Pure Test 3b. 50 marks. 1 hour

6. Express $\frac{4x}{x^2-9} - \frac{2}{x+3}$ as a single fraction in its simplest form.

(Total 4 marks)

8.

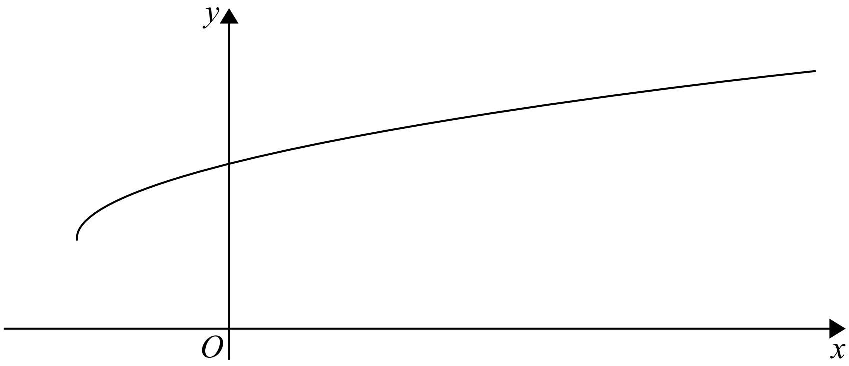

Figure 1

Figure 1 shows a sketch of part of the graph of $y = \mathrm{g}(x)$, where

$g(x) = 3 + \sqrt{x+2}, \qquad x \geq -2$

(*a*) State the range of g.

(1)

(*b*) Find $\mathrm{g}^{-1}(x)$ and state its domain.

(3)

(*c*) Find the exact value of x for which

$$\mathrm{g}(x) = x$$

(4)

(*d*) Hence state the value of a for which

$$\mathrm{g}(a) = \mathrm{g}^{-1}(a)$$

(1)

(Total 9 marks)

9. (*a*) Write $5 \cos \theta - 2 \sin \theta$ in the form $R \cos (\theta + \alpha)$, where R and α are constants,

$R > 0$ and $0 \leqslant \alpha < \dfrac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

(*b*) Show that the equation

$$5 \cot 2x - 3 \operatorname{cosec} 2x = 2$$

can be rewritten in the form

$$5 \cos 2x - 2 \sin 2x = c$$

where c is a positive constant to be determined.

(2)

(*c*) Hence or otherwise, solve, for $0 \leqslant x < \pi$,

$$5 \cot 2x - 3 \operatorname{cosec} 2x = 2$$

giving your answers to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

(Total 9 marks)

10.

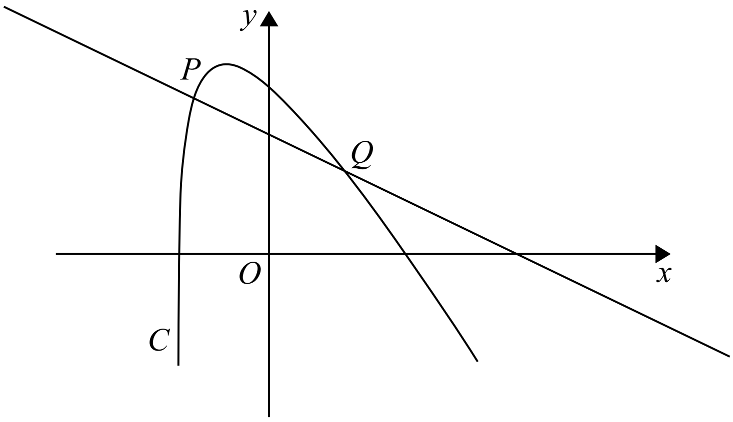

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 2\ln(2x + 5) - \frac{3x}{2}, \qquad x > -2.5$$

The point P with x coordinate –2 lies on C.

(a) Find an equation of the normal to C at P. Write your answer in the form $ax + by = c$, where a, b and c are integers.

(5)

The normal to C at P cuts the curve again at the point Q, as shown in Figure 2.

(b) Show that the x coordinate of Q is a solution of the equation

$$x = \frac{20}{11}\ln(2x + 5) - 2$$

(3)

The iteration formula

$$x_{n+1} = \frac{20}{11}\ln(2x_n + 5) - 2$$

can be used to find an approximation for the x coordinate of Q.

(c) Taking $x_1 = 2$, find the values of x_2 and x_3, giving each answer to 4 decimal places.

(2)

(Total 10 marks)

11. Given that a and b are positive constants,

(*a*) on separate diagrams, sketch the graph with equation

(i) $y = |2x - a|$

(ii) $y = |2x - a| + b$

Show, on each sketch, the coordinates of each point at which the graph crosses or meets the axes.

(4)

Given that the equation

$$|2x - a| + b = \frac{3}{2}x + 8$$

has a solution at $x = 0$ and a solution at $x = c$,

(*b*) find c in terms of a.

(4)

(Total 8 marks)

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12. (i) Given $y = 2x(x^2 - 1)^5$, show that

(*a*) $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{g}(x)(x^2 - 1)^4$ where $\mathrm{g}(x)$ is a function to be determined.

(4)

(*b*) Hence find the set of values of x for which $\frac{\mathrm{d}y}{\mathrm{d}x} \geqslant 0$

(2)

(ii) Given

$$x = \ln(\sec 2y), \qquad 0 < y < \frac{\pi}{4}$$

find $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a function of x in its simplest form.

(4)

(Total 10 marks)
