Question Number	Scheme	Marks
1.	$x = 3t - 4,\ \ y = 5 - \dfrac{6}{t},\quad t > 0$	
(a)	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = 3,\quad \dfrac{\mathrm{d}y}{\mathrm{d}t} = 6t^{-2}$	
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{6t^{-2}}{3}\ \left\{= \dfrac{6}{3t^2} = 2t^{-2} = \dfrac{2}{t^2}\right\}$	M1 A1 isw
		[2]
(b)	$\left\{t = \dfrac{1}{2} \Rightarrow\right\}\ P\left(-\dfrac{5}{2}, -7\right)$	B1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2}{\left(\frac{1}{2}\right)^2}$ **and** either • $y - "-7" = "8"\left(x - "-\frac{5}{2}"\right)$ • $"-7" = ("8")("-\frac{5}{2}") + c$ So, $y = (\text{their } m_{\mathrm{T}})x + "c"$	M1
	T: $y = 8x + 13$	A1 **cso**
		[3]
(c)	$\left\{t = \dfrac{x+4}{3} \Rightarrow\right\}\ y = 5 - \dfrac{6}{\left(\dfrac{x+4}{3}\right)}$	M1 A1 o.e.
	$\Rightarrow\ y = 5 - \dfrac{18}{x+4} \ \Rightarrow\ \ y = \dfrac{5(x+4) - 18}{x+4}$	
	So, $y = \dfrac{5x+2}{x+4},\ \ \{x > -4\}$	A1 **cso**
		[3]
		8

Question Number	Scheme	Marks
2.	$\left\{(2+kx)^{-3} = 2^{-3}\left(1+\frac{kx}{2}\right)^{-3} = \frac{1}{8}\left(1+(-3)\left(\frac{kx}{2}\right)+\frac{(-3)(-3-1)}{2!}\left(\frac{kx}{2}\right)^2+...\right)\right\}$, $k>0$	
(a)	$\{A=\}\ \frac{1}{8}$	B1 **cao**
		[1]
(b)	$\left(\frac{1}{8}\right)\frac{(-3)(-4)}{2!}\left(\frac{k}{2}\right)^2$	M1 M1 o.e.
	$\left\{\text{So, } \left(\frac{1}{8}\right)\frac{(-3)(-4)}{2!}\left(\frac{k}{2}\right)^2 = \frac{243}{16} \Rightarrow \frac{3}{16}k^2 = \frac{243}{16} \Rightarrow k^2 = 81\right\}$	
	So, $k=9$	A1 **cso**
		[3]
(c)	$''\left(\frac{1}{8}\right)''(-3)\left(\frac{k}{2}\right)$	M1
	$\left\{\text{So, } B = \left(\frac{1}{8}\right)(-3)\left(\frac{9}{2}\right) \Rightarrow\right\}\ \underline{B=-\frac{27}{16}}$	A1 **cso**
		[2]
		6

Question Number	Scheme	Marks
3.	x | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1 y | 2 | ***1.8625426...*** | 1.71830 | 1.56981 | 1.41994 | 1.27165	
(a)	$\{\text{At } x = 0.2,\}\ y = 1.86254$ (5 dp)	B1 **cao**
		[1]
(b)	$\frac{1}{2}(0.2)\underline{\left[2+1.27165+2\left(\text{their } 1.86254+1.71830+1.56981+1.41994\right)\right]}$	B1 o.e.
		M1
	$\left\{=\frac{1}{10}(16.41283)\right\} = 1.641283 = 1.6413$ (4 dp)	A1
		[3]
(c)	$\left\{u = \mathrm{e}^x \text{ or } x = \ln u \Rightarrow\right\}$	
	$\frac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x$ **or** $\frac{\mathrm{d}u}{\mathrm{d}x} = u$ **or** $\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{1}{u}$ **or** $\mathrm{d}u = u\,\mathrm{d}x$ etc., **and** $\int \frac{6}{(\mathrm{e}^x+2)}\mathrm{d}x = \int \frac{6}{(u+2)u}\,\mathrm{d}u$	B1 *
	$\{x=0\} \Rightarrow a = \mathrm{e}^0 \Rightarrow \underline{a=1}$ $\{x=1\} \Rightarrow b = \mathrm{e}^1 \Rightarrow \underline{b=\mathrm{e}}$	B1
		[2]
(d)	$\frac{6}{u(u+2)} \equiv \frac{A}{u} + \frac{B}{(u+2)}$	M1
	$\Rightarrow 6 \equiv A(u+2) + Bu$ $u = 0 \ \triangleright\ A = 3$ $u = -2 \ \triangleright\ B = -3$	A1
	$\int \frac{6}{u(u+2)}\,\mathrm{d}u = \int\left(\frac{3}{u} - \frac{3}{(u+2)}\right)\mathrm{d}u$	M1
	$= 3\ln u - 3\ln(u+2)$ or $= 3\ln 2u - 3\ln(2u+4)$	A1 ft

	$\left\{\text{So}\left[3\ln u - 3\ln(u+2)\right]_1^{\text{e}}\right\}$ $=\left(3\ln(\text{e}) - 3\ln(\text{e}+2)\right) - \left(3\ln 1 - 3\ln 3\right)$	dM1
	$= 3 - 3\ln(\text{e}+2) + 3\ln 3$ **or** $3(1 - \ln(\text{e}+2) + \ln 3)$ **or** $3 + 3\ln\left(\frac{3}{\text{e}+2}\right)$ **or** $3\ln\left(\frac{\text{e}}{\text{e}+2}\right) - 3\ln\left(\frac{1}{3}\right)$ **or** $3 - 3\ln\left(\frac{\text{e}+2}{3}\right)$ **or** $3\ln\left(\frac{3\text{e}}{\text{e}+2}\right)$ **or** $\ln\left(\frac{27\text{e}^3}{(\text{e}+2)^3}\right)$	A1 **cso**
		[6]
		12

Question Number	Scheme	Marks
4.	$4x^2 - y^3 - 4xy + 2^y = 0$	
(a)	$\left\{\frac{d}{dx}\times\right\}\ 8x - 3y^2\frac{dy}{dx} - 4y - 4x\frac{dy}{dx} + 2^y \ln 2\frac{dy}{dx} = 0$	M1 A1 M1 B1
	$8(-2) - 3(4)^2\frac{dy}{dx} - 4(4) - 4(-2)\frac{dy}{dx} + 2^4 \ln 2\frac{dy}{dx} = 0$	dM1
	$-16 - 48\frac{dy}{dx} - 16 + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} = 0$	
	$\frac{dy}{dx} = \frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$ or exact equivalent	A1 **cso**
		[6]
(b)	e.g. $m_N = \frac{-40+16\ln 2}{-32}$ or $\frac{40-16\ln 2}{32}$	M1
	• $y - 4 = \left(\frac{40-16\ln 2}{32}\right)(x - - 2)$ Cuts y-axis $\Rightarrow x = 0 \Rightarrow y - 4 = \left(\frac{40-16\ln 2}{32}\right)(2)$ $4 = \left(\frac{40-16\ln 2}{32}\right)(-2) + c$	M1
	$\left\{\Rightarrow c = 4 + \frac{40-16\ln 2}{16}, \text{ so } y = \frac{104-16\ln 2}{16} \Rightarrow\right\}$	
	y (or c) $= \frac{13}{2} - \ln 2$	A1 **cso isw**
		[3]
		9

Question Number	Scheme	Marks
5.	$\frac{\mathrm{d}h}{\mathrm{d}t} = k\sqrt{(h-9)},\ \ 9 < h \leq 200;\quad h = 130,\ \frac{\mathrm{d}h}{\mathrm{d}t} = -1.1$	
(a)	$-1.1 = k\sqrt{(130-9)} \Rightarrow k = ...$	M1
	so, $k = -\frac{1}{10}$ or -0.1	A1
		[2]
(b)	$\int \frac{\mathrm{d}h}{\sqrt{(h-9)}} = \int k\,\mathrm{d}t$	B1
	$\int (h-9)^{-\frac{1}{2}}\,\mathrm{d}h = \int k\,\mathrm{d}t$	
	$\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt\,(+c)$	M1 A1
	$\{t = 0,\ h = 200 \Rightarrow\}\ \ 2\sqrt{(200-9)} = k(0) + c$	M1
	$\Rightarrow c = 2\sqrt{191} \Rightarrow 2(h-9)^{\frac{1}{2}} = -0.1t + 2\sqrt{191}$ $\{h = 50 \Rightarrow\}\ \ 2\sqrt{(50-9)} = -0.1t + 2\sqrt{191}$ $t = ...$	dM1
	$t = 20\sqrt{191} - 20\sqrt{41}$ or $t = 148.3430145... = 148$ (minutes) (nearest minute)	A1 cso
		[6]
		8

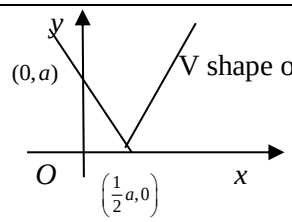
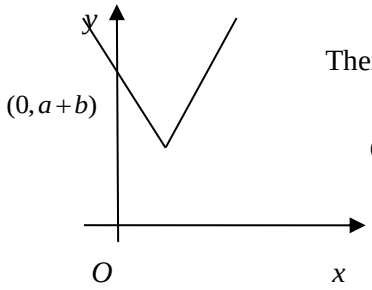
Question Number	Scheme	Marks
6.	$x^2-9=(x+3)(x-3)$	B1
	$\dfrac{4x}{x^2-9}-\dfrac{2}{(x+3)}=\dfrac{4x-2(x-3)}{(x+3)(x-3)}$	M1
	$=\dfrac{2x+6}{(x+3)(x-3)}$	A1
	$=\dfrac{2\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$	
	$=\dfrac{2}{(x-3)}$	A1
		(4)

Question Number	Scheme	Marks
7.(a)	$e^{3x-9}=8\Rightarrow 3x-9=\ln 8$	M1
	$\Rightarrow x=\dfrac{\ln 8+9}{3},=\ln 2+3$	A1, A1
		(3)
(b)	$\ln(2y+5)=2+\ln(4-y)$	
	$\ln\left(\dfrac{2y+5}{4-y}\right)=2$	M1
	$\left(\dfrac{2y+5}{4-y}\right)=e^2$	M1
	$2y+5=e^2(4-y)\Rightarrow 2y+e^2y=4e^2-5\Rightarrow y=\dfrac{4e^2-5}{2+e^2}$	dM1, A1
		(4)
		7 marks

Question Number	Scheme	Marks
8.(a)	$y \geq 3$	B1 (1)
(b)	$y = 3 + \sqrt{x+2} \Rightarrow y - 3 = \sqrt{x+2} \Rightarrow x = (y-3)^2 - 2$	M1 A1
	$\Rightarrow g^{-1}(x) = (x-3)^2 - 2, \quad \text{with } x \geq 3$	A1 (3)
(c)	$g(x) = x \Rightarrow 3 + \sqrt{x+2} = x$	
	$\Rightarrow x + 2 = (x-3)^2 \Rightarrow x^2 - 7x + 7 = 0$	M1, A1
	$\Rightarrow x = \frac{7 \pm \sqrt{21}}{2} \Rightarrow x = \frac{7 + \sqrt{21}}{2}$ only	M1, A1 (4)
(d)	$a = \frac{7 + \sqrt{21}}{2}$	B1 ft (1) **9 marks**

Question Number	Scheme	Marks
9.(a)	$R = \sqrt{29}$	B1
	$\tan\alpha = \frac{2}{5} \Rightarrow \alpha = \text{awrt } 0.381$	M1A1
		(3)
(b)	$5\cot 2x - 3\text{cosec}\, 2x = 2 \Rightarrow 5\frac{\cos 2x}{\sin 2x} - \frac{3}{\sin 2x} = 2$	M1
	$\Rightarrow 5\cos 2x - 2\sin 2x = 3$	A1
		(2)
(c)	$5\cos 2x - 2\sin 2x = 3 \Rightarrow \cos(2x + 0.381) = \frac{3}{\sqrt{29}}$	M1
	$2x + 0.381 = \arccos\left(\frac{3}{\sqrt{29}}\right) \Rightarrow x = \ldots$	dM1
	$x = \text{awrt } 0.30, 2.46$	A1A1
		(4)
		(9 marks)

Question Number	Scheme	Marks
10. (a)	At P $x=-2 \Rightarrow y=3$	B1
	$\frac{dy}{dx}=\frac{4}{2x+5}-\frac{3}{2}$	M1, A1
	$\left.\frac{dy}{dx}\right\|_{x=-2}=\frac{5}{2} \Rightarrow$ Equation of normal is $y-'3'=-\frac{2}{5}\left(x-(-2)\right)$	M1
	$\Rightarrow 2x+5y=11$	A1
		(5)
(b)	Combines $5y+2x=11$ and $y=2\ln(2x+5)-\frac{3x}{2}$ to form equation in x	
	$5\left(2\ln(2x+5)-\frac{3x}{2}\right)+2x=11$	M1
	$\Rightarrow x=\frac{20}{11}\ln(2x+5)-2$	dM1 A1*
		(3)
(c)	Substitutes $x_1=2 \Rightarrow x_2=\frac{20}{11}\ln 9-2$	M1
	Awrt $x_2=1.9950$ and $x_3=1.9929.$	A1
		(2)
		(10 marks)

Question Number	Scheme	Marks
11.(a)(i)	 V shape on x - axis **or** coordinates $\left(\frac{1}{2}a, 0\right)$ **and** $(0, a)$ Correct shape, position and coordinates	B1 B1
(ii)	 Their "V" shape translated up or $(0, a+b)$ Correct shape, position and $(0, a+b)$	B1ft B1 **(4)**
(b)	States or uses $a+b=8$	B1
	Attempts to solve $\lvert 2x-a\rvert+b=\frac{3}{2}x+8$ in either x or with $x = c$ $2c-a+b=\frac{3}{2}c+8 \Rightarrow kc=\text{f}(a,b)$	M1
	Combines $kc=\text{f}(a,b)$ with $a+b=8 \quad \Rightarrow c=4a$	dM1 A1 (4) **(8 marks)**

Question Number	Scheme	Marks
12(i) (a)	$y = 2x\left(x^2-1\right)^5 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \left(x^2-1\right)^5 \times 2 + 2x \times 10x\left(x^2-1\right)^4$	M1A1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \left(x^2-1\right)^4\left(2x^2-2+20x^2\right) = \left(x^2-1\right)^4\left(22x^2-2\right)$	M1 A1
		(4)
(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} \ldots 0 \Rightarrow \left(22x^2-2\right) \ldots 0 \Rightarrow$ critical values of $\pm\dfrac{1}{\sqrt{11}}$	M1
	$x \ldots -\dfrac{1}{\sqrt{11}}$ $x \,,, \dfrac{1}{\sqrt{11}}$	A1
		(2)
(ii)	$x = \ln(\sec 2y) \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \dfrac{1}{\sec 2y} \times 2\sec 2y \tan 2y$	B1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2\tan 2y} = \dfrac{1}{2\sqrt{\sec^2 2y - 1}} = \dfrac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$	M1 M1 A1
		(4)
		10 marks