1 (i) Simplify

$$\sqrt{48} - \frac{6}{\sqrt{3}}$$

Write your answer in the form $a\sqrt{3}$, where a is an integer to be found.

(2)

(ii) Solve the equation

$$3^{6x-3} = 81$$

Write your answer as a rational number.

(3)

(Total 5 marks)

---

2 Given

$$y = 3\sqrt{x} - 6x + 4, \qquad x > 0$$

(*a*) find $\int y \, dx$, simplifying each term.

(3)

(*b*) (i) Find $\frac{dy}{dx}$

(ii) Hence find the value of x such that $\frac{dy}{dx} = 0$

(4)

(Total 7 marks)

---

3 $$\mathrm{f}(x) = x^2 - 10x + 23$$

(*a*) Express $\mathrm{f}(x)$ in the form $(x + a)^2 + b$, where a and b are constants to be found.

(2)

(*b*) Hence, or otherwise, find the exact solutions to the equation

$$x^2 - 10x + 23 = 0$$

(2)

(*c*) Use your answer to part (*b*) to find the larger solution to the equation

$$y - 10y^{0.5} + 23 = 0$$

Write your solution in the form $p + q\sqrt{r}$, where p, q and r are integers.

(2)

(Total 6 marks)

---

4 Each year, Andy pays into a savings scheme. In year one he pays in £600. His payments increase by £120 each year so that he pays £720 in year two, £840 in year three and so on, so that his payments form an arithmetic sequence.

(*a*) Find out how much Andy pays into the savings scheme in year ten.

(2)

Kim starts paying money into a different savings scheme at the same time as Andy. In year one she pays in £130. Her payments increase each year so that she pays £210 in year two, £290 in year three and so on, so that her payments form a different arithmetic sequence.

At the end of year N, Andy has paid, in total, twice as much money into his savings scheme as Kim has paid, in total, into her savings scheme.

(*b*) Find the value of N.

(5)

(Total 7 marks)

5

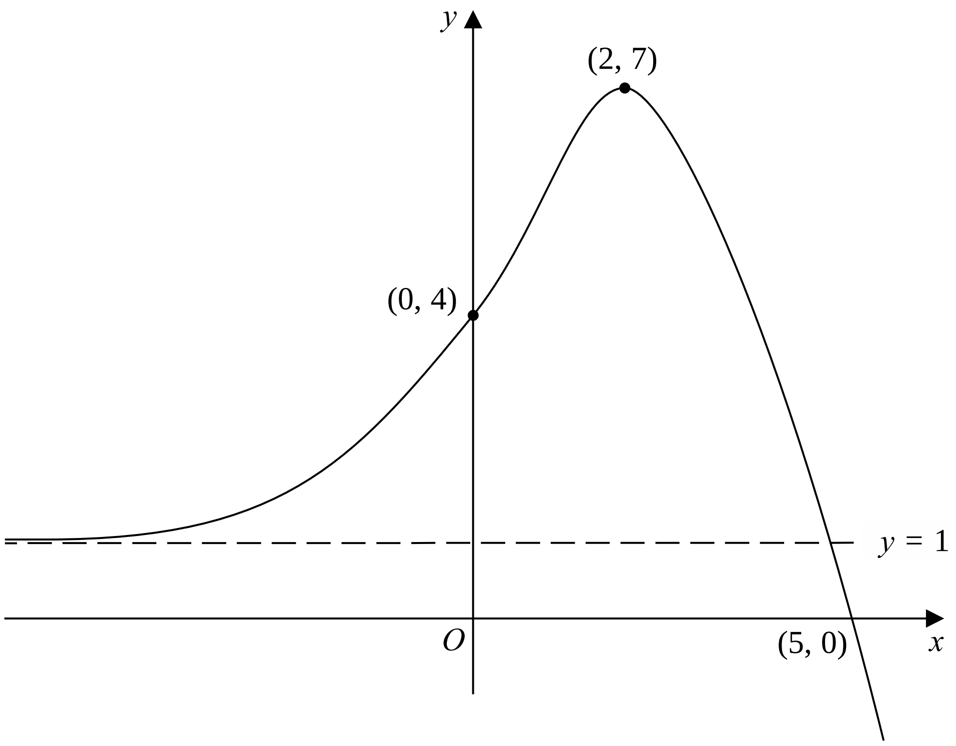


Figure 1

Figure 1 shows the sketch of a curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve crosses the y-axis at (0, 4) and crosses the x-axis at (5, 0).

The curve has a single turning point, a maximum, at (2, 7).

The line with equation $y = 1$ is the only asymptote to the curve.

(*a*) State the coordinates of the turning point on the curve with equation $y = \mathrm{f}(x - 2)$.

(1)

(*b*) State the solution of the equation $\mathrm{f}(2x) = 0$

(1)

(*c*) State the equation of the asymptote to the curve with equation $y = \mathrm{f}(-x)$.

(1)

Given that the line with equation $y = k$, where k is a constant, meets the curve $y = \mathrm{f}(x)$ at only one point,

(*d*) state the set of possible values for k.

(2)

(Total 5 marks)

6 A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4$$

$$a_{n+1} = \frac{a_n}{a_n + 1}, \qquad n \geqslant 1, n \in \mathbb{N}$$

(*a*) Find the values of a_2, a_3 and a_4

Write your answers as simplified fractions.

(3)

Given that

$$a_n = \frac{4}{pn + q}, \text{ where } p \text{ and } q \text{ are constants}$$

(*b*) state the value of p and the value of q.

(2)

(*c*) Hence calculate the value of N such that $a_N = \dfrac{4}{321}$

(2)

(Total 7 marks)

---

7 The equation $20x^2 = 4kx - 13kx^2 + 2$, where k is a constant, has no real roots.

(*a*) Show that k satisfies the inequality

$$2k^2 + 13k + 20 < 0$$

(4)

(*b*) Find the set of possible values for k.

(4)

(Total 8 marks)

8

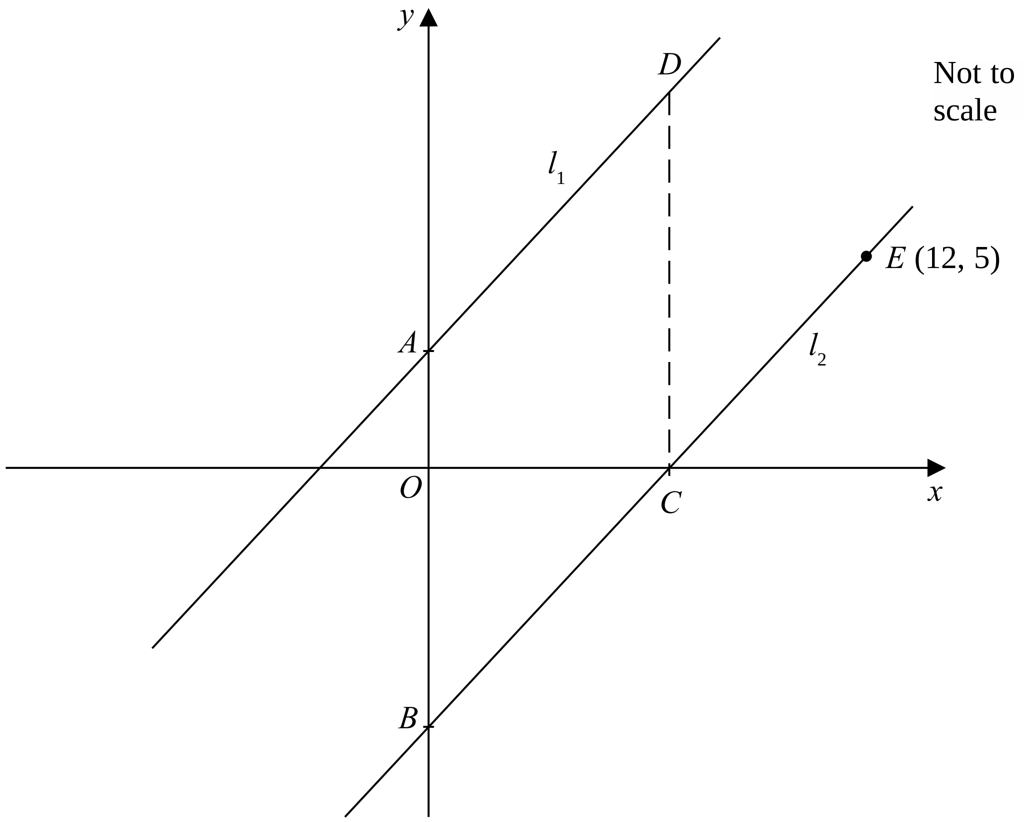


Figure 2

Figure 2 shows the straight line l_1 with equation $4y = 5x + 12$

(*a*) State the gradient of l_1

(1)

The line l_2 is parallel to l_1 and passes through the point E (12, 5), as shown in Figure 2.

(*b*) Find the equation of l_2. Write your answer in the form $y = m\,x + c$, where m and c are constants to be determined.

(3)

The line l_2 cuts the x-axis at the point C and the y-axis at the point B.

(*c*) Find the coordinates of

(i) the point B,

(ii) the point C.

(2)

The line l_1 cuts the y-axis at the point A.

The point D lies on l_1 such that $ABCD$ is a parallelogram, as shown in Figure 2.

(*d*) Find the area of $ABCD$.

(2)

(Total 8 marks)

9 The curve C has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}'(x) = (x - 3)(3x + 5)$$

Given that the point $P\ (1, 20)$ lies on C,

(*a*) find $\mathrm{f}(x)$, simplifying each term.

(5)

(*b*) Show that

$$\mathrm{f}(x) = (x - 3)^2(x + A)$$

where A is a constant to be found.

(3)

(*c*) Sketch the graph of C. Show clearly the coordinates of the points where C cuts or meets the x-axis and where C cuts the y-axis.

(4)

(Total 12 marks)

10

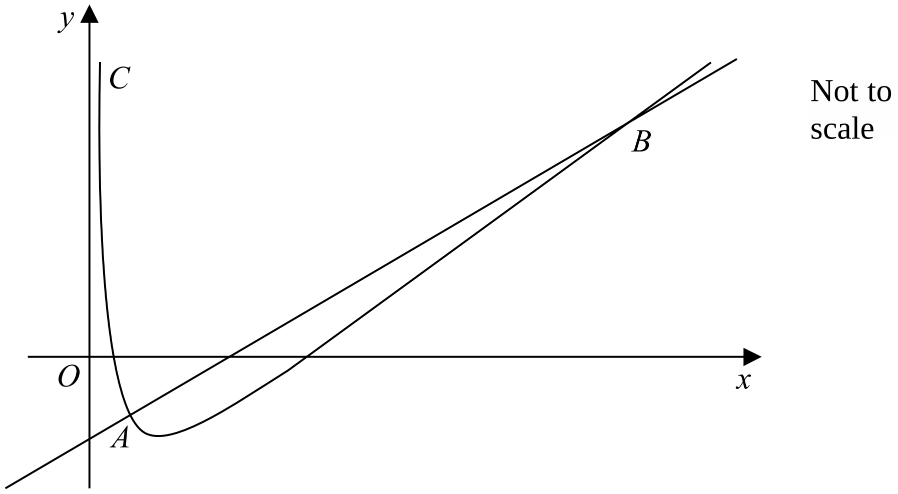


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{1}{2}x + \frac{27}{x} - 12, \quad x > 0$$

The point A lies on C and has coordinates $\left(3, -\frac{3}{2}\right)$.

(a) Show that the equation of the normal to C at A can be written as $10y = 4x - 27$

(5)

The normal to C at A meets C again at the point B, as shown in Figure 3.

(b) Use algebra to find the coordinates of B.

(5)

(Total 10 marks)

1

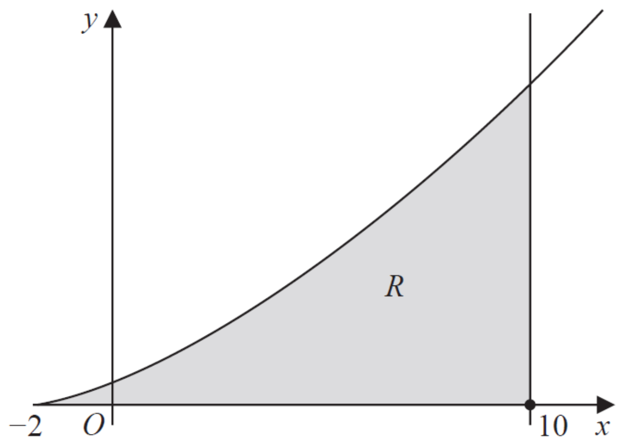


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \frac{(x+2)^{\frac{3}{2}}}{4}, \qquad x \geqslant -2$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line with equation $x = 10$

The table below shows corresponding values of x and y for $y = \frac{(x+2)^{\frac{3}{2}}}{4}$

(*a*) Complete the table, giving values of y corresponding to $x = 2$ and $x = 6$

x	-2	2	6	10
y	0			$6\sqrt{3}$

(1)

(*b*) Use the trapezium rule, with all the values of y from the completed table, to find an approximate value for the area of R, giving your answer to 3 decimal places.

(4)

(Total 5 marks)

2 (*a*) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(2 + kx)^7$$

where k is a non-zero constant. Give each term in its simplest form.

(4)

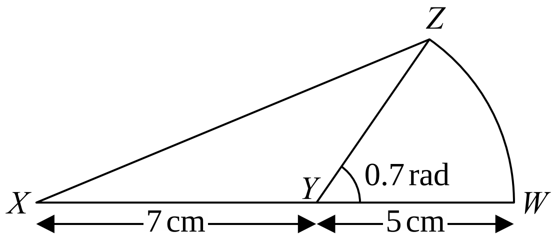
Given that the coefficient of x^3 in this expansion is 1890

(*b*) find the value of k.

(3)

(Total 7 marks)

4



Not to scale

Figure 2

Figure 2 shows a flag *XYWZX*.

The flag consists of a triangle *XYZ* joined to a sector *ZYW* of a circle with radius 5 cm and centre *Y*.

The angle of the sector, angle *ZYW*, is 0.7 radians.

The points *X*, *Y* and *W* lie on a straight line with $XY = 7$ cm and $YW = 5$ cm.

Find

(*a*) the area of the sector *ZYW* in cm^2,

(2)

(*b*) the area of the flag, in cm^2, to 2 decimal places,

(3)

(*c*) the length of the perimeter, *XYWZX*, of the flag, in cm to 2 decimal places.

(4)

(Total 9 marks)