## SECTION A: STATISTICS

## Answer ALL questions.

1. An engineer believes that there is a relationship between the $\mathrm{CO}_{2}$ emissions and fuel consumption for cars.

A random sample of 40 different car models (old and new) was taken and the $\mathrm{CO}_{2}$ emission figures, $e$ grams per kilometre, and fuel consumption, $f$ miles per gallon, were recorded, as shown in Figure 1. The engineer calculates the product moment correlation coefficient for the 40 cars and obtains $r=-0.803$.


Figure 1
(a) State what is measured by the product moment correlation coefficient.
(b) State, with a reason, whether a linear regression model based on these data is reliable or not for a car when the fuel consumption is 60 mpg .
(c) For the linear regression model $e=198-1.71 \times f$ write down the explanatory variable.
(d) State the definition of a hypothesis test.
(e) Test at $1 \%$ significance level whether or not the product moment correlation coefficient for $\mathrm{CO}_{2}$ emissions and fuel consumption is less than zero. State your hypotheses clearly.
2. The table below shows the number of gold, silver and bronze medals won by two teams in an athletics competition.

|  | Gold | Silver | Bronze |
| :--- | :---: | :---: | :---: |
| Team $\boldsymbol{A}$ | 29 | 17 | 18 |
| Team $\boldsymbol{C}$ | 21 | 23 | 17 |

The events $G, S$ and $B$ are that a medal is gold, silver or bronze respectively. Let $A$ be the event that team A won a medal and $C$ team C won a medal. A medal winner is selected at random. Find
(a) $\mathrm{P}(G)$,
(b) $\mathrm{P}\left([A \cap S]^{\prime}\right)$.
(c) Explain, showing your working, whether or not events $S$ and $A$ are statistically independent. Give reasons for your answer.
(d) Determine whether or not events $B$ and $C$ are mutually exclusive. Give a reason for your answer.
(e) Given that $30 \%$ of the gold medal winners are female, $60 \%$ of the silver medal winners are female and $40 \%$ of the bronze medal winners are female, find the probability that a randomly selected medal winner is female.
(Total 10 marks)
3. The heights of a population of men are normally distributed with mean $\mu \mathrm{cm}$ and standard deviation $\sigma \mathrm{cm}$. It is known that $20 \%$ of the men are taller than 180 cm and $5 \%$ are shorter than 170 cm .
(a) Sketch a diagram to show the distribution of heights represented by this information.
(b) Find the value of $\mu$ and $\sigma$.
(c) Three men are selected at random, find the probability that they are all taller than 175 cm .

## SECTION B: MECHANICS

## Answer ALL questions.

4. Figure 5 shows an object of 3 kg sitting on a plane inclined at an angle $\theta$ to the horizontal. The coefficient of friction between the object and the plane is $\mu$. The system is in limiting equilibrium.


Figure 5
(a) Draw a diagram showing all the forces acting on the object. Describe the origin of each force using words.
(b) By resolving forces in two perpendicular directions, show that $\mu=\tan \theta$.
(c) Hence, determine whether or not the object slips if $\mu=0.3$ and $\theta=30^{\circ}$.
(d) As $\theta$ approaches $90^{\circ}$, state whether an object of any mass could remain in equilibrium. Explain your answer.
5. A car travels along a long, straight road for one hour, starting from rest. After $t$ hours, its acceleration is $a \mathrm{~km} \mathrm{~h}^{-2}$, where $a=180-360 t$.
(a) Find the speed of the car, in $\mathrm{km} \mathrm{h}^{-1}$ in terms of $t$.

The speed limit is $40 \mathrm{~km} \mathrm{~h}^{-1}$.
(b) Find the range of times during which the car is breaking the speed limit. Give your answer in minutes.
(c) Find the average speed of the car over the whole journey.
(Total 11 marks)
6. A ball is launched from the origin with speed $1 \mathrm{~m} \mathrm{~s}^{-1}$. Its velocity vector makes an angle $\theta$ above the horizontal. It travels over flat ground and is modelled as a particle moving freely under gravity, as shown in Figure 4.
(In this question, take $g=10 \mathrm{~m} \mathrm{~s}^{-2}$.)


Figure 4
(a) Find the horizontal and vertical displacements of the particle at time $t$ seconds. You should give your answer in terms of $\theta$ and $t$.
(b) Show that the horizontal distance travelled by the particle before it hits the ground is $\frac{\sin 2 \theta}{10}$.
(c) Find the value $\theta$ for which the horizontal distance travelled is a maximum.
(d) Describe one limitation of this model.

