4

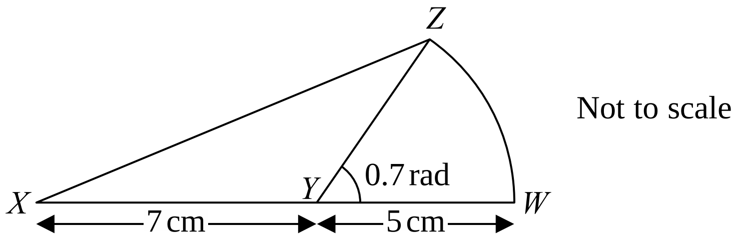

Figure 2

Figure 2 shows a flag *XYWZX*.

The flag consists of a triangle *XYZ* joined to a sector *ZYW* of a circle with radius 5 cm and centre *Y*.

The angle of the sector, angle *ZYW*, is 0.7 radians.

The points *X*, *Y* and *W* lie on a straight line with *XY* = 7 cm and *YW* = 5 cm.

Find

(*a*) the area of the sector *ZYW* in cm^2,

(2)

(*b*) the area of the flag, in cm^2, to 2 decimal places,

(3)

(*c*) the length of the perimeter, *XYWZX*, of the flag, in cm to 2 decimal places.

(4)

(Total 9 marks)

---

5 The circle *C* has equation

$$x^2 + y^2 - 2x + 14y = 0$$

Find

(*a*) the coordinates of the centre of *C*,

(2)

(*b*) the exact value of the radius of *C*,

(2)

(*c*) the *y* coordinates of the points where the circle *C* crosses the *y*-axis.

(2)

(*d*) Find an equation of the tangent to *C* at the point (2, 0), giving your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

(4)

(Total 10 marks)

---

6 A geometric series with common ratio $r = -0.9$ has sum to infinity 10 000.

For this series,

(*a*) find the first term,

(2)

(*b*) find the fifth term,

(2)

(*c*) find the sum of the first twelve terms, giving this answer to the nearest integer.

(3)

(Total 7 marks)

---

7 (i) Find the value of y for which

$$1.01^{y-1} = 500$$

Give your answer to 2 decimal places.

(2)

(ii) Given that

$$2 \log_4 (3x + 5) = \log_4 (3x + 8) + 1, \qquad x > -\frac{5}{3}$$

(*a*) show that

$$9x^2 + 18x - 7 = 0$$

(4)

(*b*) Hence solve the equation

$$2 \log_4 (3x + 5) = \log_4 (3x + 8) + 1, \qquad x > -\frac{5}{3}$$

(2)

(Total 8 marks)

---

8 *In this question solutions based entirely on graphical or numerical methods are not acceptable.*

(i) Solve for $0 \leqslant x < 360°$,

$$4 \cos (x + 70°) = 3$$

giving your answers in degrees to one decimal place.

(4)

(ii) Find, for $0 \leqslant \theta < 2\pi$, all the solutions of

$$6 \cos^2 \theta - 5 = 6 \sin^2 \theta + \sin \theta$$

giving your answers in radians to 3 significant figures.

(5)

(Total 9 marks)

9

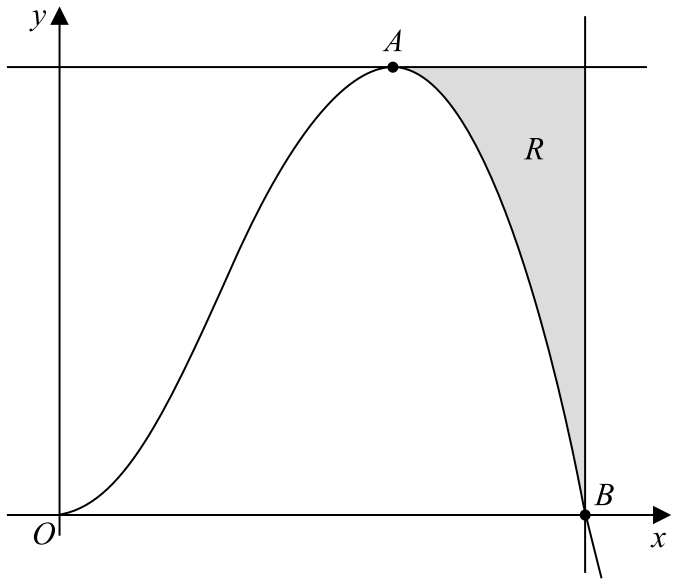

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 7x^2 (5 - 2\sqrt{x}), \qquad x \geqslant 0$$

The curve has a turning point at the point A, where $x > 0$, as shown in Figure 3.

(a) Using calculus, find the coordinates of the point A.

(5)

The curve crosses the x-axis at the point B, as shown in Figure 3.

(b) Use algebra to find the x coordinate of the point B.

(2)

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line through A parallel to the x-axis and the line through B parallel to the y-axis.

(c) Use integration to find the area of the region R, giving your answer to 2 decimal places.

(5)

(Total 12 marks)

1 Given $y = 2x(3x - 1)^5$,

(a) find $\frac{dy}{dx}$, giving your answer as a single fully factorised expression.

(4)

(b) Hence find the set of values of x for which $\frac{dy}{dx} \leqslant 0$

(2)

(Total 6 marks)

---

2 The function f is defined by

$$f(x) = \frac{6}{2x+5} + \frac{2}{2x-5} + \frac{60}{4x^2-25}, \qquad x > 4$$

(*a*) Show that $f(x) = \dfrac{A}{Bx+C}$ where A, B and C are constants to be found.

(4)

(*b*) Find $f^{-1}(x)$ and state its domain.

(3)

(Total 7 marks)

---

3 The value of a car is modelled by the formula

$$V = 16\,000e^{-kt} + A, \qquad t \geqslant 0, t \in \mathbb{R}$$

where V is the value of the car in pounds, t is the age of the car in years, and k and A are positive constants.

Given that the value of the car is £17 500 when new and £13 500 two years later,

(*a*) find the value of A,

(1)

(*b*) show that $k = \ln\left(\dfrac{2}{\sqrt{3}}\right)$

(4)

(*c*) Find the age of the car, in years, when the value of the car is £6000.

Give your answer to 2 decimal places.

(4)

(Total 9 marks)

4

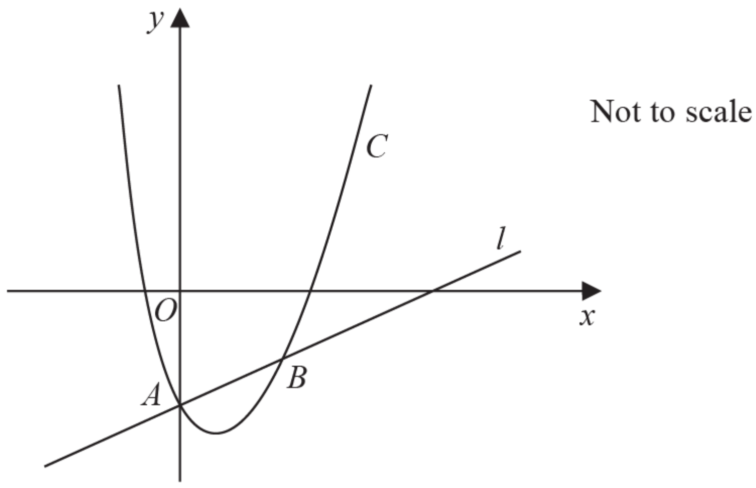

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = e^{-2x} + x^2 - 3$$

The curve C crosses the y-axis at the point A.

The line l is the normal to C at the point A.

(a) Find the equation of l, writing your answer in the form $y = mx + c$, where m and c are constants.

(5)

The line l meets C again at the point B, as shown in Figure 1.

(b) Show that the x coordinate of B is a solution of

$$x = \sqrt{1 + \frac{1}{2}x - e^{-2x}}$$

(2)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{1}{2}x_n - e^{-2x_n}}$$

with $x_1 = 1$

(c) find x_2 and x_3 to 3 decimal places.

(2)

(Total 9 marks)

5

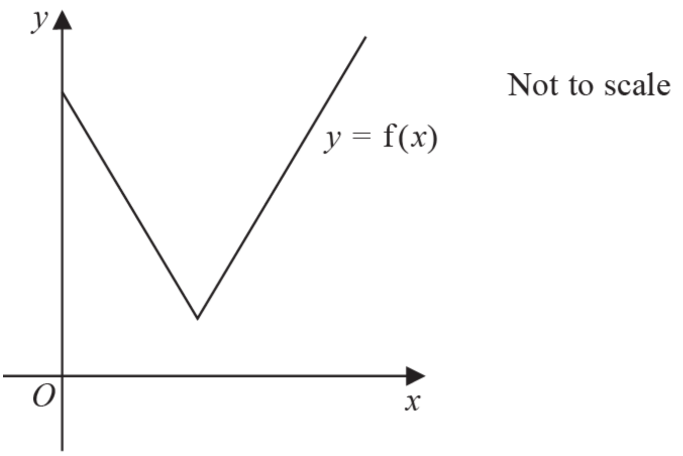

Figure 2

Figure 2 shows part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|5 - x| + 3, \qquad x \geqslant 0$$

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has exactly one root,

(*a*) state the set of possible values of k.

(2)

(*b*) Solve the equation $\mathrm{f}(x) = \frac{1}{2}x + 10$

(4)

The graph with equation $y = \mathrm{f}(x)$ is transformed onto the graph with equation $y = 4\mathrm{f}(x - 1)$.
The vertex on the graph with equation $y = 4\mathrm{f}(x - 1)$ has coordinates (p, q).

(*c*) State the value of p and the value of q.

(2)

(Total 8 marks)

6 (i) Using the identity for tan ($A \pm B$), solve, for $-90° < x < 90°$,

$$\frac{\tan 2x + \tan 32°}{1 - \tan 2x \tan 32°} = 5$$

Give your answers, in degrees, to 2 decimal places.

(4)

(ii) (*a*) Using the identity for tan ($A \pm B$), show that

$$\tan(3\theta - 45°) \equiv \frac{\tan 3\theta - 1}{1 + \tan 3\theta}, \qquad \theta \neq (60n + 45)°, \ n \in \mathbb{Z}$$

(2)

(*b*) Hence solve, for $0 < \theta < 180°$,

$$(1 + \tan 3\theta)\tan(\theta + 28°) = \tan 3\theta - 1$$

(5)

(Total 11 marks)