SECTION A: STATISTICS

Answer ALL questions.

An engineer believes that there is a relationship between the CO_2 emissions and fuel consumption for cars.

A random sample of 40 different car models (old and new) was taken and the CO_2 emission figures, e grams per kilometre, and fuel consumption, f miles per gallon, were recorded, as shown in Figure 1. The engineer calculates the product moment correlation coefficient for the 40 cars and obtains $r = -0.803$.

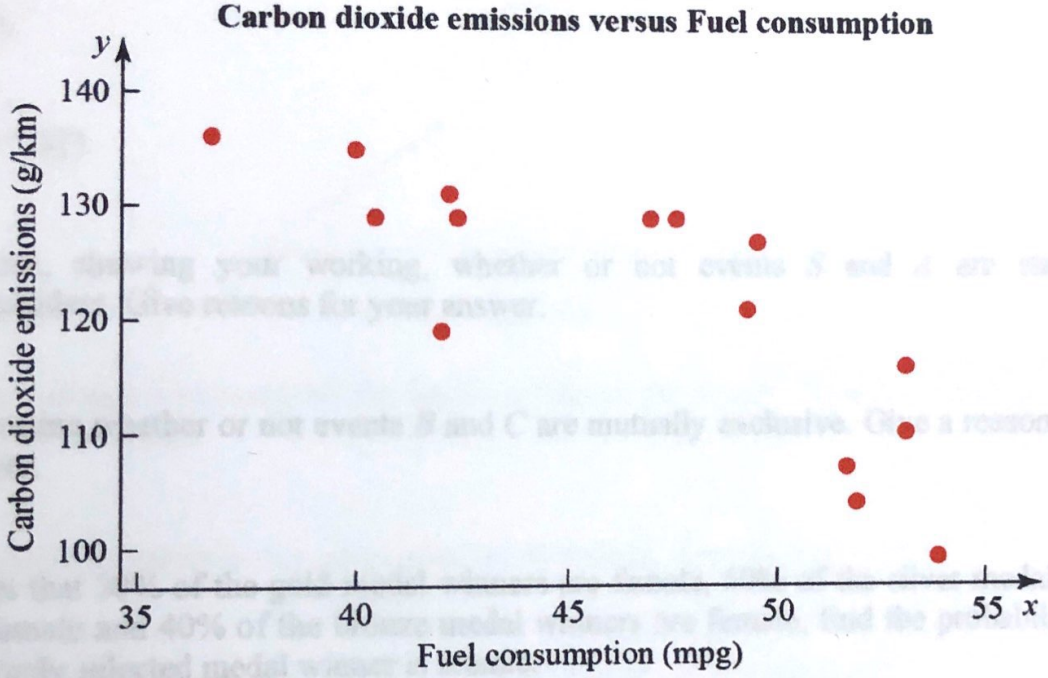


Figure 1

(a) State what is measured by the product moment correlation coefficient.

(1)

(b) State, with a reason, whether a linear regression model based on these data is reliable or not for a car when the fuel consumption is 60 mpg.

(1)

(c) For the linear regression model $e = 198 - 1.71 \times f$ write down the explanatory variable.

(1)

(d) State the definition of a hypothesis test.

(1)

(e) Test at 1% significance level whether or not the product moment correlation coefficient for CO_2 emissions and fuel consumption is less than zero. State your hypotheses clearly.

(3)

(Total 7 marks)

2. The table below shows the number of gold, silver and bronze medals won by two teams in an athletics competition.

	Gold	Silver	Bronze
Team *A*	29	17	18
Team *C*	21	23	17

The events G, S and B are that a medal is gold, silver or bronze respectively. Let A be the event that team A won a medal and C team C won a medal. A medal winner is selected at random. Find

(a) P(G),

(2)

(b) P($[A \cap S]'$).

(2)

(c) Explain, showing your working, whether or not events S and A are statistically independent. Give reasons for your answer.

(2)

(d) Determine whether or not events B and C are mutually exclusive. Give a reason for your answer.

(2)

(e) Given that 30% of the gold medal winners are female, 60% of the silver medal winners are female and 40% of the bronze medal winners are female, find the probability that a randomly selected medal winner is female.

(2)

(Total 10 marks)

3. The heights of a population of men are normally distributed with mean μ cm and standard deviation σ cm. It is known that 20% of the men are taller than 180 cm and 5% are shorter than 170 cm.

(a) Sketch a diagram to show the distribution of heights represented by this information.

(3)

(b) Find the value of μ and σ.

(7)

(c) Three men are selected at random, find the probability that they are all taller than 175 cm.

(2)

(Total 12 marks)

SECTION B: MECHANICS

Answer ALL questions.

4. Figure 5 shows an object of 3 kg sitting on a plane inclined at an angle θ to the horizontal. The coefficient of friction between the object and the plane is μ. The system is in limiting equilibrium.

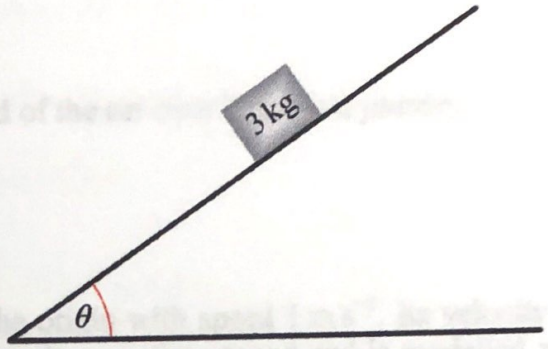


Figure 5

(a) Draw a diagram showing all the forces acting on the object. Describe the origin of each force using words.

(3)

(b) By resolving forces in two perpendicular directions, show that $\mu = \tan \theta$.

(6)

(c) Hence, determine whether or not the object slips if $\mu = 0.3$ and $\theta = 30°$.

(4)

(d) As θ approaches 90°, state whether an object of any mass could remain in equilibrium. Explain your answer.

(2)

(Total 15 marks)

5. A car travels along a long, straight road for one hour, starting from rest. After t hours, its acceleration is a km h^{-2}, where $a = 180 - 360t$.

(a) Find the speed of the car, in km h^{-1} in terms of t. **(2)**

The speed limit is 40 km h^{-1}.

(b) Find the range of times during which the car is breaking the speed limit. Give your answer in minutes. **(4)**

(c) Find the average speed of the car over the whole journey. **(5)**

(Total 11 marks)

6. A ball is launched from the origin with speed 1 m s^{-1}. Its velocity vector makes an angle θ above the horizontal. It travels over flat ground and is modelled as a particle moving freely under gravity, as shown in Figure 4.

(In this question, take $g = 10$ m s^{-2}.)

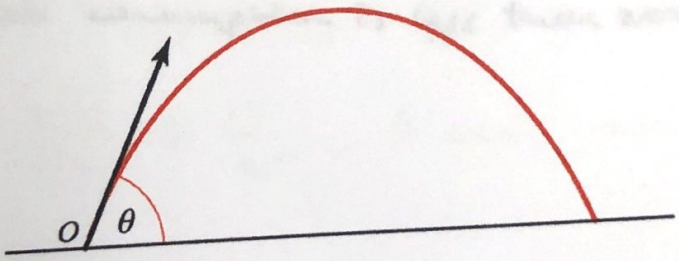


Figure 4

(a) Find the horizontal and vertical displacements of the particle at time t seconds. You should give your answer in terms of θ and t. **(4)**

(b) Show that the horizontal distance travelled by the particle before it hits the ground is $\frac{\sin 2\theta}{10}$. **(5)**

(c) Find the value θ for which the horizontal distance travelled is a maximum. **(2)**

(d) Describe one limitation of this model. **(1)**

(Total 12 marks)

Applied Test

(1) a) A linear association between e and f

b) It requires extrapolation and may be unreliable

c) Fuel consumption (f)

d) A statistical test that is used to determine whether there is enough evidence in a sample of data to infer that a certain condition is true for the entire population

e) $H_0: \rho = 0$ $H_1: \rho < 0$

critical value $= -0.3665$

since $-0.803 < -0.3665$, reject H_0

There is evidence that the PMCC for CO_2 emissions and fuel consumption is less than zero.

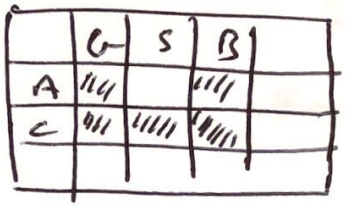
(2)

	G	S	B	
A	29	17	18	64
C	21	23	17	61
	50	40	35	125

a) $P(G) = \frac{50}{125} = \frac{2}{5}$

b) $P((A \cap S)') = 1 - \frac{17}{125}$

$= \frac{108}{125}$



c) $P(S) = \frac{40}{125}$ $P(A) = \frac{64}{125}$ $P(A \cap S) = \frac{17}{125}$

$P(S) \times P(A) = \frac{40}{125} \times \frac{64}{125} = \frac{512}{3125}$

since $\frac{512}{3125} \neq \frac{17}{125}$, A and S are not statistically independent

d) $P(B) = \frac{35}{125}$ $P(C) = \frac{61}{125}$ $P(B \cup C) = \frac{18+17+23+21}{125} = \frac{79}{125}$

$P(B) + P(C) = \frac{35}{125} + \frac{61}{125} = \frac{96}{125}$

since $\frac{96}{125} \neq \frac{79}{125}$, B and C are not mutually exclusive

e) $P(\text{Female and Gold}) = 0.3 \times \frac{50}{125} = \frac{15}{125}$

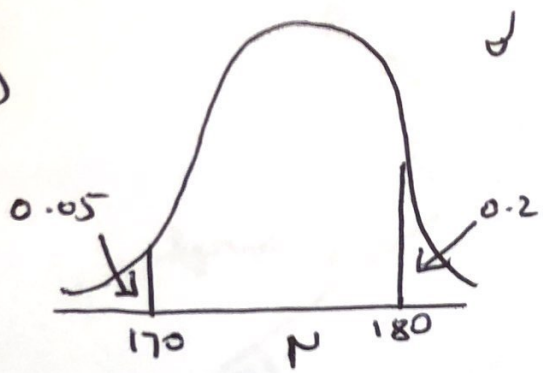
$P(\text{Female and Silver}) = 0.6 \times \frac{40}{125} = \frac{24}{125}$

$P(\text{Female and Bronze}) = 0.4 \times \frac{35}{125} = \frac{14}{125}$

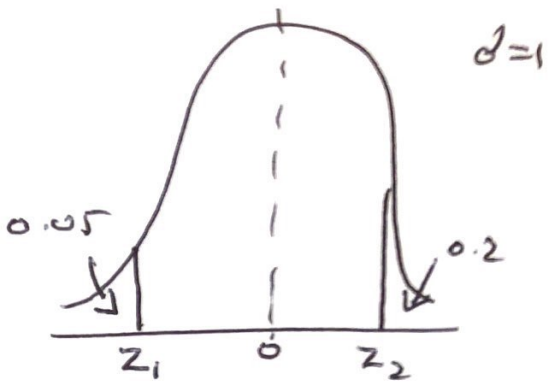
$\therefore P(\text{Female}) = \frac{15+24+14}{125} = \frac{53}{125}$

(3)

a)



$X \sim N(\mu, \sigma)$



$Z \sim N(0, 1)$

From table, $z_1 = -1.6449$

$z_2 = 0.8416$

b)

$$Z = \frac{X - \mu}{\sigma}$$

$\therefore -1.6449 = \frac{170 - \mu}{\sigma}$ and $0.8416 = \frac{180 - \mu}{\sigma}$

$\therefore \mu - 1.6449\sigma = 170$ and $\mu + 0.8416\sigma = 180$

$\therefore \mu = 176.6$ and $\sigma = 4.0217$ (calculator)

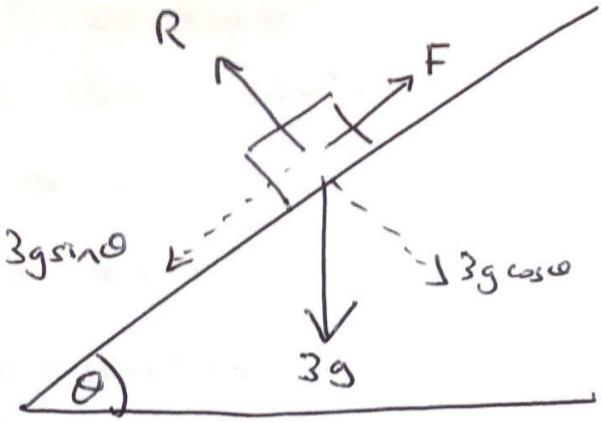
c) $P(\text{one man is taller than } 175) = 0.65606$

$\therefore P(\text{three men are taller than } 175) = 0.65606^3$

$= 0.282$ (3sf)

④

a)



3g = weight

R = Normal Reaction

F = Friction

b)

$R = 3g\cos\theta$

$F = 3g\sin\theta$

"$F = \mu R$" $\quad 3g\sin\theta = \mu 3g\cos\theta$

$\therefore \quad \sin\theta = \mu\cos\theta$

$\therefore \quad \mu = \tan\theta$

c) if $\mu = 0.3$ and $\tan\theta = 30°$

$F = 3g\tan 30$, $\quad R = 3g\cos 30$

object slips if $\quad 3g\sin 30 > F$

i.e. if $\quad 3g\sin 30 > 0.3 \times 3g\cos 30$

i.e. if $\quad \tan 30 > 0.3$

but $\quad \tan 30 = 0.5773$

$\therefore$ object slips

d) No object would remain in equilibrium, since the normal reaction would approach zero.

(5) a) $a = 180 - 360t$

$\therefore v = 180t - 180t^2 + c$

$t = 0, v = 0 \Rightarrow c = 0$

$\therefore v = 180t - 180t^2$

b) $180t - 180t^2 = 40$

$\therefore 9t^2 - 9t + 2 = 0$

$\therefore t = 2/3, 1/3$

when $t = 0$, $v = 0$ $\therefore$ car is breaking the speed limit when

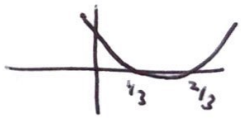
$\frac{1}{3} < t < \frac{2}{3}$

(OR

$180t - 180t^2 > 40$

$\therefore 180t^2 - 180t + 40 < 0$

$\therefore 9t^2 - 9t + 2 < 0$



$\frac{1}{3} < t < \frac{2}{3}$)

c) Average speed

$= \frac{\text{total distance}}{\text{total time}}$

$x = 90t^2 - 60t^3 + c_1$

$t = 0, x = 0 \Rightarrow c_1 = 0$

$\therefore x = 90t^2 - 60t^3$

$t = 1 \Rightarrow x = 30$

$\therefore$ total distance $= \frac{30}{1} = 30 \text{ km h}^{-1}$

(6)

	→	↑
s	x	y
u	$\cos\theta$	$\sin\theta$
v		
a	0	-10
t		

a)

→ $s = ut + \frac{1}{2}at^2 \Rightarrow x = t\cos\theta$

↑ $s = ut + \frac{1}{2}at^2 \Rightarrow y = t\sin\theta - 5t^2$

b) if $y = 0$, $t\sin\theta - 5t^2 = 0$

$\therefore t = 0$ or $t = \frac{1}{5}\sin\theta$

$\therefore x = \frac{1}{5}\sin\theta\cos\theta$

$= \frac{1}{10}2\sin\theta\cos\theta$

$= \frac{\sin 2\theta}{10}$

c) Max x occurs when $\frac{dx}{d\theta} = 0 \Rightarrow \frac{1}{5}\cos 2\theta = 0 \Rightarrow 2\theta = 90°, 270°, \ldots$

$\Rightarrow \theta = 45°$ ($\theta < 90$

d) Air resistance