SECTION A: STATISTICS

Answer ALL questions.

1. An engineer believes that there is a relationship between the CO_2 emissions and fuel consumption for cars.

A random sample of 40 different car models (old and new) was taken and the CO_2 emission figures, e grams per kilometre, and fuel consumption, f miles per gallon, were recorded, as shown in Figure 1. The engineer calculates the product moment correlation coefficient for the 40 cars and obtains $r = -0.803$.

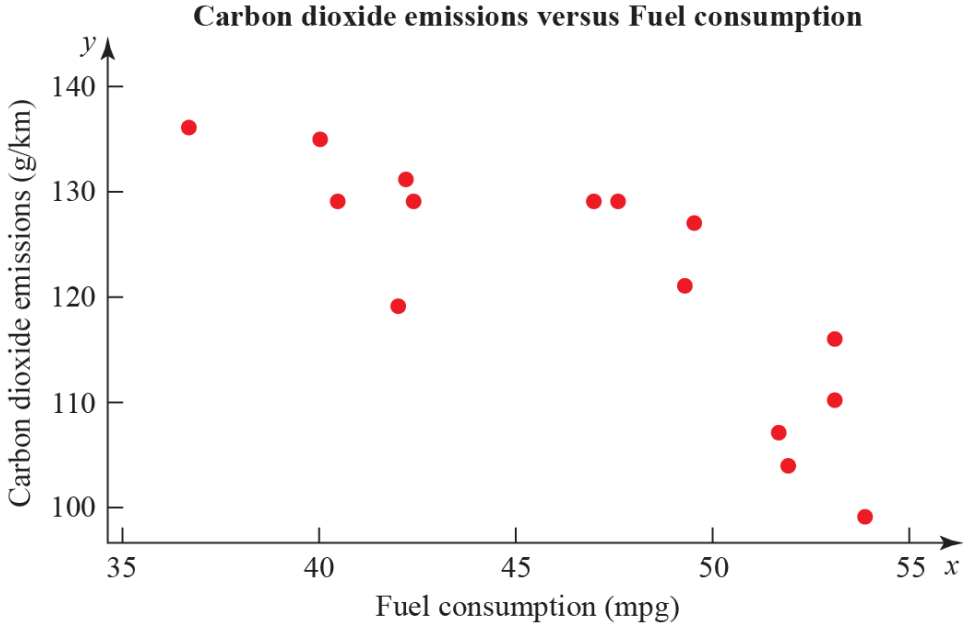


Figure 1

(a) State what is measured by the product moment correlation coefficient.

(1)

(b) State, with a reason, whether a linear regression model based on these data is reliable or not for a car when the fuel consumption is 60 mpg.

(1)

(c) For the linear regression model $e = 198 - 1.71 \times f$ write down the explanatory variable.

(1)

(d) State the definition of a hypothesis test.

(1)

(e) Test at 1% significance level whether or not the product moment correlation coefficient for CO_2 emissions and fuel consumption is less than zero. State your hypotheses clearly.

(3)

(Total 7 marks)

2. The table below shows the number of gold, silver and bronze medals won by two teams in an athletics competition.

	Gold	**Silver**	**Bronze**
Team *A*	29	17	18
Team *C*	21	23	17

The events G, S and B are that a medal is gold, silver or bronze respectively. Let A be the event that team A won a medal and C team C won a medal. A medal winner is selected at random. Find

(a) P(G),

(2)

(b) P($[A \cap S]'$).

(2)

(c) Explain, showing your working, whether or not events S and A are statistically independent. Give reasons for your answer.

(2)

(d) Determine whether or not events B and C are mutually exclusive. Give a reason for your answer.

(2)

(e) Given that 30% of the gold medal winners are female, 60% of the silver medal winners are female and 40% of the bronze medal winners are female, find the probability that a randomly selected medal winner is female.

(2)

(Total 10 marks)

3. The heights of a population of men are normally distributed with mean μ cm and standard deviation σ cm. It is known that 20% of the men are taller than 180 cm and 5% are shorter than 170 cm.

(a) Sketch a diagram to show the distribution of heights represented by this information.

(3)

(b) Find the value of μ and σ.

(7)

(c) Three men are selected at random, find the probability that they are all taller than 175 cm.

(2)

(Total 12 marks)

SECTION B: MECHANICS

Answer ALL questions.

4. Figure 5 shows an object of 3 kg sitting on a plane inclined at an angle θ to the horizontal. The coefficient of friction between the object and the plane is μ. The system is in limiting equilibrium.

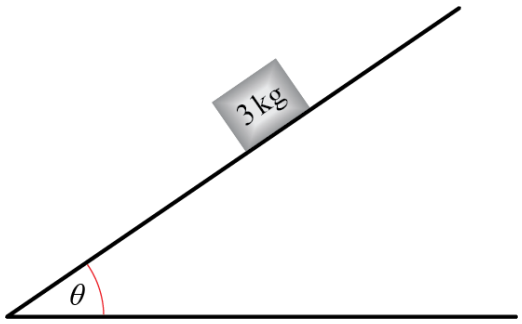


Figure 5

(a) Draw a diagram showing all the forces acting on the object. Describe the origin of each force using words.

(3)

(b) By resolving forces in two perpendicular directions, show that $\mu = \tan \theta$.

(6)

(c) Hence, determine whether or not the object slips if $\mu = 0.3$ and $\theta = 30°$.

(4)

(d) As θ approaches 90°, state whether an object of any mass could remain in equilibrium. Explain your answer.

(2)

(Total 15 marks)

5. A car travels along a long, straight road for one hour, starting from rest. After t hours, its acceleration is a km h^{-2}, where $a = 180 - 360t$.

(a) Find the speed of the car, in km h^{-1} in terms of t.

(2)

The speed limit is 40 km h^{-1}.

(b) Find the range of times during which the car is breaking the speed limit. Give your answer in minutes.

(4)

(c) Find the average speed of the car over the whole journey.

(5)

(Total 11 marks)

6. A ball is launched from the origin with speed 1 m s^{-1}. Its velocity vector makes an angle θ above the horizontal. It travels over flat ground and is modelled as a particle moving freely under gravity, as shown in Figure 4.

(In this question, take $g = 10$ m s^{-2}.)

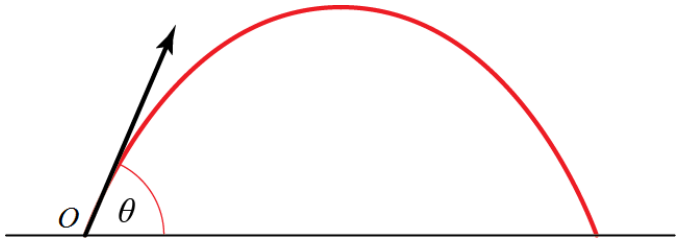


Figure 4

(a) Find the horizontal and vertical displacements of the particle at time t seconds. You should give your answer in terms of θ and t.

(4)

(b) Show that the horizontal distance travelled by the particle before it hits the ground is $\frac{\sin 2\theta}{10}$.

(5)

(c) Find the value θ for which the horizontal distance travelled is a maximum.

(2)

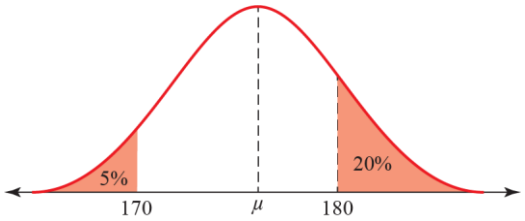
(d) Describe one limitation of this model.

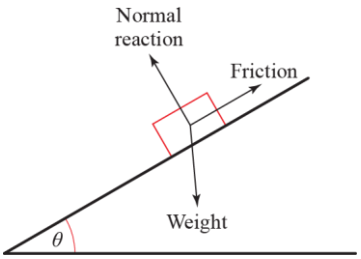
(1)

(Total 12 marks)

1	Scheme	Marks
a	Linear association between *e* and *f*.	**B1**
		(1)
b	It requires extropolation and hence it may be unreliable.	**B1**
		(1)
c	Fuel consumption (*f*)	**B1**
		(1)
d	A hypothesis test is a statistical test that is used to determine whether there is enough evidence in a sample of data to infer that a certain condition is true for the entire population.	**B1**
		(1)
e	$H_0 : \rho = 0$, $H_1 : \rho < 0$ Critical value = -0.3665 $-0.803 < -0.3665$ (test statistic in critical region) Reject H_0 There is evidence that the product moment correlation coeficient for CO_2 emissions and fuel consumption is less than zero.	**B1** **M1** **A1**
		(3)
		(7 marks)
Notes		

2	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	$\frac{29+21}{29+21+17+23+18+17}=\frac{50}{125}$	**M1**	1.1b	2nd Calculate probabilities from relative frequency tables and real data.
	$=0.4$	**A1**	1.1b	
		(2)		
b	$\frac{125-17}{125}=\frac{108}{125}$	**M1**	3.1a	4th Understand set notation.
	$=0.864$	**A1**	1.1b	
		(2)		
c	$\mathrm{P}(S\cap A)=\frac{17}{125}=0.136\neq \mathrm{P}(S)\times \mathrm{P}(A)=\frac{40}{125}\times\frac{64}{125}=0.163\ldots$	**M1**	2.1	4th Understand and use the definition of independence in probability calculations.
	So, S and A are not statistically independent.	**A1**	2.4	
		(2)		
d	B and C are not mutally exclusive	**B1**	2.2a	3rd Understand and use the definition of mutually exclusive in probability calculations.
	Being in team C does not exclude the possibility of winning a bronze medal	**B1**	2.4	
		(2)		
e	$\frac{15+24+14}{125}=\frac{53}{125}$	**M1**	3.1b	5th Calculate conditional probabilities using formulae.
	$=0.424$	**A1**	1.1b	
		(2)		
				(10 marks)

3	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	 bell shaped	**B1**	1.2	5th Understand the basic features of the normal distribution including parameters, shape and notation.
	170, 180 on axis	**B1**	1.1b	
	5% and 20%	**B1**	1.1b	
		(3)		
b	$P(X < 170) = 0.05$ $\frac{170-\mu}{\sigma} = -1.6449$ $\mu = 170 + 1.6449\sigma$ $P(X > 180) = 0.2$ $\mu = 180 - 0.8416\sigma$ Solving simultaneously gives: $\mu = 176.615\ldots$ (awrt 176.6) and $\sigma = 4.021\ldots$(awrt 4.02)	**M1** **B1** **B1** **B1** **M1** **A1** **A1**	3.3 3.4 1.1b 3.4 1.1b 1.1b 1.1b	7th Find unknown means and/or standard deviations for normal distributions.
		(7)		
c	P(All three are taller than 175 cm) = $0.656\ldots^3$	**M1**	1.1b	5th Understand informally the link to probability distributions.
	= 0.282… (using calculator) awrt 0.282	**A1**	1.1b	
		(2)		
				(12 marks)

4	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	 B1 for each correct force with correct label.	**B3**	2.5	3rd Draw force diagrams.
		(3)		
b	Resolve horizontally/vertically or along/perp to plane.	**M1**	1.1b	7th The concept of limiting equilibrium.
	$R = 3g\cos\theta$	**A1**	1.1b	
	$F = 3g\sin\theta$	**A1**	1.1b	
	Limiting equilibrium means $\mu R = F$ $\mu R = 3\mu g\cos\theta$	**A1**	1.1b	
	$3\mu g\cos\theta = 3g\sin\theta$	**M1**	1.1b	
	$\mu = \tan\theta$	**A1**	1.1b	
		(6)		
c	$\tan 30 = 0.577\ldots$	**A1**	3.1a	7th The concept of limiting equilibrium.
	For limiting equilibrium, $\mu = 0.577\ldots$	**M1**	3.1a	
	But $\mu = 0.3$ so less friction.	**M1**	3.1a	
	Hence the object slips.	**A1**	3.2a	
		(4)		
d	No object would remain in equilibrium, because normal reaction becomes zero.	**B1** **A1**	3.2a	7th The concept of limiting equilibrium.
		(2)		
				(15 marks)

5	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Integrate a w.r.t. t	**M1**	1.1a	5th Use integration to determine functions for velocity and/or displacement.
	$a = 180t - 180t^2$	**A1**	1.1b	
		(2)		
b	$180t - 180t^2 > 40$	**M1**	3.1a	7th Solve general kinematics problems in less familiar contexts.
	$20(3t-2)(3t-1) < 0$	**A1**	1.1b	
	$\frac{1}{3} < t < \frac{2}{3}$	**A1**	2.4	
	Breaking the speed limit between 20 and 40 minutes.	**A1**	3.2a	
		(4)		
c	Integrate v w.r.t. t	**M1**	1.1a	5th Use integration to determine functions for velocity and/or displacement.
	$x = 90t^2 - 60t^3\ (+C)$	**A1**	1.1b	
	When $t = 1$, $x = 30$	**A1**	3.1b	
	Average speed $= \frac{\text{distance}}{\text{time}}$	**M1**	1.1b	
	$30\ \text{km h}^{-1}$	**A1**	1.1b	
		(5)		
				(11 marks)

6	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Use of $s = ut + \frac{1}{2}at^2$	**M1**	1.1a	6th Resolve velocity into horizontal and vertical components.
	Initial velocity is $(\cos\theta, \sin\theta)$	**A1**	3.4	
	$x = t\cos\theta$	**A1**	1.1b	
	$y = t\sin\theta - 5t^2$	**B1**	1.1b	
		(4)		
b	Solve $y = 0$ for t	**M1**	3.4	5th Model horizontal projection under gravity.
	$t(\sin\theta - 5t) = 0$	**A1**	1.1b	
	$t = 0$ or $t = \frac{\sin\theta}{5}$	**A1**	1.1b	
	$t = 0$ is initial position so $t = \frac{\sin\theta}{5}$	**M1**	2.4	
	$x = \frac{\cos\theta\sin\theta}{5} = \frac{2\sin\theta\cos\theta}{10} = \frac{\sin 2\theta}{10}$	**A1**	1.1b	
		(5)		
c	Sketch of $\sin 2\theta$ or other legitimate method.	**M1**	2.2a	6th Resolve velocity into horizontal and vertical components.
	Maximum is at $\theta = 45°$	**A1**	2.4	
		(2)		
d	Correct limitation. For example, air resistance.	**B1**	3.5b	3rd Understand assumptions common in mathematical modelling.
		(1)		
				(12 marks)