
y	2	1.3041		0.9089	1.2958

(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(*c*) Use calculus to find the exact area of S.

Give your answer in the form $\frac{a}{b} + \ln c$, where a, b and c are integers.

(6)

(*d*) Hence calculate the percentage error in using your answer to part (*b*) to estimate the area of S. Give your answer to one decimal place.

(2)

(*e*) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

12(a)	0.9242 exactly	B1 **(1)**
(b)	Strip width =0.5	B1
	Area $\approx \frac{0.5}{2}\left((2+1.2958+2\times(1.3041+'0.9242'+0.9089)\right)$	M1
	$=2.393$	A1 **(3)**
(c)	$\int \frac{x^2 \ln x}{3} - 2x + 4 \, dx$	
	$= \frac{x^3}{9}\ln x - \int \frac{x^3}{9} \times \frac{1}{x} dx, \quad -x^2 + 4x$	M1A1, B1
	$= \frac{x^3}{9}\ln x - \frac{x^3}{27}\left(-x^2 + 4x\right)$	A1
	$\text{Area} = \left[\frac{x^3}{9}\ln x - \frac{x^3}{27} - x^2 + 4x\right]_1^3 = (3\ln 3 - 1 - 9 + 12) - \left(-\frac{1}{27} - 1 + 4\right)$	dM1
	$= \ln 27 - \frac{26}{27}$	A1 **(6)**
(d)	% error $= \pm \frac{\lvert real - approx \rvert}{real} \times 100$ =Accept awrt ±2.6%	M1A1 **(2)**
(e)	Increase the number of 'strips'	B1 **(1)** **(13 marks)**

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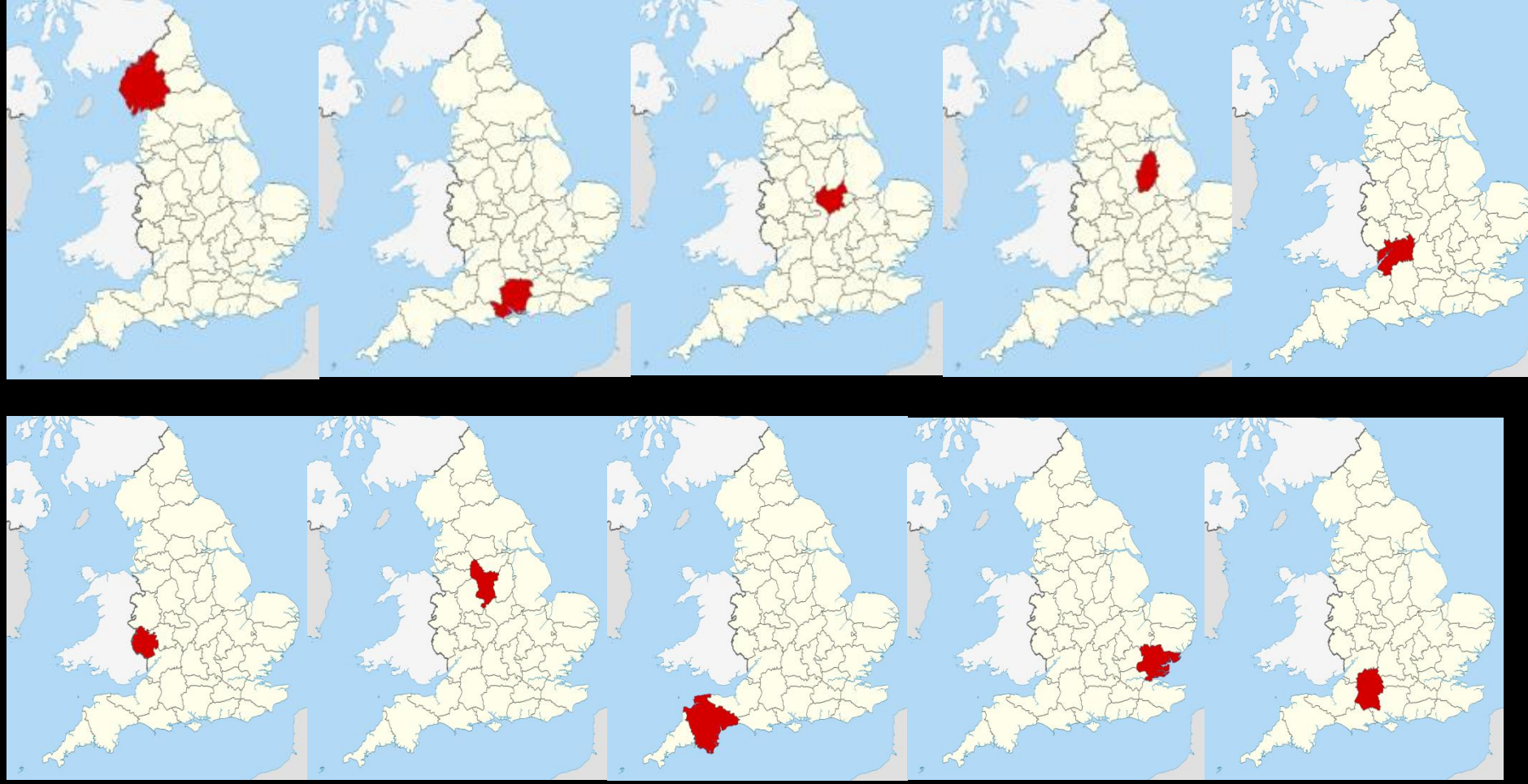
10

Name The County



1 = Cumbria, 2 = Hampshire, 3 = Leicestershire, 4 = Nottinghamshire, 5 = Gloucestershire,
6 = Herefordshire, 7 = Derbyshire, 8 = Devon, 9 = Essex, 10 = Wiltshire

9.

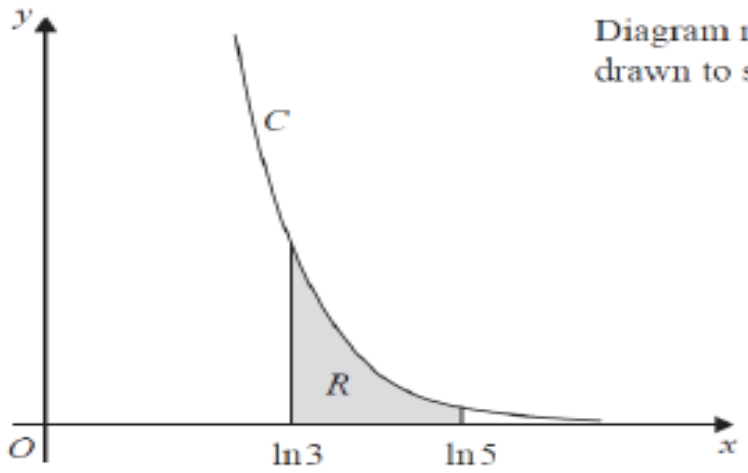


Diagram not drawn to scale

Figure 2

The curve C has parametric equations

$$x = \ln(t+2), \quad y = \frac{4}{t^2} \quad t > 0$$

The finite region R, shown shaded in Figure 2, is bounded by the curve C, the x-axis and the lines with equations $x = \ln 3$ and $x = \ln 5$.

(a) Show that the area of R is given by the integral

$$\int_1^3 \frac{\square}{t^2(t+\square)}\,dt$$

State p

and q

(3)

(b) Hence find an exact value for the area of R.

Write your answer in the form $(a + \ln b)$, where a and b are rational numbers.

(7)

(c) Find a cartesian equation of the curve C in the form $y = f(x)$.

(2)

9 (a)

$\frac{dx}{dt} = \frac{1}{t+2}$, Area of R $= \int y\,dx = \int \frac{4}{t^2} \times \frac{1}{(t+2)}(dt)$ B1, M1

Correct proof with limits and no errors Area $= \int_1^3 \frac{4}{t^2(t+2)} dt$ A1*

(3)

(b)

$\frac{4}{t^2(t+2)} = \frac{A}{t} + \frac{B}{t^2} + \frac{C}{(t+2)}$ or $\frac{4}{t^2(t+2)} = \frac{A}{t^2} + \frac{B}{(t+2)}$ p=4, q=2 B1

$4 = At(t+2) + B(t+2) + Ct^2$

Sub $t = 0 \Rightarrow B = 2$

Sub $t = -2 \Rightarrow C = 1$

Compare t^2 $A + C = 0 \Rightarrow A = -1$ M1A1

$\int_1^3 \frac{4}{t^2(t+2)} dt = \int_1^3 \frac{-1}{t} + \frac{2}{t^2} + \frac{1}{(t+2)} dt = \left[-\ln t - \frac{2}{t} + \ln(t+2) \right]_1^3$ M1A1

$= \left(-\ln 3 - \frac{2}{3} + \ln 5 \right) - \left(-\ln 1 - \frac{2}{1} + \ln 3 \right)$

$= \ln\left(\frac{5}{9}\right) + \frac{4}{3}$ dM1A1

(7)

(c)

Sub $t = e^x - 2$ into $y = \frac{4}{t^2} \Rightarrow y = \frac{4}{(e^x - 2)^2}$, $(x > \ln 2)$ M1A1

(2)

(12 marks)

Solve these equations

For every equation, solve for $0 \leq x \leq 360°$

1) sin x + cos x = 0
2) cot x = 0
3) sin x = 0.5
4) sec x = 0
5) 3 sin x + 4 cos x = 6

Solve these equations

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1) sin x + cos x = 0
2) cot x = 0
3) sin x = 0.5
4) sec x = 0
5) 3 sin x + 4 cos x = 6

1) 135, 315
2) 90, 270
3) 30, 150
4) No solutions
5) No solutions

8.

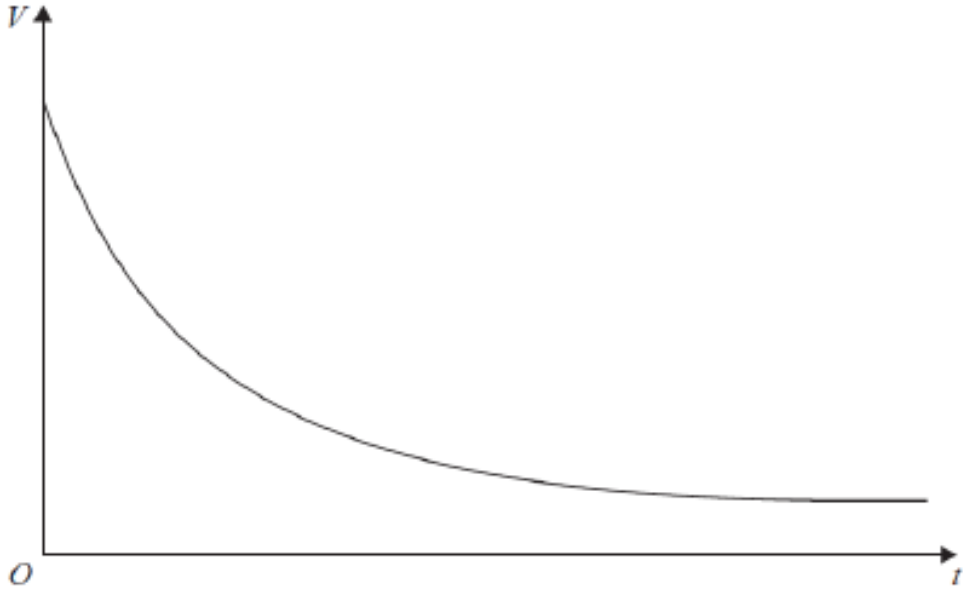


Figure 1

The value of Lin's car is modelled by the formula

$$V = 18\,000e^{-0.2t} + 4000e^{-0.1t} + 1000, \quad t \geq 0$$

where the value of the car is V pounds when the age of the car is t years.

A sketch of t against V is shown in Figure 1.

(*a*) State the range of V.

(2)

According to this model,

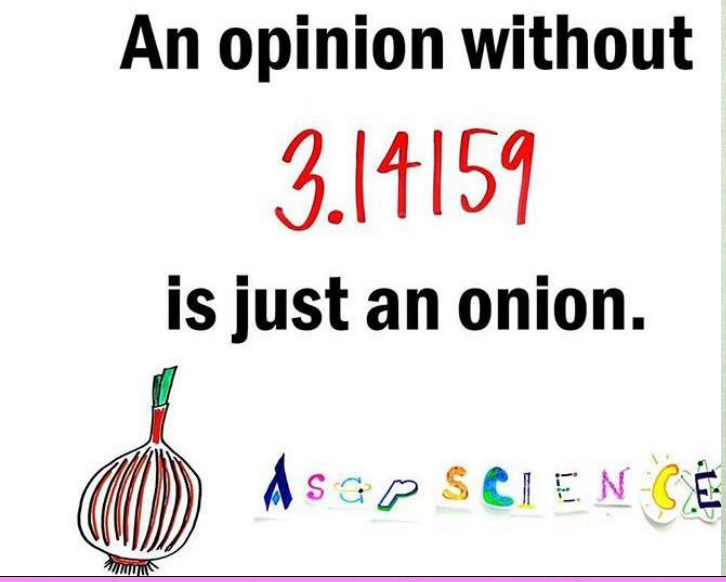
(*b*) find the rate at which the value of the car is decreasing when $t = 10$.
Give your answer in pounds per year.

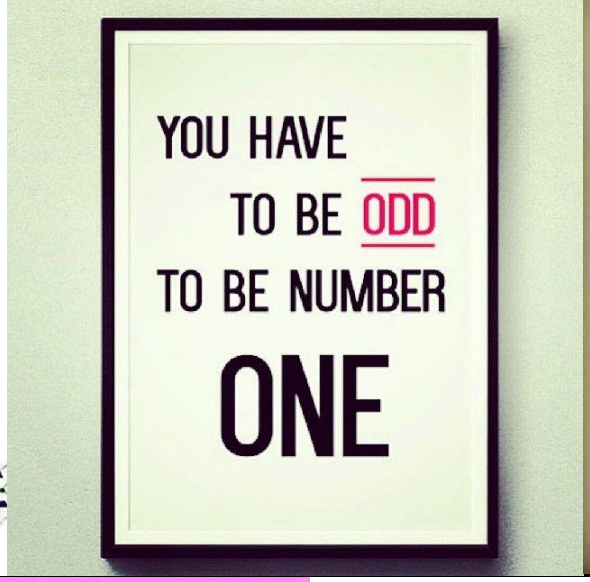
(3)

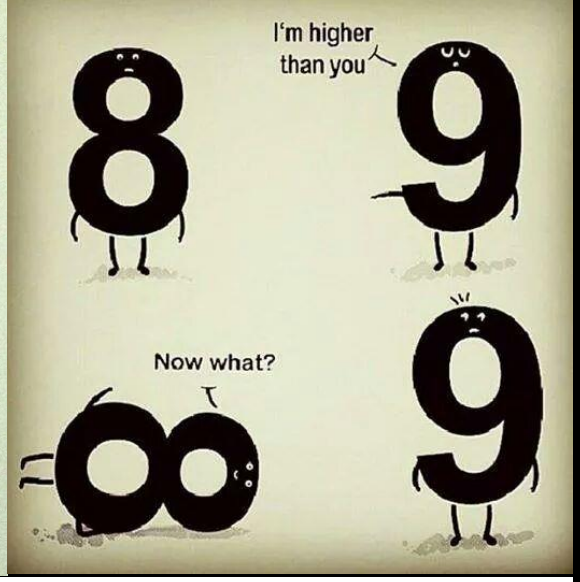
(*c*) Calculate the exact value of t when $V = 15\,000$.

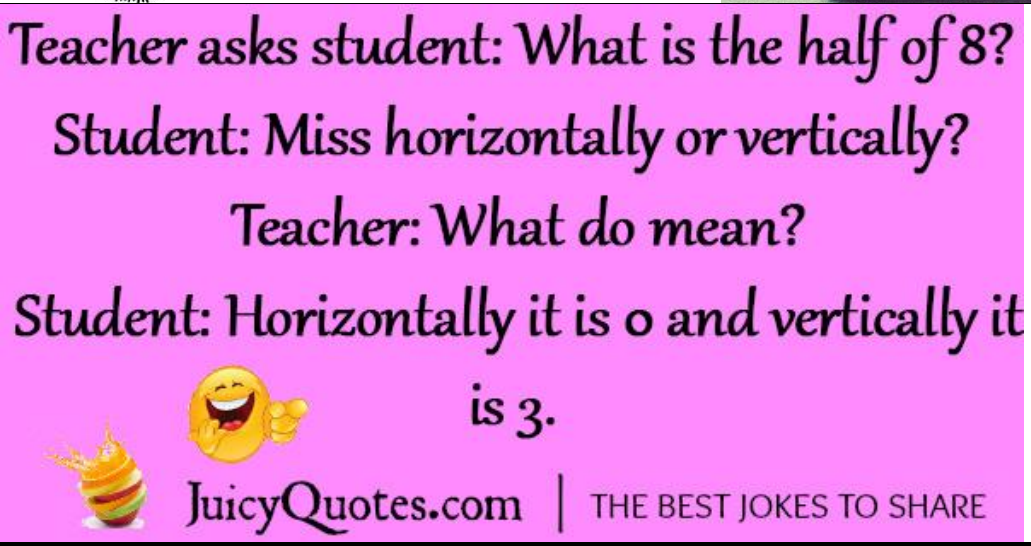
(4)

8 (a)	$1000 < V \leq 23000$	B1,B1 **(2)**
(b)	$\frac{dV}{dt} = 18000 \times -0.2e^{-0.2t} + 4000 \times -0.1e^{-0.1t}$	M1
	$\left.\frac{dV}{dt}\right\|_{t=10} = 18000 \times -0.2e^{-2} + 4000 \times -0.1e^{-1} = \text{awrt}(-)634$	M1A1 **(3)**
(c)	$15000 = 18000e^{-0.2t} + 4000e^{-0.1t} + 1000$ $0 = 9e^{-0.2t} + 2e^{-0.1t} - 7$ $0 = (9e^{-0.1t} - 7)(e^{-0.1t} + 1)$	M1A1
	$9e^{-0.1t} = 7 \Rightarrow t = 10\ln\left(\frac{9}{7}\right)$ oe	dM1A1 **(4)** **(9 marks)**









How many of these jokes are funny?

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