Question Number	Scheme		Marks
1.(i) **Way 1**	$\sqrt{48}=\sqrt{16}\sqrt{3}$ or $\frac{6}{\sqrt{3}}=6\frac{\sqrt{3}}{3}$	Writes one of the terms of the given expression correctly in terms of $\sqrt{3}$	M1
	$\Rightarrow \sqrt{48}-\frac{6}{\sqrt{3}}=2\sqrt{3}$	A correct answer of $2\sqrt{3}$. A correct answer with **no** working implies both marks.	A1
			(2)
(i) **Way 2**	$\sqrt{48}=2\sqrt{12}$ or $\frac{6}{\sqrt{3}}=\sqrt{12}$	Writes one of the terms of the given expression correctly in terms of $\sqrt{12}$	M1
	$2\sqrt{12}-\sqrt{12}=\sqrt{12}=2\sqrt{3}$	A correct answer of $2\sqrt{3}$. A correct answer with **no** working implies both marks.	A1
			(2)
(i) **Way 3**	$\sqrt{48}=\frac{12}{\sqrt{3}}$ or $\sqrt{48}=\frac{\sqrt{144}}{\sqrt{3}}$	Writes $\sqrt{48}$ correctly as $\frac{12}{\sqrt{3}}$ or $\frac{\sqrt{144}}{\sqrt{3}}$	M1
	$\frac{12}{\sqrt{3}}-\frac{6}{\sqrt{3}}=\frac{6}{\sqrt{3}}=2\sqrt{3}$	A correct answer of $2\sqrt{3}$. A correct answer with **no** working implies both marks.	A1
			(2)
(i) **Way 4**	$\sqrt{48}-\frac{6}{\sqrt{3}}=\frac{\sqrt{3}\sqrt{48}-\ldots}{\sqrt{3}}=\frac{12-\ldots}{\sqrt{3}}$ or $\sqrt{48}-\frac{6}{\sqrt{3}}=\frac{\sqrt{3}\sqrt{48}-\ldots}{\sqrt{3}}=\frac{\sqrt{144}-\ldots}{\sqrt{3}}$	Writes $\sqrt{48}$ correctly as $\frac{12}{\sqrt{3}}$ or $\frac{\sqrt{144}}{\sqrt{3}}$	M1
	$\frac{12-6}{\sqrt{3}}=\frac{6}{\sqrt{3}}=2\sqrt{3}$	A correct answer of $2\sqrt{3}$. A correct answer with **no** working implies both marks.	A1
			(2)

(ii) **Way 1**	$81 = 3^4$ or $\log_3 81 = 6x - 3$	For $81 = 3^4$ or $\log_3 81 = 6x - 3$. This may be implied by subsequent work.	B1
	$3^{6x-3} = 3^4$ or $\log_3 81 = 6x - 3$ $\Rightarrow 4 = 6x - 3 \Rightarrow x = ...$	Solves an equation of the form 6*x* – 3 = *k* where *k* is their power of 3.	M1
	$\Rightarrow x = \frac{4+3}{6} = \frac{7}{6}$	$\frac{7}{6}$ or $1\frac{1}{6}$ or $1.1\dot{6}$ with a dot over the 6	A1
			(3)
Way 2	$3 = 81^{\frac{1}{4}}$	For $3 = 81^{\frac{1}{4}}$. This may be implied by subsequent work.	B1
	$81^{\frac{6x-3}{4}} = 81 \Rightarrow \frac{6x-3}{4} = 1 \Rightarrow x = ...$	Solves an equation of the form *k*(6*x* – 3) = 1 where *k* is their power of 81.	M1
	$\Rightarrow x = \frac{4+3}{6} = \frac{7}{6}$	$\frac{7}{6}$ or $1\frac{1}{6}$ or $1.1\dot{6}$ with a dot over the 6	A1

			(3)
Way 3	$81 = 9^2$ **and** $3 = 9^{\frac{1}{2}}$	For $81 = 9^2$ **and** $3 = 9^{\frac{1}{2}}$. This may be implied by subsequent work.	B1
	$9^{\frac{6x-3}{2}} = 9^2 \Rightarrow \frac{6x-3}{2} = 2 \Rightarrow x = ...$	Solves an equation of the form *p*(6*x* – 3) = *q* where *p* is their power of 9 for the 3 and *q* is their power of 9 for the 81.	M1
	$\Rightarrow x = \frac{4+3}{6} = \frac{7}{6}$	$\frac{7}{6}$ or $1\frac{1}{6}$ or $1.1\dot{6}$ with a dot over the 6	A1
			(3)
Way 4	$3^{6x-3} = 3^{6x} \times 3^{-3}$	For writing 3^{6x-3} correctly in terms of 3^{6x}	B1
	$3^{6x} = 81 \times 3^3 = 3^7$ $\Rightarrow 6x = 7 \Rightarrow x = ...$	Solves an equation of the form 6*x* = *k* where *k* is their $3^3 \times 81$ written as a power of 3.	M1
	$\Rightarrow x = \frac{7}{6}$	$\frac{7}{6}$ or $1\frac{1}{6}$ or $1.1\dot{6}$ with a dot over the 6	A1
			(3)
Way 5	$\log 3^{6x-3} = \log 81$	Takes logs of both sides	B1
	$6x - 3 = \frac{\log 81}{\log 3}$ $6x - 3 = 4 \Rightarrow x = ...$	Solves an equation of the form 6*x* – 3 = *k* where *k* is their $\frac{\log 81}{\log 3}$	M1
	$\Rightarrow x = \frac{7}{6}$	$\frac{7}{6}$ or $1\frac{1}{6}$ or $1.1\dot{6}$ with a dot over the 6	A1
			(3)
			(5 marks)

Note:

The question does not specify the form of the final answer in (b) and so if answers are left un-simplified as e.g. $\frac{\log_3 81 + 3}{6}$, $\frac{\log_3 2187}{6}$ **then allow full marks if correct.**

Question Number	Scheme		Marks
2.(a)	$2x^{1.5}-3x^2+4x+c$	M1: For $x^n \to x^{n+1}$ i.e. $x^{1.5}$ or x^2 or x seen (not for "+ *c*")	M1A1A1
		A1: For two out of three terms correct un-simplified or simplified (Ignore + *c* for this mark)	
		A1: cao $2x^{1.5}-3x^2+4x+c$. All correct and simplified and on one line including "+ c". Allow $\sqrt{x^3}$ for $x^{1.5}$ but not x^1 for *x*.	
	Ignore any spurious integral signs.		
			(3)
(b)(i)	**Mark (b)(i) and (ii) together and must be differentiating the original function not their answer to part (a)**		
	$\frac{3}{2}x^{-0.5}-6$	M1: For $x^n \to x^{n-1}$ i.e. $x^{0.5} \to x^{-0.5}$ or $6x \to 6$	M1A1
		A1: For $\frac{3}{2}x^{-0.5}-6$ or equivalent. May be un-simplified. Allow $\frac{3/2}{\sqrt{x}}-6$.	
			(2)
(ii)	$\frac{3}{2}x^{-0.5}-6=0 \Rightarrow x^n=\ldots$	Sets their $\frac{dy}{dx}=0$ (may be implied by their working) and reaches $x^n = C$ (including n = 1) **with correct processing allowing sign errors only** – this may be implied by e.g. $\sqrt{x}=\frac{1}{4}$ or $\frac{1}{\sqrt{x}}=4$.	M1
	$x=\frac{1}{16}$ cso	Allow equivalent fractions e.g. $\frac{9}{144}$ or 0.0625. If other solutions are given (e.g. likely to be x = 0 or	A1

Question Number	Scheme		Marks
		$x = -1/16$) then this mark should be withheld.	
			(2)
			(7 marks)

Question Number	Scheme		Marks
3.(a)	$x^2-10x+23=(x\pm5)^2\pm A$	For an attempt to complete the square. Note that if their *A* = 23 then this is M0 by the General Principles.	M1
	$(x-5)^2-2$	Correct expression. Ignore "= 0".	A1
			(2)
(b)	$(x\pm5)^2-A\Rightarrow x=\ldots$ or $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\Rightarrow x=\ldots$ $\left(x=\frac{10\pm\sqrt{10^2-4(1)(23)}}{2}\right)$	Uses their completion of the square for **positive** *A* or uses the correct quadratic formula to obtain at least one value for *x*	M1
	$x=5\pm\sqrt{2}$	Correct exact values. If using the quadratic formula must reach as far as $\frac{10\pm\sqrt{8}}{2}$	A1
			(2)
(c)	$(5\pm\sqrt{2})^2=27+10\sqrt{2}$	Attempts to square **any** solution from part (b). Allow poor squaring e.g. $(5+\sqrt{2})^2=25+2=27$. **Do not allow** for substituting e.g. $5+\sqrt{2}$ into $x^2-10x+23$.	M1
	$=27+10\sqrt{2}$	Accept equivalent forms such as $27+\sqrt{200}$. If any extra answers are given, this mark should be withheld.	A1
			(2)
	Allow candidates to start again:		
	$y-10y^{0.5}+23=0\Rightarrow y^{0.5}=\frac{10\pm\sqrt{10^2-4\times23}}{2}=5\pm\sqrt{2}$ $y=(5\pm\sqrt{2})^2=\ldots$		M1

Question Number	Scheme		Marks
	$= 27 + 10\sqrt{2}$	Accept equivalent forms such as $27 + \sqrt{200}$. If any extra answers are given, this mark should be withheld.	A1
			(6 marks)

Question Number	Scheme		Marks
4 (a)	$a+(n-1)d=600+9\times120$	This mark is for: $600+9\times120$ or $600+8\times120$	M1
	$=(£)1680$	1680 with or without the "£"	A1
	Answer only scores both marks		
	<u>**Listing**</u> M1: Lists ten terms starting £600, £720, £840, £960, … A1: Identifies the 10th term as $(£)1680$		
			(2)
(b)	**Allow the use of *n* instead of *N* throughout in (b)**		
	$d=80$ for Kim	Identifies or uses $d=80$ for Kim	B1
	$\frac{N}{2}\{2\times600+(N-1)\times120\}$ OR $\frac{N}{2}\{2\times130+(N-1)\times80\}$	Attempts a sum formula for Andy or Kim. A correct formula must be seen or implied with: *a* = 600, *d* = 120 for Andy **or** *a* = 130, *d* = 80 for Kim. If B0 was scored, allow M1 here if Kim's incorrect "*d*" is used.	M1
	$\frac{N}{2}\{2\times600+(N-1)\times120\}=2\times\frac{N}{2}\{2\times130+(N-1)\times80\}$ A <u>**correct**</u> equation in any form		A1
	$20N=360\Rightarrow N=...$	Proceeds to find a value for *N*. (Allow if it leads to *N* < 0) **Dependent on the first method mark and must be an equation that uses Andy's and Kim's sum.**	dM1
	$(N=)18$	Ignore *N*/*n* = 0 and if a correct value of *N* is seen, isw any further reference to years etc.	A1
	See below for listing approach		
	If you see *N* = 18 with no working send to Review		
			(5)
			(7 marks)

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Andy	600	1320	2160	3120	4200	5400	6720	8160	9720	11400	13200	15120	17160	19320	21600	24000	26520	29160
Kim	130	340	630	1000	1450	1980	2590	3280	4050	4900	5830	6840	7930	9100	10350	11680	13090	14580
Kimx2	260	680	1260	2000	2900	3960	5180	6560	8100	9800	11660	13680	15860	18200	20700	23360	26180	29160

B1: States or uses $d = 80$ for Kim

M1: Attempts to find the total savings for Andy or Kim – must see the correct pattern for Andy (600, 1320, 2160,…) or Kim (130, 340, 630,…) (or Kimx2)

A1: Correct totals for Andy and Kim (or Kimx2) at least as far as n = 18

M1: Identifies when Andy's total = 2xKim's total

A1: N = 18

Question Number	Scheme		Marks
5(a)	(4, 7)	Accept (4, 7) or x = 4, y = 7 or a sketch of $y = \mathrm{f}(x-2)$ with a maximum point marked at (4, 7). (Condone missing brackets) There should be no other coordinates.	B1
			(1)
(b)	(x =) 2.5	Allow (2.5, 0) (condone missing brackets) but no other values or points. Allow a sketch of f(2x) with the only x-intercept marked at x = 2.5 (Allow (0, 2.5) marked in the correct place.	B1
			(1)
(c)	$y = 1$ (oe e.g. $y - 1 = 0$)	Must be an equation and not just '1' and no other asymptotes stated.	B1
			(1)
(d)	$k \le 1$ **or** $k = 7$	Either of $k \le 1$ or $k = 7$	B1

Question Number	Scheme		Marks
		Accept either of $y \leq 1$ or $y = 7$ Note that *k* = 7 may sometimes be seen embedded in e.g. *k* = 0, 1, 7 and can score B1 here.	
	$k \leq 1 \quad k = 7$	Both correct and in terms of *k* with no other solutions.	B1
			(2)
			(5 marks)

Question Number	Scheme		Marks
6 (a)	$a_1 = 4 \Rightarrow a_2 = \frac{4}{4+1}$	Attempts to use the given recurrence relation correctly at least once e.g. $a_2 = \frac{4}{4+1}$ or $a_3 = \frac{\text{their } a_2}{(\text{their } a_2)+1}$ or $a_4 = \frac{\text{their } a_3}{(\text{their } a_3)+1}$. May be implied by their term(s).	M1
	$\frac{4}{5}, \frac{4}{9}, \frac{4}{13}$	A1: Two of $\frac{4}{5}, \frac{4}{9}, \frac{4}{13}$ which may be un-simplified. Accept for example $0.8, \frac{0.8}{1.8}, \ldots$ or $\frac{4}{5}, \frac{\frac{4}{5}}{1+\frac{4}{5}}, \ldots$	A1A1
		A1: $\frac{4}{5}, \frac{4}{9}, \frac{4}{13}$ (Allow 0.8 for $\frac{4}{5}$)	
			(3)
(b)	*p* = 4 or e.g. $4 = \frac{4}{p+q}, \ "\frac{4}{5}" = \frac{4}{2p+q}$ $\Rightarrow p = \ldots \ or \ q = \ldots$	$a_n = \frac{4}{4n \pm ..}$ or *p* = 4 **OR** Uses 2 terms to set up and solve two **<u>correct equations</u> for their fractions** in *p* and *q* to obtain a value for *p* or a value for *q*.	M1
	$a_n = \frac{4}{4n-3} \Rightarrow$ *p* = 4 and $q = -3$	Either $a_n = \frac{4}{4n-3}$ OR *p* = 4 and $q = -3$	A1
	Correct answer only scores both marks.		
			(2)
(c)	$\frac{4}{4N-3} = \frac{4}{321} \Rightarrow N = \ldots$	Solves their $\frac{4}{pN+q} = \frac{4}{321}$ to obtain a value for *N* or *n*.	M1
	$(N =) 81$	Cao (ignore what they use for *N*)	A1
	Allow trial and improvement if 81 is clearly identified and then award both marks following a correct answer in (b) but just trying random values is M0		
			(2)

Question Number	Scheme	Marks
		(7 marks)

Question Number	Scheme		Marks
7(a)	$b^2 - 4ac = (4k)^2 - 4(-2)(20+13k)$	Attempts to use $b^2 - 4ac$ with $a = \pm(20 \pm 13k)$, $b = \pm 4k$, $c = \pm 2$. This could be as part of the quadratic formula or as $b^2 < 4ac$ or as $b^2 > 4ac$ or as $\sqrt{b^2 - 4ac}$ etc. If it is part of the quadratic formula only look for use of $b^2 - 4ac$. There must be no x's. If they gather to the lhs, condone the omission of the "– " on the "4*k*".	M1
	$(4k)^2 - 4(-2)(20+13k)$	For a correct un-simplified expression.	A1
	$b^2 - 4ac < 0$ $\Rightarrow (4k)^2 - 4(-2)(20+13k) < 0$	Uses $b^2 - 4ac < 0$ or e.g. $b^2 < 4ac$ with their values of *a*, *b* and *c* in terms of *k*. The " < 0" must appear before the final printed answer but can appear as $b^2 - 4ac < 0$ at the start.	M1
	$16k^2 + 160 + 104k < 0$ $\Rightarrow 2k^2 + 13k + 20 < 0$ *	Reaches the printed answer with no errors, including bracketing errors, or contradictory statements and sufficient working shown. Note that the statement $(20 + 13k)x^2 - 4kx - 2 < 0$ or starting with e.g. $20x^2 < 4kx - 13kx^2 + 2$	A1*

Question Number	Scheme		Marks
		would be an error.	
			(4)
(b)	$2k^2+13k+20=0 \Rightarrow k=...$ e.g. $(2k+5)(k+4)=0 \Rightarrow k=...$	Attempt to solve the **given** quadratic to find 2 values for *k*. See general guidance.	M1
	$\Rightarrow k=-\frac{5}{2},-4$	Both correct. May be implied by e.g. $k<-\frac{5}{2}$, $k<-4$ or seen on a sketch. If they use the quadratic formula allow $\frac{-13\pm 3}{4}$ for this mark but not $\sqrt{9}$ for 3 and allow e.g. $-\frac{13}{4}\pm\frac{3}{4}$ if they complete the square.	A1
	$-4<k<-\frac{5}{2}$ Allow equivalent values e.g. $-\frac{10}{4}$ i.e. the critical values must be in the form $\frac{a}{b}$ where *a* and *b* are integers	M1: Chooses 'inside' region for **their critical values** i.e. $\text{Lower Limit} < k < \text{Upper Limit}$ or e.g. $\text{Lower Limit} \le k \le \text{Upper Limit}$ A1: Allow $k\in(-4,-\frac{5}{2})$ or just $(-4,-\frac{5}{2})$ and allow $k>-4$ **and** $k<-2.5$ and $-\frac{5}{2}>k>-4$ but $k>-4,\ k<-\frac{5}{2}$ scores M1A0. $-\frac{5}{2}<k<-4$ is M0A0	M1A1
	Allow working in terms of *x* in (b) but the answer must be in terms of *k* for the final mark.		
			(4)
			(8 marks)

Question Number	Scheme		Marks
8(a)	$\frac{5}{4}$ oe	$\frac{5}{4}$ or exact equivalents such as 1.25 but **not** $\frac{5}{4}x$.	B1
			(1)

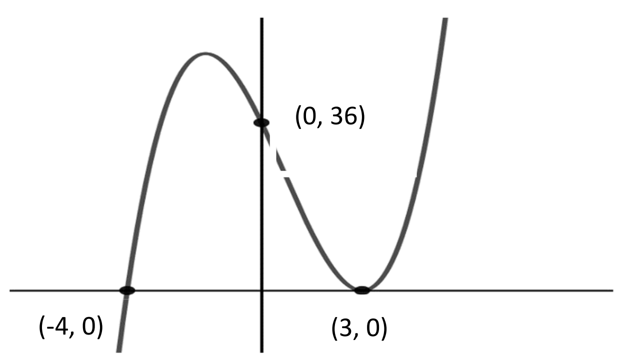
Question Number	Scheme		Marks
(b)	$y=\frac{5}{4}x+c$	Uses a line with a parallel gradient $\frac{5}{4}$ oe or their gradient from part (a). Evidence is $y="\frac{5}{4}"x+c$ or similar.	M1
	$(12,5)\Rightarrow\ 5="\frac{5}{4}"\times 12+c\Rightarrow\ c=\,..$	Method of finding an equation of a line with numerical gradient and passing through $(12,5)$. Score even for the perpendicular line. Must be seen in part (a).	M1
	$y=\frac{5}{4}x-10$	Correct equation. Allow $-\frac{40}{4}$ for -10	A1
			(3)
(c)	$(B=)(0,-10)$	$(B=)(0,-10)$ Follow through on their 'c'. Allow also if –10 is marked in the correct place on the diagram. Allow *x* = 0, *y* = -10 (the *x* = 0 may be seen "embedded" but not just *y* = -10 with no evidence that *x* = 0)	B1ft
	$(C=)(8,0)$	$(C=)(8,0)$ Correct coordinates. Allow also if 8 is marked in the correct place on the diagram. Allow *y* = 0, *x* = 8 (the *y* = 0 may be seen "embedded" but not just *x* = 8 with no evidence that *y* = 0)	B1
	Do not penalise lack of "0" twice so penalise it at the first occurrence but check the diagram if necessary.		
			(2)

(d) **Way 1**	Area of Parallelogram = $(3+'10')\times'8'$	Uses area of parallelogram = $bh = (3+'10')\times''8''$ Follow through on their 10 and their 8	M1
	= 104	cao	A1
	Correct answer only scores both marks		**(2)**
(d) **Way 2**	Trapezium *AOCD* + Triangle *OCB* $=\frac{1}{2}(3+3+'10')\times'8'+\frac{1}{2}\times'8''\times'10'$	A correct method using their values for *AOCD* + *OCB*.	M1
	= 104	cao	A1
			(2)
(d) **Way 3**	2 Triangles + Rectangle $=2\times\frac{1}{2}('8''\times'10')+'8''\times 3$	A correct method using their values for 2x*OBC* + rectangle.	M1
	= 104	cao	A1
			(2)
(d) **Way 4**	Triangle *ACD* + Triangle *ACB* $=2\times\frac{1}{2}('10'+3)\times'8'$	A correct method using their values for *ACD* + *ABC*.	M1
	= 104	cao	A1
			(2)
			(8 marks)

Question Number	Scheme		Marks
9.(a)	$(x-3)(3x+5)=3x^2-4x-15$ Allow $3x^2+5x-9x-15$	Correct expansion simplified or un-simplified.	B1
	$\mathrm{f}(x)=x^3-2x^2-15x+c$	M1: $x^n \to x^{n+1}$ for any term. Follow through on incorrect indices but not for "+ *c*" A1: All terms correct. Need not be simplified. No need for + *c* here.	M1A1
	$x=1, y=20 \Rightarrow 20=1-2-15+c$ $\Rightarrow c=36$	Substitutes *x* = 1 and *y* = 20 into their f(*x*) to find *c*. Must have + *c* at this stage. **Dependent on the first method mark.**	**d**M1
	$(\mathrm{f}(x)=)x^3-2x^2-15x+36$	Cao $(\mathrm{f}(x)=)x^3-2x^2-15x+36$ (All together and on one line)	A1
			(5)
(b)	*A* = 4	Correct value (may be implied)	B1
Way 1	$\mathrm{f}(x)=(x-3)^2(x+A)=(x^2-6x+9)(x+A)$ $\mathrm{f}(x)=x^3+(A-6)x^2+(9-6A)x+9A$ $A-6=-2 \Rightarrow A=4 \quad 9-6A=-15 \Rightarrow A=4 \quad 9A=36 \Rightarrow A=4$ M1: Expands $(x-3)^2(x+A)$ and compares coefficients with their f(*x*) from part (a) to **form 3 equations** and attempts to solve **at least two of them** in an attempt to show that *A* is the same in each case or substitutes their *A* to show that the coefficients are the same. A1: Fully correct proof – must use all 3 coefficients		M1A1
			(3)
Way 2	*A* = 4	Correct value (may be implied)	B1
	$\mathrm{f}(x)=(x-3)^2(x+4)=(x^2-6x+9)(x+4)$ $=x^3-6x^2+4x^2+9x-24x+36=x^3-2x^2-15x+36$ M1: Expands $(x-3)^2(x+"4")$ fully in an attempt to show that the expansion gives the same expression found as found in part (a) A1: Fully correct proof (Condone invisible brackets here e.g. around *x* + 4 provided sufficient working is shown)		M1A1

Question Number	Scheme		Marks
			(3)
Way 3	$A = 4$	Correct value (may be implied)	B1
	$(x^3 - 2x^2 - 15x + 36) \div (x-3) = x^2 + x - 12$ $(x^2 + x - 12) \div (x-3) = x+4$ or $(x^2 + x - 12) = (x+4)(x-3)$ M1: Divides their f(*x*) from part (a) by (*x* – 3) and divides their quotient by (*x* – 3) in an attempt to establish the value of *A*. Alternatively divides their f(*x*) from part (a) by $(x-3)^2$ (Allow $x^2 \pm 6x \pm 9$) in an attempt to establish the value of *A*. A1: Fully correct proof		M1A1
			(3)
	Note that this is an acceptable proof: *A* = 4 (may be implied) $x^3 - 2x^2 - 15x + 36 = (x-3)(x^2 + x - 12)$ $= (x-3)(x-3)(x+4)$ $= (x-3)^2(x+4)$		

Remember to check the last page for their sketch

Question Number	Scheme	Marks
9(c)		
	A positive cubic shape. Its position is not important but must be a curve and not straight lines and the "ends" must not clearly turn back in on themselves.	B1
	Touches at the point (3, 0) (could be a maximum). Accept 3 marked on the	B1

	x-axis and accept (0, 3) as long as it is in the correct place. Allow (3, 0) in the body of the script but it must correspond with the sketch. If ambiguous, the sketch takes precedence.	
	Crosses or reaches the *x*-axis at $(-4,0)$. Accept -4 marked on the *x*-axis and accept (0, -4) as long as it is in the correct place. FT on their $-A$ from part (b) and allow "–*A*" and allow a "made up" *A*. Allow $(-4,0)$ in the body of the script but it must correspond with the sketch. If ambiguous, the sketch takes precedence.	B1ft
	Crosses the *y*-axis at (0, 36) **and with a maximum in the second quadrant**. Accept 36 marked on the *y* – axis and accept (36, 0) as long as it is in the correct place. FT on their numerical '*c*' **<u>from part (a) only</u>**. Allow (0, 36) in the body of the script but it must correspond with the sketch. If ambiguous, the sketch takes precedence.	B1ft
		(4)
		(12 marks)

Question Number	Scheme		Marks
10(a)	$\frac{dy}{dx}=\frac{1}{2}-\frac{27}{x^2}$	M1: $\frac{1}{2}$ or $-\frac{27}{x^2}$ A1: $\frac{dy}{dx}=\frac{1}{2}-\frac{27}{x^2}$ oe e.g. $\frac{1}{2}x^0-27x^{-2}$	M1A1
	$x=3\Rightarrow\frac{dy}{dx}=\frac{1}{2}-\frac{27}{9}=\left(-\frac{5}{2}\right)$	Substitutes *x* = 3 into their $\frac{dy}{dx}$ to obtain a numerical gradient	M1
	$m_T=-\frac{5}{2}\Rightarrow m_N=-1\div-\frac{5}{2}$ $\Rightarrow y-\left(-\frac{3}{2}\right)=\frac{2}{5}(x-3)$	The correct method to find the equation of a normal. Uses $-\frac{1}{m_T}$ with $\left(3,-\frac{3}{2}\right)$ where m_T has come from calculus. If using *y* = *mx* + *c* must reach as far as *c* = …	M1
	$10y=4x-27*$	Cso (correct equation must be seen in (a))	A1*
			(5)
(b)	$\frac{1}{2}x+\frac{27}{x}-12=\frac{4x-27}{10}$ or $y=\frac{10y+27}{8}+\frac{108}{10y+27}-12$	Equate equations to produce an equation just in *x* or just in *y*. Do not allow e.g. $\frac{1}{2}x^2+27-12x=\frac{4x-27}{10}$ Unless $\frac{1}{2}x+\frac{27}{x}-12=\frac{4x-27}{10}$ was seen previously. **Allow sign slips only.**	M1
	$x^2-93x+270=0$ or $20y^2-636y-999=0$	Correct 3 term quadratic equation (or any multiple of). Allow terms on both sides e.g. $x^2-93x=-270$ (The "= 0" may be implied by their attempt to solve)	A1
	$(x-90)(x-3)=0\Rightarrow x=\ldots$ or $x=\frac{93\pm\sqrt{93^2-4\times270}}{2}$ or $(10y-333)(2y+3)=0\Rightarrow y=\ldots$ or	Attempt to solve a 3TQ (see general guidance) leading to at least one for *x* or *y*. **Dependent on the first method mark.**	dM1

Question Number	Scheme		Marks
	$y=\dfrac{636\pm\sqrt{636^2-4\times20\times(-999)}}{2\times20}$		
	$x=90$ **or** y = 33.3 oe	Cso. The x must be 90 and the y an equivalent number such as e.g. $\frac{333}{10}$	A1
	$x=90$ **and** y = 33.3 oe	Cso. The x must be 90 and the y an equivalent number such as e.g. $\frac{333}{10}$	A1
			(5)
			(10 marks)

Question Number	Scheme	Marks
1.		
(a)	$\{\text{At } x=2,\}\ y=2$ and $\{\text{At } x=6,\}\ y=4\sqrt{2}$ or $2\sqrt{8}$ or awrt 5.7	B1 cao
		(1)
(b)	$\frac{1}{2}\times4$; or $h=4$	B1 oe
	$\underline{\left\{0+6\sqrt{3}+2\left(their2+their4\sqrt{2}\right)\right\}}$ For structure of $\underline{\{................\}}$	M1A1ft
	$\frac{1}{2}\times4\ \underline{\left\{0+6\sqrt{3}+2\left(2+4\sqrt{2}\right)\right\}}$ $\{=2(25.706)=51.412..\}$ = awrt 51.412	A1
		(4)
		(5 marks)

x	-2	2	6	10
y	0	**2**	**$4\sqrt{2}$**	$6\sqrt{3}$

Question Number	Scheme	Marks

Notes

(a) B1: 2 and $4\sqrt{2}$ or $2\sqrt{8}$ or awrt 5.7 (or any correct unsimplified surd equivalent given as the final answer to part (a)) These may be stated as a final answer and not appear in the table, or may appear in the table. If a correct surd appears in the working (unsimplified) and is then simplified to give an incorrect answer to (a) which is used in the table and in part (b) then this is B0.

(b) B1: for using $\frac{1}{2}\times 4$ or 2 or equivalent or for stating h

M1: requires the correct $\{......\}$ bracket structure.

It needs the first bracket to contain first y value (as this is zero it may be omitted) **plus** last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if values used in brackets are x values instead of y values

A1ft: for the correct bracket $\{......\}$ following through candidate's y values found in part (a).

A1: for answer which rounds to 51.412 then isw

NB: Separate trapezia may be used : B1 for 4, M1 for $\frac{1}{2}h(a+b)$ used 3 times (and A1ft if it is all correct)

Then A1 as before.

Special case: Bracketing mistake $2\times(0+6\sqrt{3})+2\left(2+4\sqrt{2}\right)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). An answer of 36.098 usually indicates this error.

Question Number	Scheme	Marks
	$\left(2+kx\right)^7$	
2. (a)	$2^7 + {}^7C_1 2^6(kx) + {}^7C_2 2^5 (kx)^2 + {}^7C_3 2^4 (kx)^3 \ldots$	
	First term of 128	B1
	$\left({}^7C_1 \times \ldots \times x\right) + \left({}^7C_2 \times \ldots \times x^2\right) + \left({}^7C_3 \times \ldots \times x^3\right)\ldots$	M1
	$=(128\,..) + 448kx + 672k^2x^2 + 560k^3x^3..$	A1, A1
		(4)
(b)	$560k^3 = 1890$	M1
	$k^3 = \dfrac{1890}{560}$ *so* $k =$	dM1
	$k = 1.5$ o.e.	A1
		(3)
		(7marks)
Alternative method For (a)	$\left(2+kx\right)^7 = 2^7(1 + \frac{kx}{2})^7$ $2^7(1 + {}^7C_1(\frac{k}{2}x) + {}^7C_2(\frac{k}{2}x)^2 + {}^7C_3(\frac{k}{2}x)^3 \ \ldots)$ Scheme is applied exactly as before	

Notes

(a)

B1: The constant term should be 128 in their expansion (should not be followed by other constant terms)

M1: Two of the three binomial coefficients must be correct and must be with the correct power of x. Accept $^7C_1$ or $\binom{7}{1}$ or 7 **as a coefficient, and** $^7C_2$ or $\binom{7}{2}$ or 21 **as another and** $^7C_3$ or $\binom{7}{3}$ or 35 **as another……..**

Pascal's triangle may be used to establish coefficients.

A1: Two of the final three terms correct (i.e. two of $448kx + 672k^2x^2 + 560k^3x^3..$).
A1: All three final terms correct. (Accept answers without + signs, can be listed with commas or appear on separate lines)

e.g. The common error $=(128\,..\,) + 448kx + 672kx^2 + 560kx^3..$ would earn B1, M1, A0, A0, so 2/4 Then would gain a maximum of 1/3 in part (b)

If extra terms are given then isw

If the **final** answer is given as $=(128\,..\,) + 448kx + 672(kx)^2 + 560(kx)^3..$ with correct brackets and no errors are seen, this may be given full marks. If they continue and remove the brackets wrongly then they lose the accuracy marks.

Special case using Alternative Method: Uses $2\ (1 + \frac{kx}{2})^7$ is likely to result in a maximum mark of B0M1A0A0 then M1M1A0

If the correct expansion is seen award the marks and isw

(b)

M1: Sets their **Coefficient** of x^3 equal to 1890. They should have an equation which does not include a power of x. This mark may be recovered if they continue on to get $k = 1.5$

dM1: This mark depends upon the previous M mark. Divides then attempts a cube root of their answer to give k – the intention must be clear. (You may need to check on a calculator) The correct answer implies this mark.

A1: Any equivalent to 1.5 If they give – 1.5 as a second answer this is A0

Question Number	Scheme	Marks

4. (a)	**Usually answered in radians:**	
	Uses Area $ZYW = \frac{1}{2} \times 5^2 \times (angle)$, $= 12.5 \times 0.7 = 8.75$ o.e.(cm^2)	M1 A1
		(2)
(b)	Area of triangle $XYZ = \frac{1}{2} \times 7 \times 5 \times \sin Y = (11.273)$ (cm^2)	M1
	Area of whole flag = "8.75" + "11.273", = 20.02 (cm^2)	M1, A1
		(3)
(c)	$(XZ^2) = 7^2 + 5^2 - 2 \times 7 \times 5 \cos(\pi - 0.7)$, Or $(XZ^2) = (7 + 5\cos 0.7)^2 + (5\sin 0.7)^2$	M1,
	Use of arc length formula $s = 5\theta$ (= 3.5)	M1
	Total perimeter = $12 + "3.5" + "11.293"$	ddM1
	= 26.79 cm	A1
		(4)
		(9 marks)

Notes

(a)

M1: uses $A = 12.5\times\theta$ with θ in radians or completely correct work in degrees.

(If the angle is given as 0.7π and the formula has not been quoted correctly do not give this mark)

A1: 8.75 or $\frac{35}{4}$ or equivalent (do not need to see units)

(b)

M1 for use of $A=\frac{1}{2}\times 7\times 5\times \sin Y$ (where Y = 0.7 or attempt at (π– 0.7) they give the same answer) Do not need to see 11.273 (Do not allow use of 0.7 or π– 0.7 instead of their respective sines)

This may arise from use of $A=\frac{1}{2}\times a\times b\times \sin C$ formula or from $A=\frac{1}{2}\times b\times h$ with h found by a correct method so either $A=\frac{1}{2}\times 7\times(5\sin Y)$ or $A=\frac{1}{2}\times 5\times(7\sin Y)$

This may follow a long method finding all the angles and side lengths of triangle *XYZ.* If their answer rounds to 11.3 credit should be given. E.g. $A=\frac{1}{2}\times 11.293\times 1.996$

M1 for adding two numerical areas – triangle and sector (not dependent on previous M marks)

A1 for 20.02 (do not need to see units) (Allow answers which round to 20.02 e.g. do not allow 20.05)

(c)

M1: Uses cosine rule with correct angle (allow 2.4) or uses right angle triangle with correct sides**.**

(do not need to see *XZ* = 11.293) This may be calculated in part (b)

M1: Uses arc length with correct radius (may use wrong angle)

ddM1: (Needs to have earned both previous M marks) Adds 7 + 5 + their arc length + their *XZ*

This mark should not be awarded if they use their answer for XZ^2 instead of *XZ.*

A1: 26.79 – allow awrt

Question number	Scheme	Marks
5	**You may mark (a) and (b) together** $x^2 + y^2 - 2x + 14y = 0$	

(a)	Obtain LHS as $\underline{(x \pm 1)^2} + \underline{\underline{(y \pm 7)^2}} = \ldots$	M1
	Centre is $(1, -7)$.	A1
		(2)
(b)	Uses $r^2 = a^2 + b^2$ or $r = \sqrt{a^2 + b^2}$ where their centre was at $(\pm a, \pm b)$	M1
	$r = \sqrt{50}$ or $5\sqrt{2}$	A1 **(2)**
(c)	Substitute $x = 0$ in either form of equation of circle and solve resulting quadratic to give $y =$	M1
	$y^2 + 14y = 0$ *so* $y = 0$ *and* -14 **or** $\underline{\underline{(y \pm 7)^2 - 49}} = 0$ *so* $y = 0$ *and* -14	A1
		(2)
(d)	Gradient of radius joining centre to (2,0) is $\frac{"-7"-0}{"1"-2}$ $(=7)$	M1
	Gradient of tangent is $\frac{-1}{m}$ $(=-\frac{1}{7})$	M1
	So equation is $y - 0 = -\frac{1}{7}(x-2)$ and so $x + 7y - 2 = 0$	M1, A1
		(4)
		(10 marks)
	Alternative Methods which may be seen	
(a)	Method 2: Comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$ to write down centre $(-g, -f)$ directly. Condone sign errors for this M mark**.** Centre is $(1, -7)$.	M1 A1
		(2)
(b)	Method 2: Using $\sqrt{g^2 + f^2 - c}$. So $r = \sqrt{50}$ or $5\sqrt{2}$	M1 A1
		(2)
(d)	Method 3: Using Implicit Differentiation	
	$2x + 2y\frac{dy}{dx} - 2 + 14\frac{dy}{dx} = 0$ **or** $2(x-1) + 2(y+7)\frac{dy}{dx} = 0$	M1
		M1

	$\frac{dy}{dx} = ...\left(\frac{2-2x}{14+2y} = \frac{-2}{14}\right)$	M1, A1
		(4)
	So equation is $y - 0 = -\frac{1}{7}(x-2)$ and so $x + 7y - 2 = 0$	
	Method 4: Making y the subject of the formula and differentiating	
	$y = -7 \pm \sqrt{\{50-(x-1)^2\}}$ so $\frac{dy}{dx} = \pm\frac{1}{2} \times -2(x-1)\{50-(x-1)^2\}^{-\frac{1}{2}}$	M1
	At $x = 2$, $\frac{dy}{dx} = \mp\frac{1}{7}$	M1 (contd next page)
	So equation is $y - 0 = \mp\frac{1}{7}(x-2)$	M1
	Chooses $\frac{dy}{dx} = -\frac{1}{7}$ and so $x + 7y - 2 = 0$	A1

Notes

(a)

M1: as in scheme and can be implied by $(\pm 1, \pm 7)$ even if this follows some poor working.

A1: (1, –7)

(b)

M1 : Uses $r^2 = a^2 + b^2$ or $r = \sqrt{a^2 + b^2}$ where their centre was at $(\pm a, \pm b)$

A1**:** √50 or 5√2 not 50 only

Special case: if centre is given as (–1 , –7) or (1, 7) or (-1, 7) or coordinates given wrong way round- allow M1A1 for $r = 5\sqrt{2}$ worked correctly. r^2 = "1"+"49"

If they get $r = 5\sqrt{2}$ after wrong statements such as r^2 = "–1"+"–49" then this is M0A0

$r = 5\sqrt{2}$ with no working earns M1A1 as there is no wrong work.

(c)

M1: As in the scheme – allow for just one value of y

A1: Accept (0, 0), (0, -14) or $y = 0$, $y = -14$ or just 0 and -14

(d) Method 1:

M1: Correct method for gradient – if no method shown answer must be correct to earn this mark

If x and y coordinates are confused and fraction is upside down this is M0 even if the formula is quoted as there is no evidence of understanding.

M1: Correct negative reciprocal of their gradient

M1: Line equation through (2,0) with changed gradient so if they use $y = mx + c$ they need to use

(2, 0) to find c

A1: For any multiple of the answer in the scheme. (The answer must be an equation so if "=0" is missing this is A0)

(d) Method 3:

M1: Correct implicit differentiation (no errors)

M1: Rearranges their differentiated expression and substitutes $x = 2$, $y = 0$ to obtain gradient – allow slips. (It should be $\frac{dy}{dx} = \frac{2-2x}{14+2y} = \left(\frac{-2}{14}\right)$)

If there is no y term this mark may be earned for substitution of $x = 2$ as $y = 0$ is not needed

M1: Line equation through (2,0) with their obtained gradient so if they use $y = mx + c$ they need to use (2, 0) to find c

A1: For any multiple of the answer in the scheme (The answer must be an equation so if "=0" is missing this is A0)

Method 4:

M1: Correct rearrangement and differentiation (no errors)

M1: Substitutes $x = 2$ to obtain gradient – allow minus and plus.

M1: Line equation through (2,0) with their obtained gradient so if they use $y = mx + c$ they need to use (2, 0) to find c

A1: For any multiple of the answer in the scheme (The answer must be an equation so if "= 0" is missing this is awarded A0)

Question Number	Scheme	Marks
6.(a)	$10000 = \dfrac{a}{1-(-0.9)}$	M1
	$a = 19000$	A1
		(2)
(b)	Use ar^4	M1
	$19000 \times (-0.9)^4 = 12465.9$ (accept awrt 12466)	A1
		(2)
(c)	$S = \dfrac{a(1-r^{12})}{1-r}$ or lists and adds their first twelve terms with their *a*	M1
	$S = \dfrac{"19000"(1-(-0.9)^{12})}{1-(-0.9)}$ or $S = 10000(1-(-0.9)^{12})$	A1ft
	= 7176 only	A1cso
		(3)
		[7]

Notes

(a) M1: Correct use of formula for sum to infinity as above, or states correct formula and makes small slip such as replacing *r* with 0.9 instead of – 0.9

A1: Correct answer

(b) M1: Correct use of formula with $n - 1 = 4$, allow 0.9 instead of -0.9 here. Condone invisible brackets.

A1: accept awrt 12466 (even following use of 0.9) Correct answer implies M1A1 even with no method shown. Accept correct equivalents such as mixed or improper fractions

(c) M1: Correct use of formula with power 12 (or adds 12 terms) with their *a* (not 10000) and $r = +0.9$ or -0.9

A1ft: Correct unsimplified with their a and with r = +0.9 or -0.9 or for listing method as follows

19000 + -17100 + 15390 + -13851 + 12465.9 + -11219.31 + 10097.379 + -9087.6411 + 8178.87699 + -7360.989291 + 6624.890362 + -5962.401326 = (Do not follow through for listing method)

A1cso: 7176 only

Special case: $S = \frac{a(1-r^n)}{1-r}$ so $S = \frac{"19000"(1+(0.9)^{12})}{1+(0.9)}$ is M1A0A0

Whereas $S = \frac{"19000"(1+(0.9)^{12})}{1+(0.9)}$ on its own with no formula quoted is M0A0A0

$S = \frac{"19000"(1--0.9^{12})}{1--0.9}$ should have M1 (bod) then final two A marks depend on whether answer is correct so if this is followed by 7176 the A1A1 should be awarded. If it is followed by 12824 then A0A0 is implied.

Question Number	Scheme	Marks
7. (i)	Use of power rule so $(y-1)\log 1.01 = \log 500$ or $(y-1) = \log_{1.01} 500$	M1
	625.56	A1
		(2)
(ii) (a)	Ignore labels (a) and (b) in part ii and mark work as seen	
	$\log_4 (3x+5)^2 =$ Applies power law of logarithms	M1
	Uses $\log_4 4 = 1$ or $4^1 = 4$	M1
	Uses quotient or product rule so e.g. $\log(3x+5)^2 = \log 4(3x+8)$ or $\log\frac{(3x+5)^2}{(3x+8)} = 1$	M1
	Obtains with no errors $9x^2 + 18x - 7 = 0$ *	A1* cso
		(4)
(b)	Solves given or "their" quadratic equation by any of the standard methods	M1
	Obtains $x = \frac{1}{3}$ and $-\frac{7}{3}$ and rejects $-\frac{7}{3}$ to give just $\frac{1}{3}$	A1
		(2)
		[8]

Notes

(i)

M1: Applies power law of logarithms correctly or changes base (Allow missing brackets)

A1: Accept answers which round to 625.56 (This may follow 624.56 + 1 = or may follow $y = \log_{1.01} 505$ or $\frac{\log 505}{\log 1.01}$ or may appear with no working)

(ii) (a)

M1: Applies power law of logarithms $2\log_4(3x+5) = \log_4(3x+5)^2$

M1: Uses $\log_4 4 = 1$ or $4^1 = 4$

M1: Applying the subtraction or addition law of logarithms **correctly** to make **two** log **terms into one** log term in x (*see note below)

A1cso: This is a given answer and needs a correct algebraic statement such as $9x^2 + 30x + 25 = 4(3x+8)$ followed by a conclusion, such as $9x^2 + 18x - 7 = 0$

(ii) (b)

M1: Solves by factorisation or by completion of the square or by correct use of formula (see general principles)

A1: Needs to find two answers and reject one to give the correct $\frac{1}{3}$ (This may be indicated by underlining just the 1/3 for example).

Special case: States $\frac{\log(3x+5)^2}{\log(3x+8)} = \log\frac{(3x+5)^2}{(3x+8)} = 1$, loses the third M mark in part ii(a) and the A1 cso

Question Number	Scheme	Marks
8. (i)	$4\cos(x+70°)=3$	
	$\cos(x+70°)=0.75$, so $x+70°=41.4(1)°$	M1A1
	$x = 248.6°$ or $331.4°$	M1 A1
		(4)
(ii)	$6\cos^2\theta - 5 = 6\sin^2\theta + \sin\theta$ so $6(1-\sin^2\theta) - 5 = 6\sin^2\theta + \sin\theta$	M1
	$12\sin^2\theta + \sin\theta - 1 = 0$	A1
	$(4\sin\theta - 1)(3\sin\theta + 1) = 0$ *so* $\sin\theta =$	M1
		A1 A1
	$\theta = 0.253,\ 2.89,\ 3.48,\ 5.94$	
		(5)
		[9]

Notes

(i)

M1: Divides by 4 and then uses inverse cosine
A1: Any Correct answer for $x+70°$ or for x (not necessarily in the range) Accept awrt 41.4

Or ($x =$) -28.6. If an intermediate answer here is not seen the final correct answers imply this mark.

M1: One correct answer (awrt) so awrt 331.4 or 248.6

A1: Both answers – accept awrt (Lose this mark for extra answers in the range) Ignore extra answers outside the range.

4.3 radians and 5.8 radians is special case: M1A0M1A0

(ii)

M1: Uses $\cos^2\theta = 1-\sin^2\theta$

A1: correct three term quadratic – any equivalent - so $12\sin^2\theta + \sin\theta = 1$ is acceptable

M1: Solves their quadratic to give values for sin θ (implied if arcsin is used on their answer(s))

1st A1: Need two correct angles (accept awrt)

A1: All four solutions correct accept awrt 3 sf and ignore subsequent rounding or copying errors. (Extra solutions in range lose this A mark, but outside range - ignore)

Special case: All four angles correct but in degrees (awrt 14.5, 166, 199, 341) gets A1 A0

Question Number	Scheme	Marks
9. (a)	$\frac{dy}{dx} = 70x - 35x^{\frac{3}{2}}$	M1A1
	Put $\frac{dy}{dx} = 0$ to give $70x - 35x^{\frac{3}{2}} = 0$ *so* $x^{\frac{1}{2}} = 2$	M1
	$x = 4$	A1
	$y = 112$	A1
		(5)
(b) (Way 1)	When $y = 0$, $35x^2 = 14x^{\frac{5}{2}}$ *and* $x^{\frac{1}{2}} = \frac{35}{14}$ *or* $5 = 2\sqrt{x}$ *so* $\sqrt{x} = \frac{5}{2}$	M1

	$x = \frac{25}{4}$	A1 **(2)**
(b) (Way 2)	When $y = 0$, $35x^2 = 14x^{\frac{5}{2}}$ *so* $1225x^4 = 196x^5$ *or* $5 = 2\sqrt{x}$ *so* $25 = 4x$ $x = \frac{25}{4}$ or $x = \frac{1225}{196}$	M1 A1 **(2)**
(c) Way 1	$\int 35x^2 - 14x^{\frac{"5"}{2}} dx = \frac{35}{3}x^3 - \frac{14x^{\frac{"7"}{2}}}{\frac{"7"}{2}} \; (+c)$ $\left[\frac{35}{3}x^3 - 4x^{\frac{7}{2}}\right]_4^{\frac{25}{4}} = 406.901.. - 234.667 = 172.23$ Hence Area = "*their* $112 \times (6\frac{1}{4} - 4)$" − "172.23" or "252" – " 172.23" 79.77	M1A1ft dM1 ddM1 A1 **(5)**
(c) Way 2	$\int "112" - \{35x^2 - 14x^{\frac{5}{2}}\} dx = (112x) - \frac{35}{3}x^3 + \frac{14x^{\frac{7}{2}}}{\frac{7}{2}} \; (+c)$ $\left[(112x) - (\frac{35}{3}x^3 - 4x^{\frac{7}{2}})\right]_{"4"}^{"\frac{25}{4}"}$ with correct use of limits Integrated their 112 to give $112x$ with correct use of limits 79.77	M1A1ft dM1 ddM1 A1 **(5)** **[12]**

Notes

(a)

M1: Attempt at differentiation after multiplying out - may be awarded for $70x$ term correct

(If product rule is used it must be of correct form i.e. $\frac{dy}{dx} = 7x^2\left(-2kx^{k-1}\right) + 14x(5-2x^k)$)

A1: the derivative must be completely correct but may be unsimplified

For product rule this is $\frac{dy}{dx} = 7x^2\left(-x^{-\frac{1}{2}}\right) + 14x(5-2\sqrt{x})$

M1: uses derivative = 0 to find $x^k =$ or $x =$ with correct work for their equation (even without fractional powers)

A1: obtains $x = 4$ then

A1: for $y = 112$ (may be credited if seen in part (a) or in part(c))

(b)

Way 1 (Dividing first)

M1: Puts $y = 0$ and obtains expression of the form $x^k = A$ (where k is not equal to 1) after correct algebra for their equation (may be a sign slip)

A1: Obtains $x = 6.25$ or equivalent correct answer

(b)

Way 2 (dealing with fractional power first i.e. Squaring)

M1: Puts $y = 0$ and squares each term correctly for their equation obtaining expression of the form

$A^2 x^m = B^2 x^n$ after correct algebra

A1: Obtains $x = 6.25$ or equivalent correct answer

(c)

Way 1

M1: Correct integration of one of their terms – e.g. see x^2 term integrated correctly (not just raised power)

A1ft: completely correct integral for their power which must have been a fraction **(may be unsimplified)**

dM1: (dependent on previous M) substituting their 25/4 and their 4 and subtracting

ddM1 (depends on both method marks) **Correct** method **to obtain shaded area** so their rectangle minus their area under curve

A1: Accept answers which round to 79.77

(c)

Way 2

M1: Attempt at integration – x^2 term integrated correctly

A1ft: completely correct integral for second and their third terms (provided one has a fractional power) (ignore sign errors) **(may be unsimplified)**

dM1: (dependent on previous M) substituting their 25/4 and their 4 and subtracting (either way) ddM1 (depends on both method marks) **Correct** method **to obtain shaded area** so their 112 integrated correctly and correct signs for the other two terms in the integrand

A1: Accept answers which round to 79.77

Answer with no working – send to review

If they have the wrong fractional power on their second term after expansion in part (a) (usually 3/2), all the method marks are available throughout the question and the A1ft is available in (c). The A mark in part (b) may also be accessible. Maximum score is likely to be 8/12

If they have the trivial power 1 on their second term, then two method marks are available in (a) and three method marks are available in part (c) Maximum score is likely to be 5/12

M1: Uses the product rule $vu'+uv'$ with $u=2x$ and $v=(3x-1)^5$ or vice versa to achieve an expression of the form $A(3x-1)^5+Bx(3x-1)^4, \quad A,B>0$

Question Number	Scheme	Marks
1.(a)	$y=2x(3x-1)^5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=2(3x-1)^5+30x(3x-1)^4$	M1A1
	$\Rightarrow \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=2(3x-1)^4\{(3x-1)+15x\}=2(3x-1)^4(18x-1)$	M1A1
		(4)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}\leqslant 0 \Rightarrow 2(3x-1)^4(18x-1)\leqslant 0 \Rightarrow x\leqslant \frac{1}{18} \quad x=\frac{1}{3}$	B1ft, B1
		(2)
		(6 marks)

Condone slips on the $(3x-1)$ and $2x$ terms but misreads on the question must be of equivalent difficulty. If in doubt use review.

Eg: $y=2x(3x+1)^5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=2(3x+1)^5+30x(3x+1)^4$ can potentially score 1010 in (a) and 11 in (b)

Eg: $y=2x(3x+1)^{15} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=2(3x+1)^{15}+90x(3x+1)^{14}$ can potentially score 1010 in (a) and 11 in (b)

Eg: $y=2(3x+1)^5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=30(3x+1)^4$ is 0000 even if attempted using the product rule (as it is easier)

A1: A correct un-simplified expression. You may never see the lhs which is fine for all marks.

M1: Scored for taking a common factor of $(3x-1)^4$ out of $A(3x-1)^5 \pm Bx^n(3x-1)^4$ where $n=1$ or 2,to reach a form $(3x-1)^4\{........\}$ You may condone one slip in the $\{........\}$

Alternatively they take out a common factor of $2(3x-1)^4$ which can be scored in the same way

Example of one slip $2(3x-1)^5+30x(3x-1)^4=(3x-1)^4\{(3x-1)+30x\}$

If a different form is reached, see examples above, it is for equivalent work.

A1: Achieves a fully factorised simplified form $2(3x-1)^4(18x-1)$ which may be awarded in (b)

(b)

B1ft: For a final answer of either $x \leqslant \frac{1}{18}$ or $x = \frac{1}{3}$ Condone $x \leqslant \frac{2}{36}$ $x \leq 0.0\dot{5}$ $x = 0.\dot{3}$

Do not allow $x = \frac{1}{3}$ if followed by $x \leqslant \frac{1}{3}$ Follow through on a linear factor of $(Ax+B) \leqslant 0 \Rightarrow x\ldots$ where $A, B \neq 0$. Watch for negative *A*'s where the inequality would reverse.

It may be awarded within an equality such as $\frac{1}{3} \leqslant$ $x \leqslant \frac{1}{18}$

B1: **For a final answer** of $x \leqslant \frac{1}{18}$ oe (and) $x = \frac{1}{3}$ oe with no other solutions. Ignore any references to and/or here. Misreads can score these marks

Question Number	Scheme	Marks
2(a)	$4x^2 - 25 \to (2x+5)(2x-5)$	B1
	$\frac{6}{2x+5} + \frac{2}{2x-5} + \frac{60}{4x^2-25} = \frac{6(2x-5)+2(2x+5)+60}{(2x+5)(2x-5)}$	M1
	$= \frac{16x+40}{(2x+5)(2x-5)}$	A1
	$= \frac{8(2x+5)}{(2x+5)(2x-5)} = \frac{8}{(2x-5)}$	A1
		(4)
(b)	$f(x) = \frac{8}{2x-5} \Rightarrow y = \frac{8}{2x-5} \Rightarrow 2xy - 5y = 8 \Rightarrow x = \frac{8+5y}{2y}$	M1
	$\Rightarrow f^{-1}(x) = \frac{8+5x}{2x}$ oe	A1
	$0 < x < \frac{8}{3}$	B1ft
		(3)
		(7 marks)

Alternative solutions to part (a)

2(a) ALT I	$4x^2 - 25 = (2x+5)(2x-5)$	B1

	$\frac{6}{2x+5}+\frac{2}{2x-5}=\frac{16x-20}{4x^2-25}$	
	$\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{60}{4x^2-25}=\frac{16x-20+60}{4x^2-25}$	M1
	$=\frac{16x+40}{4x^2-25}$	A1
	$=\frac{8(2x+5)}{(2x+5)(2x-5)}=\frac{8}{2x-5}$	A1

2(a) ALT II	$4x^2-25=(2x+5)(2x-5)$	B1
	$\frac{60}{4x^2-25}=\frac{-6}{2x+5}+\frac{6}{2x-5}$	M1
	$\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{60}{4x^2-25}=\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{-6}{2x+5}+\frac{6}{2x-5}$	A1
	$=\frac{8}{(2x-5)}$	A1

(a)

B1: For **factorising** $4x^2-25 \to (2x+5)(2x-5)$ This can occur anywhere in the solution.

Note that it is possible to score this mark for expanding $(2x+5)(2x-5) \to 4x^2-25$ and then cancelling by $4x^2-25$. Both processes are required by this route. It can be implied if you see the correct intermediate form. (See A1)

M1: For combining the three fractions with a common denominator. The denominator must be correct and at least one numerator must have been adapted correctly . Accept as separate fractions. Condone missing brackets.

Accept $\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{60}{4x^2-25}=\frac{6(2x-5)\left(4x^2-25\right)+2(2x+5)\left(4x^2-25\right)+60(2x+5)(2x-5)}{(2x+5)(2x-5)\left(4x^2-25\right)}$

Condone $\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{60}{4x^2-25}=\frac{6(2x-5)+2+60}{(2x+5)(2x-5)}$ correct denominator, one numerator adapted correctly

Alternatively uses partial fractions $\frac{60}{4x^2-25}=\frac{A}{2x+5}+\frac{B}{2x-5}$ leading to values for A and B

A1: A correct intermediate form of $\frac{\text{simplified linear}}{\text{quadratic}}$ most likely to be $\frac{16x+40}{(2x+5)(2x-5)}$

Sometimes the candidate may write out the simplified numerator separately. In cases like this, you can award this A mark without explicitly seeing the fraction as long as a correct denominator is seen.

Using the partial fraction method, it is for $\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{60}{4x^2-25}=\frac{6}{2x+5}+\frac{2}{2x-5}+\frac{-6}{2x+5}+\frac{6}{2x-5}$

A1: Further factorises and cancels (all of which may be implied) to reach the answer $\frac{8}{2x-5}$

This is not a given answer so condone slips in bracketing etc.

(b)

M1: Attempts to change the subject of the formula for a function of the form $y=\frac{A}{Bx+C}$

Condone attempts on an equivalent made up equation for candidates who don't progress in part (a).

As a minimum expect to see multiplication by $(Bx+C)$ leading to *x* (or a replaced *y*) =

Alternatively award for 'inverting' Eg. $y=\frac{A}{Bx+C}$ to $\frac{Bx+C}{A}=\frac{1}{y}$ leading to *x* (or a replaced *y*) =

A1: $\mathrm{f}^{-1}(x)=\frac{8+5x}{2x}$ or $y=\frac{8+5x}{2x}$ or equivalent. Accept $y=\frac{4}{x}+\frac{5}{2}$ Condone $\mathrm{F}^{-1}(x)=\frac{8+5x}{2x}$

Condone $y=\frac{1}{2}\left(\frac{8}{x}+5\right)$ and $y=\frac{8}{2x}+\frac{5}{2}$ **BUT NOT** $y=\frac{\frac{8}{x}+5}{2}$ (fractions within fractions)

You may isw after a correct answer.

B1ft: $0<x<\frac{8}{3}$ or alternative forms such as $0<Domain<\frac{8}{3}$ Domain = $\left(0,\frac{8}{3}\right)$ or $\frac{8}{3}>x>0$

Do not accept $0<y<\frac{8}{3}$ or $0<\mathrm{f}^{-1}(x)<\frac{8}{3}$

Follow through on their $y=\frac{A}{Bx+C}$ so accept $0<x<\frac{A}{4B+C}$

Question Number	Scheme	Marks
3(a)	$A = 1500$	B1 **(1)**
(b)	Sub $t = 2, V = 13500 \Rightarrow 16000e^{-2k} = 12000$ $\Rightarrow e^{-2k} = \dfrac{3}{4} \quad (0.75)$ oe $\Rightarrow k = -\dfrac{1}{2}\ln\dfrac{3}{4}, = \ln\sqrt{\dfrac{4}{3}} = \ln\left(\dfrac{2}{\sqrt{3}}\right)$	M1 A1 dM1, A1* **(4)**
(c)	Sub $6000 = 16000e^{-\ln\left(\frac{2}{\sqrt{3}}\right)T} + '1500' \Rightarrow e^{-\ln\left(\frac{2}{\sqrt{3}}\right)T} = C$ $\Rightarrow e^{-\ln\left(\frac{2}{\sqrt{3}}\right)T} = \dfrac{45}{160} = (0.28125)$ $\Rightarrow T = -\dfrac{\ln\left(\frac{45}{160}\right)}{\ln\left(\frac{2}{\sqrt{3}}\right)} = 8.82$	M1 A1 M1 A1 **(4)** **(9 marks)**
Alt (b)	Sub $t = 2, V = 13500 \Rightarrow 13500 = 16000e^{-2k} + '1500' \Rightarrow 1600e^{-2k} = 1200$ $\Rightarrow \ln 1600 - 2k = \ln 1200$ $\Rightarrow k = -\dfrac{1}{2}\ln\dfrac{1200}{1600}, = \ln\sqrt{\dfrac{4}{3}} = \ln\left(\dfrac{2}{\sqrt{3}}\right)$	M1 A1 dM A1* **(4)**

You may mark parts (a) and (b) together

(a)

B1: Sight of $A = 1500$

(b)

M1: Substitutes $t = 2, V = 13500 \Rightarrow 13500 = 16000e^{-2k} + '\mathit{their}\ 1500'$ and proceeds to $Pe^{-2k} = \ldots$ or $Qe^{2k} = \ldots$
Condone slips, for example, *V* may be 1350. It is for an **attempt** to make $e^{\pm 2k}$ the subject.

A1: $e^{-2k} = \frac{3}{4}$ (0.75) or $e^{2k} = \frac{4}{3}$ $\left(1.\dot{3}\right)$ oe

dM1: For taking ln's and proceeding to *k* =... For example $k = -\frac{1}{2}\ln\frac{3}{4}$ oe

May be implied by the correct decimal answer awrt 0.144 . This mark cannot be awarded from impossible to solve equations, that is ones of the type $\Rightarrow e^{\pm 2k} = c, \quad c \leq 0$

A1*: cso $k = \ln\left(\frac{2}{\sqrt{3}}\right)$ (brackets not required) **with a correct intermediate line** of either

$$\frac{1}{2}\ln\frac{4}{3},\ \frac{1}{2}\ln 4 - \frac{1}{2}\ln 3,\ \ln\sqrt{\frac{4}{3}} \text{ or } \ln\left(\frac{3}{4}\right)^{-\frac{1}{2}}$$

Note: $e^{-2k} = \frac{3}{4} \Rightarrow e^{2k} = \frac{4}{3} \Rightarrow e^{k} = \frac{2}{\sqrt{3}}$ are perfectly acceptable steps

See scheme for alternative method when ln's are taken before e^{-2k} is made the subject.

It is also possible to substitute $k = \ln\left(\frac{2}{\sqrt{3}}\right)$ into $13500 = 16000e^{-k\times 2} + 1500$ and show that 12000 =12000 or similar. This is fine as long as a minimal conclusion (eg ✓)is given for the A1*.

(c)

M1: Sub $V = 6000 \Rightarrow 6000 = 16000e^{\pm kT} + \text{'their 1500'}$ and proceeds to $e^{\pm kT} = c, \quad c > 0$

Allow candidates to write $k =$ awrt 0.144 or leave as '*k*'. Condone slips on *k*. Eg $k = 2\ln\left(\frac{2}{\sqrt{3}}\right)$

Allow this when the = sign is replaced by any inequality.

If the candidate attempts to simplify the exponential function score for $\left(\frac{2}{\sqrt{3}}\right)^{\pm T} = c, \quad c > 0$

A1: $e^{-\ln\left(\frac{2}{\sqrt{3}}\right)T} = \frac{45}{160} = (0.28125),\ e^{-kT} = \frac{45}{160}$ or $\left(\frac{2}{\sqrt{3}}\right)^{-T} = \frac{45}{160}$ Condone inequalities for =

Allow solutions from rounded values (3sf). Eg. $e^{-0.144T} = 0.281$

M1: Correct order of operations using ln's and division leading to a value of T. It is implied by awrt 8.8

$\left(\frac{2}{\sqrt{3}}\right)^{-T} = \frac{45}{160} \Rightarrow -T = \log_{\frac{2}{\sqrt{3}}}\frac{45}{160}$ is equivalent work for this M mark.

A1: cso 8.82 only following correct work. Note that this is not awrt

Allow a solution using an inequality as long as it arrives at the solution 8.82.

....

There may be solutions using trial and improvement. Score (in this order) as follows

M1: Trial at value of $V = 16000e^{-0.144t} + 1500$ (oe) at either t = 8 or t = 9 and shows evidence

$V_{t=8} = awrt\ 6500\ V_{t=9} = awrt\ 5900$ This may be implied by the subsequent M1

M1: Trial at value of $V = 16000e^{-0.144t} + 1500$ (oe) at either t = 8.81 or t = 8.82 and shows evidence. (See below for answers. Allow to 2sf)

A1: Correct answers for V at **both** t = 8.81 **and** t = 8.82 $V_{t=8.81} = awrt\ 6006 \quad V_{t=8.82} = awrt\ 5999$

A1: Correctly deduces 8.82 with all evidence.

Hence candidates who **just** write down 8.82 will score 1, 1, 0, 0

....

Question Number	Scheme	Marks
4.(a)	$\frac{dy}{dx} = -2e^{-2x} + 2x$	M1A1
	At $x = 0$ $\frac{dy}{dx} = -2 \Rightarrow \frac{dx}{dy} = \frac{1}{2}$	M1
	Equation of normal is $y - (-2) = \frac{1}{2}(x - 0) \Rightarrow y = \frac{1}{2}x - 2$	M1 A1
		(5)
(b)	$y = e^{-2x} + x^2 - 3$ meets $y = \frac{1}{2}x - 2$ when $e^{-2x} + x^2 - 3 = "\frac{1}{2}x - 2"$	
	$x^2 = 1 + \frac{1}{2}x - e^{-2x}$	M1
	$x = \sqrt{1 + \frac{1}{2}x - e^{-2x}}$ *	A1*
		(2)

Question Number	Scheme	Marks
(c)	$x_2 = \sqrt{1+0.5-e^{-2}}$	M1
	$x_2 = 1.168,\ x_3 = 1.220$	A1
		(2)
		(9 marks)

(a)

M1: Attempts to differentiate with $e^{-2x} \to Ae^{-2x}$ with any non -zero *A*, even 1.

Watch for $e^{-2x} \to Ae^{2x}$ which is M0 A0

A1: $\frac{dy}{dx} = -2e^{-2x} + 2x$

M1: A correct method of finding the gradient of the normal at $x = 0$

To score this the candidate must find the negative reciprocal of $\left.\frac{dy}{dx}\right|_{x=0}$

So for example candidates who find $\frac{dy}{dx} = e^{-2x} + 2x$ should be using a gradient of -1

Candidates who write down $\frac{dy}{dx} = -2$ (from their calculators?) have an opportunity to score this mark and the next.

M1: An attempt at the equation of the normal at $(0,-2)$

To score this mark the candidate must be using the point $(0,-2)$ and a gradient that has been **changed from** $\left.\frac{dy}{dx}\right|_{x=0}$

Look for $y-(-2) = changed\left|\frac{dy}{dx}\right|_{x=0} (x-0)$ or $y = mx - 2$ where *m*= $changed\left|\frac{dy}{dx}\right|_{x=0}$

If there is an attempt using $y = mx + c$ then it must proceed using $(0,-2)$ with *m*= $changed\left|\frac{dy}{dx}\right|_{x=0}$

A1: $y = \frac{1}{2}x - 2$ cso with as well as showing the correct differentiation.

So reaching $y = \frac{1}{2}x - 2$ from $\frac{dy}{dx} = -2e^{2x} + 2x$ is A0

If it is not simplified (or written in the required form) you may award this if $y=\frac{1}{2}x-2$ is seen in part (b)

(b)

M1: Equates $y=e^{-2x}+x^2-3$ and their $y=mx+c, m\neq 0$ and proceeds to $x^2=...$

Condone an attempt for this M mark where the candidate uses an adapted $y=mx+c$ in an attempt to get the printed answer.

A1*: Proceeds to $x=\sqrt{1+\frac{1}{2}x-e^{-2x}}$. It is a printed answer but you may accept a different order $x=\sqrt{1-e^{-2x}+\frac{1}{2}x}$

For this mark, the candidate must start with a normal equation of $y=\frac{1}{2}x-2$ oe found in (a). It can be awarded when the candidate finds the equation incorrectly, for example from $\frac{dy}{dx}=-2e^{2x}+2x$

(c)

M1: Sub $x_1=1$ in $x=\sqrt{1+\frac{1}{2}x-e^{-2x}}$ to find x_2. May be implied by $\sqrt{1+0.5-e^{-2}}$ oe or awrt 1.17

A1: $x_2=\text{awrt } 1.168, x_3=\text{awrt } 1.220$ 3dp. Condone 1.22 for x_3

Mark these in the order given, the subscripts are not required and incorrect ones may be ignored.

Question Number	Scheme	Marks
5(a)	Either $k>13$ or $k=3$	B1
	Both $k>13$ $k=3$	B1
		(2)
(b)	Smaller solution: $2(5-x)+3=\frac{1}{2}x+10 \Rightarrow x=\frac{6}{5}$	M1 A1
	Larger solution: $-2(5-x)+3=\frac{1}{2}x+10 \Rightarrow x=\frac{34}{3}$	M1 A1
		(4)
(c)	$(6,12)$	B1B1
		(2)
		(8 marks)

(a)

B1: Either $k>13$ or $k=3$ Condone $k \ldots 13$ instead of $k>13$ for this mark only. Also condone $y \leftrightarrow k$

Do not accept $k \geq 3$ for B1

B1: Both $k>13$, $k=3$ with no other restrictions. Accept and / or / , between the two solutions

(b)

M1: An acceptable method of finding **the smaller intersection**. The initial equation must be of the correct form and it must lead to a value of *x*. For example $2(5-x)+3=\frac{1}{2}x+10 \Rightarrow x=...$ or $5-x=\left(\frac{1}{4}x+\frac{7}{2}\right)$

A1: For $x=\frac{6}{5}$ or equivalent such as 1.2   Ignore any reference to the y coordinate

M1: An acceptable method of finding **the larger intersection**. The initial equation must be of the correct form and it must lead to a value of *x*. For example $-2(5-x)+3=\frac{1}{2}x+10 \Rightarrow x=...$ or $5-x=-\left(\frac{1}{4}x+\frac{7}{2}\right)$

A1: For $x=\frac{34}{3}$ or equivalent such as $11.\dot{3}$ Ignore any reference to the y coordinate

If there are any extra solutions in addition to the correct two, then withhold the final A1 mark.

ISW if the candidate then refers back to the range in (a) and deletes a solution

……

Alt method by squaring

M1: $2|5-x|+3=\frac{1}{2}x+10 \Rightarrow 4(5-x)^2=\left(\frac{1}{2}x+7\right)^2$ oe. In the main scheme the equation must be correct of the correct form but in this case you may condone '2' not being squared

A1: Correct 3TQ. The = 0 may be implied by subsequent work. $\frac{15}{4}x^2-47x+51=0$ oe

M1: Solves using an appropriate method $15x^2-188x+204=0 \Rightarrow (5x-6)(3x-34)=0 \Rightarrow x=..$

A1: Both $x=\frac{6}{5}$ $x=\frac{34}{3}$ and no others.

…….

(c)

B1: Accept *p* = 6 **or** *q* = 12. Allow in coordinates as *x* = 6 **or** *y* = 12.

B1: For both *p* = 6 **and** *q* = 12. Allow in coordinates as *x* = 6 **and** *y* = 12

Allow embedded within a single coordinate $(6,12)$. So for example $(2,12)$ is scored B1 B0

Question Number	Scheme	Marks
6(i)	$\dfrac{\tan 2x+\tan 32°}{1-\tan 2x\tan 32°}=5 \Rightarrow \tan(2x+32°)=5$	B1
	$\Rightarrow x=\dfrac{\arctan 5-32°}{2}$	M1
	$\Rightarrow x=\text{awrt } 23.35°, -66.65°$	A1A1
		(4)
(ii)(a)	$\tan(3\theta-45°)=\dfrac{\tan 3\theta-\tan 45°}{1+\tan 45°\tan 3\theta}=\dfrac{\tan 3\theta-1}{1+\tan 3\theta}$	M1A1*
		(2)
(b)	$(1+\tan 3\theta)\tan(\theta+28°)=\tan 3\theta-1$	
	$\Rightarrow \tan(\theta+28°)=\tan(3\theta-45°)$	B1
	$\theta+28°=3\theta-45° \Rightarrow \theta=36.5°$	M1A1

Question Number	Scheme	Marks
	$\theta+28°+180°=3\theta-45° \Rightarrow \theta=126.5°$	dM1A1 **(5)** **(11 marks)**
6(i) ALT 1	$\dfrac{\tan 2x+\tan 32°}{1-\tan 2x\tan 32°}=5 \Rightarrow \tan 2x=\dfrac{5-\tan 32°}{1+5\tan 32°}=awrt1.06$	B1
	$\Rightarrow x=\dfrac{\arctan\left(\dfrac{5-\tan 32°}{1+5\tan 32°}\right)}{2}$	M1
	$\Rightarrow x=23.35°,-66.65°$	A1A1 **(4)**
6(ii) ALT 2	$\dfrac{\tan 2x+\tan 32°}{1-\tan 2x\tan 32°}=5 \Rightarrow \dfrac{2\tan x}{1-\tan^2 x}+\tan 32°=5-5\times\dfrac{2\tan x}{1-\tan^2 x}\tan 32°$	
	$\Rightarrow (5-\tan 32°)\tan^2 x+(2+10\tan 32°)\tan x+\tan 32°-5=0$	
	OR $\Rightarrow awrt\ 4.38\tan^2 x+8.25\tan x-4.38=0$	B1
	Quadratic formula $\Rightarrow \tan x=0.4316,-2.3169 \Rightarrow x=..$	M1
	$\Rightarrow x=23.35°,-66.65°$	A1 A1 **(4)**

(i)

B1: Stating or implying by subsequent work $\tan(2x+32°)=5$

M1: Scored for the correct order of operations from $\tan(2x\pm 32°)=5$ to x = .. $x=\dfrac{\arctan 5\pm 32°}{2}$

This may be implied by one correct answer

A1: One of awrt $x=23.3/23.4°,-66.6/-66.7°$ **One dp accuracy required for this penultimate mark.**

A1: Both of $x=\text{awrt } 23.35°,-66.65°$ and no other solutions in the range $-90°<x<90°$

Using Alt I

B1: $\tan 2x=\text{awrt}1.06$

M1: For attempting to make $\tan 2x$ the subject followed by correct inverse operations to find a value for *x* from their $\tan 2x = k$

If they write down $\tan(2x+32°) = 5$ and then the answers that is fine for all 4 marks.

Answers mixing degrees and radians can only score the first B1

(ii)(a)

M1: States or implies (just rhs) $\tan(3\theta - 45°) = \dfrac{\tan 3\theta \pm \tan 45°}{1 \pm \tan 45° \tan 3\theta}$

A1*: Complete proof with the lhs, the correct identity $\dfrac{\tan 3\theta - \tan 45°}{1 + \tan 45° \tan 3\theta}$ and either stating that $\tan 45° = 1$ or substituting $\tan 45° = 1$ (which may only be seen on the numerator) and proceeding to given answer.

It is possible to work backwards here $\dfrac{\tan 3\theta - 1}{1 + \tan 3\theta} = \dfrac{\tan 3\theta - \tan 45°}{1 + \tan 45° \tan 3\theta} = \tan(3\theta - 45°)$ with M1 A1 scored at the end. Do not allow the final A1* if there are errors.

(ii)(b)

B1: Uses (ii)(a) to state or imply that $\tan(\theta + 28°) = \tan(3\theta - 45°)$

Allow this mark for $(1 + \tan 3\theta)\tan(\theta + 28°) = (1 + \tan 3\theta)\tan(3\theta - 45°)$

M1: $\theta + 28° = 3\theta - 45° \Rightarrow \theta = ..$

We have seen two incorrect methods that should be given M0.

$\tan(\theta + 28°) = \tan(3\theta - 45°) \Rightarrow \tan(3\theta - 45°) - \tan(\theta + 28°) = 0 \Rightarrow (3\theta - 45°) - (\theta + 28°) = 0 \Rightarrow \theta = ...$

and $\tan 3\theta - \tan 45° = \tan\theta + \tan 28° \Rightarrow 3\theta - 45° = \theta + 28° \Rightarrow \theta = ...$

A1: $\theta = 36.5°$ oe such as $\dfrac{73}{2}$

dM1: A correct method of finding a 2nd solution $\theta + 28° + 180° = 3\theta - 45° \Rightarrow \theta = ..$ The previous M must have been awarded. The method may be implied by their $\theta_1 + 90°$ but only if the previous M was scored.

It is an incorrect method to substitute the acute angle into one side of $\tan(\theta + 28°) = \tan(3\theta - 45°)$

Eg. $\tan(36.5 + 28°) = \tan(3\theta - 45°)$ and use trig to find another solution.

A1: $\theta = 36.5°, 126.5°$ oe and no other solutions in the range.

The questions states 'hence' so the minimum expected working is $\tan(\theta + 28°) = \tan(3\theta - 45°)$. Full marks can be awarded when this point is reached.

(ii) (b) Alternative solution using compound angles.

……….

(ii) (b) Alternative solution using compound angles.

From the B1 mark, $\tan(\theta+28°)=\tan(3\theta-45°)$ they proceed to

$$\frac{\sin(\theta+28°)}{\cos(\theta+28°)}=\frac{\sin(3\theta-45°)}{\cos(3\theta-45°)}\Rightarrow \sin\left((3\theta-45°)-(\theta+28°)\right)=0$$ via the compound angle identity

So, M1 is gained for an attempt at one value for $\sin(2\theta-73°)=0$, condoning slips andA1 for $\theta=36.5°$

……

Question Number	Scheme	Marks
7.(a)	Applies $\frac{vu'-uv'}{v^2}$ to $y=\frac{\ln(x^2+1)}{x^2+1}$ with $u=\ln(x^2+1)$ and $v=x^2+1$	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x^2+1)\times\frac{2x}{x^2+1}-2x\ln(x^2+1)}{(x^2+1)^2}$	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2x-2x\ln(x^2+1)}{(x^2+1)^2}$	A1
		(3)
(b)	Sets $2x-2x\ln(x^2+1)=0$	M1
	$2x\left(1-\ln(x^2+1)\right)=0\Rightarrow x=\pm\sqrt{e-1},$	M1,A1
	Sub $x=\pm\sqrt{e-1},\,0$ into $f(x)=\frac{\ln(x^2+1)}{x^2+1}$	dM1
	Stationary points $\left(\sqrt{e-1},\frac{1}{e}\right),\left(-\sqrt{e-1},\frac{1}{e}\right),\underline{\underline{(0,0)}}$	A1 $\underline{\underline{\text{B1}}}$
		(6)
		(9 marks)

(a)

M1: Attempts the quotient or product rule to achieve an expression in the correct form

Using the quotient rule achieves an expression of the form $\frac{dy}{dx} = \frac{(x^2+1)\times\frac{...}{x^2+1} - 2x\ln(x^2+1)}{(x^2+1)^2}$

or the form $\frac{dy}{dx} = \frac{... - 2x\ln(x^2+1)}{(x^2+1)^2}$ where $... = A$ or Ax

or using the product rule achieves and an expression $\frac{dy}{dx} = (x^2+1)^{-1}\times\frac{...}{x^2+1} - 2x(x^2+1)^{-2}\ln(x^2+1)$

You may condone the omission of brackets ....especially on the denominator

A1: A correct un-simplified expression for $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{(x^2+1)\times\frac{2x}{x^2+1} - 2x\ln(x^2+1)}{(x^2+1)^2} \text{ or } \frac{dy}{dx} = (x^2+1)^{-1}\times\frac{2x}{x^2+1} - 2x(x^2+1)^{-2}\ln(x^2+1)$$

A1: $\frac{dy}{dx} = \frac{2x - 2x\ln(x^2+1)}{(x^2+1)^2}$ or exact simplified equivalent such as $\frac{dy}{dx} = \frac{2x(1-\ln(x^2+1))}{(x^2+1)^2}$.

Condone $\frac{dy}{dx} = \frac{2x - \ln(x^2+1)2x}{(x^2+1)^2}$ which may be a little ambiguous. The lhs $\frac{dy}{dx} =$ does not need to be seen.

You may assume from the demand in the question that is what they are finding.

ISW can be applied here.

(b)

M1: Sets the numerator of their $\frac{dy}{dx}$, which must contain at least two terms, equal to 0

M1: For solving an equation of the form $\ln(x^2+1) = k, \quad k > 0$ to get at least one non- zero value of *x*. Accept decimal answers. $x = awrt \pm 1.31$ The equation must be legitimately obtained from a numerator = 0

A1: Both $x = \pm\sqrt{e-1}$ scored from ± a correct numerator Condone $x = \pm\sqrt{e^1 - 1}$

dM1: Substitutes any of their non zero solutions to $\frac{dy}{dx} = 0$ into $f(x) = \frac{\ln(x^2+1)}{x^2+1}$ to find at least one '*y*' value. It is dependent upon both previous M's

A1: Both $\left(\sqrt{e-1}, \frac{1}{e}\right), \left(-\sqrt{e-1}, \frac{1}{e}\right)$ oe or the equivalent with $x = ..., y = ...$ ln e must be simplified

Condone $\left(\sqrt{e^1-1},\frac{1}{e^1}\right),\left(-\sqrt{e^1-1},\frac{1}{e^1}\right)$ but the *y* coordinates must be simplified as shown.

Condone $\left(\pm\sqrt{e-1},\frac{1}{e}\right)$ Withhold this mark if there are extra solutions to these apart from (0,0)

It can only be awarded from ± a correct numerator

B1 : $(0,0)$ or the equivalent $x=0, y=0$

Notes:

(1) A candidate can "recover" and score all marks in (b) when they have an incorrect denominator in part (a) or a numerator the wrong way around in (a)

(2) A candidate who differentiates $\ln\left(x^2+1\right)\rightarrow\frac{1}{x^2+1}$ will probably only score (a) 100 (b) 100000

(3) A candidate who has $\frac{vu'+uv'}{v^2}$ cannot score anything more than (a) 000 (b)100001 as they would have $k<0$

(4) A candidate who attempts the product rule to get $\frac{dy}{dx}=\left(x^2+1\right)^{-1}\times\frac{1}{x^2+1}-\left(x^2+1\right)^{-2}\ln\left(x^2+1\right)=\frac{1-\ln\left(x^2+1\right)}{\left(x^2+1\right)^2}$

can score (a) 000 (b) 110100 even though they may obtain the correct non zero coordinates.

Question Number	Scheme	Marks
8(a)	$\frac{d}{d\theta}(\sec\theta)=\frac{d}{d\theta}(\cos\theta)^{-1}=-1\times(\cos\theta)^{-2}\times-\sin\theta$	M1
	$=\frac{1}{\cos\theta}\times\frac{\sin\theta}{\cos\theta}$	

Question Number	Scheme	Marks
	$=\sec\theta\tan\theta$	A1*
		(2)
(b)	$x=\mathrm{e}^{\sec y}\Rightarrow\dfrac{\mathrm{d}x}{\mathrm{d}y}=\mathrm{e}^{\sec y}\times\sec y\tan y$ oe	M1A1
	$\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{\mathrm{e}^{\sec y}\times\sec y\tan y}$	M1
	Uses $1+\tan^2 y=\sec^2 y$ with $\sec y=\ln x\Rightarrow\tan y=\sqrt{(\ln x)^2-1}$	M1
	$\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{x\times\ln x\times\sqrt{(\ln x)^2-1}}=\dfrac{1}{x\sqrt{(\ln x)^4-(\ln x)^2}}$ oe	A1
		(5)
		(7 marks)
Alt (b)	$\ln x=\sec y\Rightarrow\dfrac{1}{x}\dfrac{\mathrm{d}x}{\mathrm{d}y}=\sec y\tan y$	**M1A1**
	$\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{x\times\sec y\tan y}$	**M1**
	Uses $1+\tan^2 y=\sec^2 y$ and $\sec y=\ln x\Rightarrow\tan y=\sqrt{(\ln x)^2-1}$	**M1**
	$\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{x\times\ln x\times\sqrt{(\ln x)^2-1}}=\dfrac{1}{x\sqrt{(\ln x)^4-(\ln x)^2}}$ oe	**A1**
		(5)

(a)

M1: Uses the chain rule to get $\pm 1\times(\cos\theta)^{-2}\times\sin\theta$

Alternatively uses the quotient rule to get $\dfrac{\cos\theta\times 0\pm 1\times\sin\theta}{\cos^2\theta}$ condoning the denominator as $\cos\theta^2$

When applying the quotient rule it is very difficult to see if the correct rule has been used. So only withhold this mark if an incorrect rule is quoted.

A1*: Completes proof with no errors (see below *) and shows line $\dfrac{1}{\cos\theta}\times\dfrac{\sin\theta}{\cos\theta}$, $\dfrac{\tan\theta}{\cos\theta}$ or $\dfrac{\sin\theta}{\cos\theta\times\cos\theta}$ before the given answer. The notation should be correct so do not allow if they start $y=\sec\theta\Rightarrow\dfrac{dy}{dx}=\sec\theta\tan\theta$

* You do not need to see $\dfrac{d}{d\theta}(\sec\theta)=...$ or $\dfrac{dy}{d\theta}$ anywhere in the solution

(b)

M1 Differentiates to get the rhs as $e^{\sec y}\times...$

A1 Completely correct differential inc the lhs $\dfrac{dx}{dy}=e^{\sec y}\times\sec y\tan y$

M1 Inverts **their** $\dfrac{dx}{dy}$ to get $\dfrac{dy}{dx}$.

The variable used **must be** consistent. Eg $\dfrac{dx}{dy}=e^{\sec y}\Rightarrow\dfrac{dy}{dx}=\dfrac{1}{e^{\sec x}}$ is M0

M1 For attempting to use $1+\tan^2 y=\sec^2 y$ with $\sec y=\ln x$

(You may condone $\ln x^2 \to 2\ln x$ for the method mark)

It may be implied by $\tan y = \sqrt{\pm(\ln x)^2 \pm 1}$ They must have a term in $\tan y$ to score this.

A valid alternative would be attempting to use $1+\cot^2 y = \text{cosec}^2 y$ with $\text{cosec} y = \dfrac{1}{\sqrt{1-\frac{1}{\ln^2 x}}}$ oe

A1 $\dfrac{dy}{dx} = \dfrac{1}{x\sqrt{(\ln x)^4 - (\ln x)^2}}$ or exact equivalents such as $\dfrac{dy}{dx} = \dfrac{1}{x\sqrt{\ln^4 x - \ln^2 x}}$

Do not isw here. Withhold this mark if candidate then writes down $\dfrac{dy}{dx} = \dfrac{1}{x\sqrt{4(\ln x) - 2(\ln x)}}$

Also watch for candidates who write $\dfrac{dy}{dx} = \dfrac{1}{x\sqrt{\ln x^4 - \ln x^2}}$ which is incorrect (without the brackets)

Question Number	Scheme	Marks
9.(a)	$R = \sqrt{5}$	B1
	$\tan\alpha = 2 \Rightarrow \alpha = \text{awrt } 1.107$	M1A1
		(3)
(b)(i)	$'40 + 9R^2' = 85$	M1A1
(ii)	$\theta = \dfrac{\pi}{2} + 1.107 \Rightarrow \theta = \text{awrt } 2.68$	B1ft
		(3)
(c)(i)	6	B1

Question Number	Scheme	Marks
(ii)	$2\theta - '1.107' = 3\pi \Rightarrow \theta = \text{awrt } 5.27$	M1A1 (3) **(9 marks)**

(a)

B1: Accept $R = \sqrt{5}$ **Do not accept** $R = \pm\sqrt{5}$

M1: For sight of $\tan\alpha = \pm 2, \tan\alpha = \pm\frac{1}{2}$. Condone $\sin\alpha = 2, \cos\alpha = 1 \Rightarrow \tan\alpha = \frac{2}{1}$

If *R* is found first, accept $\sin\alpha = \pm\frac{2}{R}, \cos\alpha = \pm\frac{1}{R}$

A1: $\alpha = \text{awrt } 1.107$. The degrees equivalent $63.4°$ is A0.

(b)(i)

M1: Attempts $'40 + 9R^2'$ OR $'40 + 3R^2'$ using their *R*.

Can be scored for sight of the statement $'40 + 9R^2'$

It can be done via calculus. The M mark will probably be awarded when $\left("\alpha" - \frac{\pi}{2}\right) = -0.464$ is substituted into $M(\theta)$

A1: 85 exactly. Without any method this scores both marks. Do not accept awrt 85.

(b)(ii)

B1ft: For awrt 2.68 or $\left(\frac{\pi}{2} + "\alpha"\right)$ A simple way would be to add 1.57 to their α to 2dp

Accept awrt $153.4°$ for candidates who work in degrees. Follow through in degrees on $90° + '\alpha'$

(c)(i)

B1: 6

(c)(ii)

M1: Using $2\theta \pm '1.107' = n\pi$ where *n* is a positive integer leading to a value for θ

In degrees for $2\theta \pm \text{their}'63.43' = 180n$ where *n* is a positive integer leading to a value for θ

Another alternative is to solve $\tan 2\theta = 2$ so score for $\frac{180n + \arctan 2}{2}$ or $\frac{\pi n + \arctan 2}{2}$

A1: $\theta =$ awrt 5.27 or if candidate works in degrees awrt $301.7°$

Question Number	Scheme	Notes	Marks
1. (a)	$\sqrt{(4-9x)} = (4-9x)^{\frac{1}{2}} = \underline{(4)^{\frac{1}{2}}}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}} = \underline{2}\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$	$\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$	<u>B1</u>
	$= \{2\}\left[1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2+\ldots\right]$	see notes	M1 A1ft
	$= \{2\}\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{9x}{4}\right)^2+\ldots\right]$		
	$= 2\left[1-\frac{9}{8}x-\frac{81}{128}x^2+\ldots\right]$	see notes	
	$= 2-\frac{9}{4}x; -\frac{81}{64}x^2+\ldots$	isw	A1; A1
			[5]
(b)	$\sqrt{310} = 10\sqrt{3.1} = 10\sqrt{(4-9(0.1))}$, so $x = 0.1$	E.g. For $10\sqrt{3.1}$ (can be implied by later working) **and** $x = 0.1$ (or uses $x = 0.1$) **Note:** $\sqrt{(100)(3.1)}$ by itself is B0	B1
	When $x = 0.1$ $\sqrt{(4-9x)} \approx 2-\frac{9}{4}(0.1) - \frac{81}{64}(0.1)^2+\ldots$	Substitutes their x, where $\lvert x\rvert < \frac{4}{9}$ into all three terms of their binomial expansion	M1
	$= 2-0.225-0.01265625 = 1.76234375$		
	So, $\sqrt{310} \approx 17.6234375 = \underline{17.623}$ (3 dp)	17.623 **cao**	A1 **cao**
	Note: the calculator value of $\sqrt{310}$ is 17.60681686... which is 17.607 to 3 decimal places		**[3]**
			8 marks

Question 1 Notes

1. (a)	**B1**	$\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$ outside brackets or $\underline{2}$ as candidate's constant term in their binomial expansion
	M1	Expands $(\ldots+kx)^{\frac{1}{2}}$ to give any 2 terms out of 3 terms simplified or un-simplified, E.g. $1+\left(\frac{1}{2}\right)(kx)$ or $\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ or $1+\ldots+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ where k is a numerical value and **where** $k \neq 1$

	A1ft	A correct simplified or un-simplified $1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2$ expansion with **consistent** (kx)
	Note	(kx), $k \neq 1$ must be consistent (on the RHS, not necessarily on the LHS) in their expansion
	Note	Award B1M1A0 for $2\left[1+\left(\frac{1}{2}\right)(-9x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{9x}{4}\right)^2+\ldots\right]$ because (kx) is not consistent
	Note	**<u>Incorrect bracketing:</u>** $2\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(-\frac{9x^2}{4}\right)+\ldots\right]$ is B1M1A0 unless recovered
	A1	$2-\frac{9}{4}x$ **(simplified fractions) or** allow $2-2.25x$ or $2-2\frac{1}{4}x$
	A1	Accept only $-\frac{81}{64}x^2$ or $-1\frac{17}{64}x^2$ or $-1.265625x^2$

	Question 1 Notes Continued	
1. (a) ctd.	**SC**	If a candidate ***would otherwise score*** 2nd A0, 3rd A0 (i.e. scores A0A0 in the final two marks to (a)) then **allow Special Case 2nd A1 for either** **SC:** $2\left[1-\frac{9}{8}x;\ldots\right]$ or **SC:** $2\left[1+\ldots-\frac{81}{128}x^2+\ldots\right]$ or **SC:** $\lambda\left[1-\frac{9}{8}x-\frac{81}{128}x^2+\ldots\right]$ or **SC:** $\left[\lambda-\frac{9\lambda}{8}x-\frac{81\lambda}{128}x^2+\ldots\right]$ (where λ can be 1 or omitted), where each term in the $[.....]$ is a simplified fraction or a decimal, **OR SC:** for $2-\frac{18}{8}x-\frac{162}{128}x^2+\ldots$ (i.e. for not simplifying their correct coefficients)
	Note	Candidates who write $2\left[1+\left(\frac{1}{2}\right)\left(\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{9x}{4}\right)^2+\ldots\right]$, where $k=\frac{9}{4}$ and not $-\frac{9}{4}$ and achieve $2+\frac{9}{4}x;-\frac{81}{64}x^2+\ldots$ will get B1M1A1A0A1
	Note	**Ignore** extra terms beyond the term in x^2
	Note	You can ignore subsequent working following a correct answer

	Note	Allow B1M1A1 for $2\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{9x}{4}\right)^2+\ldots\right]$
	Note	Allow B1M1A1A1A1 for $2\left[1+\left(\frac{1}{2}\right)\left(-\frac{9x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{9x}{4}\right)^2+\ldots\right] = 2-\frac{9}{4}x-\frac{81}{64}x^2+\ldots$
(b)	**Note**	Give B1 M1 for $\sqrt{310}\approx 10\left(2-\frac{9}{4}(0.1)-\frac{81}{64}(0.1)^2\right)$
	Note	**<u>Other alternative suitable values for *x* for</u>** $\sqrt{310}\approx\beta\sqrt{4-9(\text{their } x)}$

β	*x*	**Estimate**	β	*x*	**Estimate**
7	$-\frac{38}{147}$	17.479	14	$\frac{79}{294}$	18.256
8	$-\frac{3}{32}$	17.599	15	$\frac{118}{405}$	18.555
9	$\frac{14}{729}$	17.607	16	$\frac{119}{384}$	18.899
10	$\frac{1}{10}$	**17.623**	17	$\frac{94}{289}$	19.283
11	$\frac{58}{363}$	17.690	18	$\frac{493}{1458}$	19.701
12	$\frac{133}{648}$	17.819	19	$\frac{126}{361}$	20.150
13	$\frac{122}{507}$	18.009	20	$\frac{43}{120}$	20.625

	Note	Apply the scheme in the same way for their β and their *x* E.g. Give B1 M1 A1 for $\sqrt{310}\approx 12\left(2-\frac{9}{4}\left(\frac{133}{648}\right)-\frac{81}{64}\left(\frac{133}{648}\right)^2\right) = 17.819$ (3 dp)
	Note	Allow B1 M1 A1 for $\sqrt{310}\approx 100\left(2-\frac{9}{4}(0.441)-\frac{81}{64}(0.441)^2\right) = 76.161$ (3 dp)
	Note	Give B1 M1 A0 for $\sqrt{310}\approx 10\left(2-\frac{9}{4}(0.1)-\frac{81}{64}(0.1)^2-\frac{729}{512}(0.1)^3\right) = 17.609$ (3 dp)

	Question 1 Notes Continued	
1. (b)	**Note**	***Send to review*** using $\beta = \sqrt{155}$ and $x = \frac{2}{9}$ (which gives 17.897 (3 dp))
	Note	***Send to review*** using $\beta = \sqrt{1000}$ and $x = 0.41$ (which gives 27.346 (3 dp))

1. (a) **Alt 1**	**Alternative method 1:** Candidates can apply an alternative form of the binomial expansion $\left\{(4-9x)^{\frac{1}{2}}\right\} = (4)^{\frac{1}{2}} + \left(\tfrac{1}{2}\right)(4)^{-\frac{1}{2}}(-9x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(4)^{-\frac{3}{2}}(-9x)^2$	
	B1	$(4)^{\frac{1}{2}}$ or 2
	M1	Any two of three (un-simplified) terms correct
	A1	All three (un-simplified) terms correct
	A1	$2 - \frac{9}{4}x$ **(simplified fractions)** or allow $2 - 2.25x$ or $2 - 2\frac{1}{4}x$
	A1	Accept only $-\frac{81}{64}x^2$ or $-1\frac{17}{64}x^2$ or $-1.265625x^2$
	Note	The terms in C need to be evaluated. So ${}^{\frac{1}{2}}C_0(4)^{\frac{1}{2}} + {}^{\frac{1}{2}}C_1(4)^{-\frac{1}{2}}(-9x); + {}^{\frac{1}{2}}C_2(4)^{-\frac{3}{2}}(-9x)^2$ without further working is B0M0A0

1. (a)	**Alternative Method 2: Maclaurin Expansion** $\text{f}(x) = (4-9x)^{\frac{1}{2}}$		
	$\text{f}''(x) = -\frac{81}{4}(4-9x)^{-\frac{3}{2}}$	Correct $\text{f}''(x)$	B1
	$\text{f}'(x) = \frac{1}{2}(4-9x)^{-\frac{1}{2}}(-9)$	$\pm a(4-9x)^{-\frac{1}{2}}$; $a \neq \pm 1$	M1
		$\frac{1}{2}(4-9x)^{-\frac{1}{2}}(-9)$	A1 oe
	$\left\{\therefore \text{f}(0) = 2\ , \text{f}'(0) = -\frac{9}{4} \text{ and } \text{f}''(0) = -\frac{81}{32}\right\}$		
	So, $\text{f}(x) = 2 - \frac{9}{4}x; - \frac{81}{64}x^2 + \ldots$		A1; A1

Question Number	Scheme	Notes	Marks
2.	$x^2 + xy + y^2 - 4x - 5y + 1 = 0$		
(a)	$\left\{\frac{dy}{dx} \times\right\}$ $\underline{2x} + \left(\underline{\underline{y + x\frac{dy}{dx}}}\right) + \underline{2y\frac{dy}{dx} - 4 - 5\frac{dy}{dx}} = \underline{0}$		M1A1 B1
	$2x + y - 4 + (x + 2y - 5)\frac{dy}{dx} = 0$		dM1
	$\frac{dy}{dx} = \frac{2x + y - 4}{5 - x - 2y}$ or $\frac{4 - 2x - y}{x + 2y - 5}$	**o.e.**	A1 **cso**
			[5]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}$ $2x + y - 4 = 0$		M1
	$\{y = 4 - 2x \Rightarrow\}$ $x^2 + x(4 - 2x) + (4 - 2x)^2 - 4x - 5(4 - 2x) + 1 = 0$		dM1
	$x^2 + 4x - 2x^2 + 16 - 16x + 4x^2 - 4x - 20 + 10x + 1 = 0$		
	gives $3x^2 - 6x - 3 = 0$ **or** $3x^2 - 6x = 3$ **or** $x^2 - 2x - 1 = 0$	Correct 3TQ in terms of x	A1
	$(x - 1)^2 - 1 - 1 = 0$ and $x = ...$	Method mark for solving a 3TQ in x	ddM1
	$x = 1 + \sqrt{2},\ 1 - \sqrt{2}$	$x = 1 + \sqrt{2},\ 1 - \sqrt{2}$ only	A1
			[5]
(b) **Alt 1**	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}$ $2x + y - 4 = 0$		M1
	$\left\{x = \frac{4 - y}{2} \Rightarrow\right\}$ $\left(\frac{4 - y}{2}\right)^2 + \left(\frac{4 - y}{2}\right)y + y^2 - 4\left(\frac{4 - y}{2}\right) - 5y + 1 = 0$		dM1
	$\left(\frac{16 - 8y + y^2}{2}\right) + \left(\frac{4y - y^2}{2}\right) + y^2 - 2(4 - y) - 5y + 1 = 0$		
	gives $3y^2 - 12y - 12 = 0$ **or** $3y^2 - 12y = 12$ **or** $y^2 - 4y - 4 = 0$	Correct 3TQ in terms of y	A1
	$(y - 2)^2 - 4 - 4 = 0$ and $y = ...$ $x = \frac{4 - (2 + 2\sqrt{2})}{2}$, $x = \frac{4 - (2 - 2\sqrt{2})}{2}$	Solves a 3TQ in y **and finds at least one value for x**	ddM1
	$x = 1 + \sqrt{2},\ 1 - \sqrt{2}$	$x = 1 + \sqrt{2},\ 1 - \sqrt{2}$ only	A1
			[5]

			10
(a) **Alt 1**	$\left\{\frac{\cancel{dx}}{\cancel{dy}}\cancel{\times}\right\}$ $\underline{2x\frac{dx}{dy}} + \left(y\frac{dx}{dy} + x\right) \underline{+ 2y - 4\frac{dx}{dy} - 5} = \underline{0}$		M1A1 B1
	$x + 2y - 5 + (2x + y - 4)\frac{dx}{dy} = 0$		dM1
	$\frac{dy}{dx} = \frac{2x + y - 4}{5 - x - 2y}$ or $\frac{4 - 2x - y}{x + 2y - 5}$	**o.e.**	A1 **cso**
			[5]

	Question 2 Notes	
2. (a)	**M1**	Differentiates implicitly to include either $x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $y^2 \to 2y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $-5y \to -5\frac{\mathrm{d}y}{\mathrm{d}x}$. $\left(\text{Ignore } \frac{\mathrm{d}y}{\mathrm{d}x} = ...\right)$
	A1	$x^2 \to 2x$ **and** $y^2 - 4x - 5y + 1 = 0 \to 2y\frac{\mathrm{d}y}{\mathrm{d}x} - 4 - 5\frac{\mathrm{d}y}{\mathrm{d}x} = 0$
	B1	$xy \to y + x\frac{\mathrm{d}y}{\mathrm{d}x}$
	Note	If an extra term appears then award 1st A0
	Note	$2x + y + x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\frac{\mathrm{d}y}{\mathrm{d}x} - 4 - 5\frac{\mathrm{d}y}{\mathrm{d}x} \to 2x + y - 4 = -x\frac{\mathrm{d}y}{\mathrm{d}x} - 2y\frac{\mathrm{d}y}{\mathrm{d}x} + 5\frac{\mathrm{d}y}{\mathrm{d}x}$ will get 1st A1 (implied) as the $"=0"$ can be implied the rearrangement of their equation.
	dM1	**dependent on the previous M mark** An attempt to factorise out **all the terms in** $\frac{\mathrm{d}y}{\mathrm{d}x}$ as long as there are **at least two terms** in $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	A1 **cso**	$\frac{2x + y - 4}{5 - x - 2y}$ or $\frac{4 - 2x - y}{x + 2y - 5}$ If the candidate's solution is not completely correct, then do not give the final A mark
(b)	**M1**	Sets the numerator of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero (or the denominator of their $\frac{\mathrm{d}x}{\mathrm{d}y}$ equal to zero) o.e.
	Note	This mark can also be gained by setting $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero in their differentiated equation from (a)
	Note	**If the numerator involves one variable only then *only* the 1st M1 mark is possible in part (b).**
	dM1	**dependent on the previous M mark** Substitutes their x or their y (from their numerator = 0) into the printed equation to give an equation in one variable only
	A1 **Note**	For obtaining the correct 3TQ. E.g.: either $3x^2 - 6x - 3\ \{= 0\}$ or $-3x^2 + 6x + 3\ \{= 0\}$ This mark can also be awarded for a correct 3 term equation. E.g. either $3x^2 - 6x = 3$ $x^2 - 2x - 1 = 0$ or $x^2 = 2x + 1$ are all fine for A1
	ddM1	**dependent on the previous 2 M marks** See page 6: Method mark for solving THEIR 3-term quadratic in one variable

		<u>Quadratic Equation to solve:</u> $3x^2 - 6x - 3 = 0$ **<u>Way 1:</u>** $x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(3)(-3)}}{2(3)}$ **<u>Way 2:</u>** $x^2 - 2x - 1 = 0 \Rightarrow (x-1)^2 - 1 - 1 = 0 \Rightarrow x = ...$ **<u>Way 3:</u>** Or writes down at least one ***exact*** correct x-root **(*or one correct x-root to 2 dp*)** from ***their*** quadratic equation. This is usually found on their calculator. **<u>Way 4:</u> (Only allowed if their 3TQ can be factorised)** • $(x^2 + bx + c) = (x+p)(x+q)$, where $\lvert pq \rvert = \lvert c \rvert$, leading to $x = ...$ • $(ax^2 + bx + c) = (mx+p)(nx+q)$, where $\lvert pq \rvert = \lvert c \rvert$ and $\lvert mn \rvert = a$, leading to $x = ...$
	Note	If a candidate applies ***the alternative method*** then they also need to use their $x = \dfrac{4-y}{2}$ to find **at least one value** for x in order to gain the final M mark.
	A1	Exact values of $x = 1 + \sqrt{2},\ 1 - \sqrt{2}$ (or $1 \pm \sqrt{2}$) , **cao** Apply isw if y-values are also found.
	Note	It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for $\dfrac{dy}{dx}$) to gain all 5 marks in part (b)

	Question 2 Notes	
2. (a) **Alt 1**	**M1**	Differentiates implicitly to include either $y\frac{\mathrm{d}x}{\mathrm{d}y}$ or $x^2 \to 2x\frac{\mathrm{d}x}{\mathrm{d}y}$ or $-4x \to -4\frac{\mathrm{d}x}{\mathrm{d}y}$. $\left(\text{Ignore } \frac{\mathrm{d}x}{\mathrm{d}y} = ...\right)$
	A1	$x^2 \to 2x\frac{\mathrm{d}x}{\mathrm{d}y}$ **and** $y^2 - 4x - 5y + 1 = 0 \to 2y - 4\frac{\mathrm{d}x}{\mathrm{d}y} - 5 = 0$
	B1	$xy \to y\frac{\mathrm{d}x}{\mathrm{d}y} + x$
	Note	If an extra term appears then award 1st A0
	Note	$2x\frac{\mathrm{d}x}{\mathrm{d}y} + y\frac{\mathrm{d}x}{\mathrm{d}y} + x + 2y - 4\frac{\mathrm{d}x}{\mathrm{d}y} - 5 \;\to\; x + 2y - 5 = -2x\frac{\mathrm{d}x}{\mathrm{d}y} - y\frac{\mathrm{d}x}{\mathrm{d}y} + 4\frac{\mathrm{d}x}{\mathrm{d}y}$ will get 1st A1 (implied) as the $"=0"$ can be implied the rearrangement of their equation.
	dM1	**dependent on the previous M mark** An attempt to factorise out **all the terms in** $\frac{\mathrm{d}x}{\mathrm{d}y}$ as long as there are **at least two terms** in $\frac{\mathrm{d}x}{\mathrm{d}y}$
	A1 **cso**	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + y - 4}{5 - x - 2y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4 - 2x - y}{x + 2y - 5}$ If the candidate's solution is not completely correct, then do not give the final A mark
(a)	**Note**	Writing down ***from no working*** • $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + y - 4}{5 - x - 2y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4 - 2x - y}{x + 2y - 5}$ scores M1 A1 B1 M1 A1 • $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4 - 2x - y}{5 - x - 2y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + y - 4}{x + 2y - 5}$ scores M1 A0 B1 M1 A0
	Note	Writing $2x\,\mathrm{d}x + y\,\mathrm{d}x + x\,\mathrm{d}y + 2y\,\mathrm{d}y - 4\mathrm{d}x - 5\mathrm{d}y = 0$ scores M1 A1 B1

Question Number	Scheme	Notes	Marks
3. (i)	$\frac{13-4x}{(2x+1)^2(x+3)} \equiv \frac{A}{(2x+1)} + \frac{B}{(2x+1)^2} + \frac{C}{(x+3)}$		
(a)	$B = 6,\ C = 1$	At least one of $B = 6$ or $C = 1$	B1
		Both $B = 6$ and $C = 1$	B1
	$13 - 4x \equiv A(2x+1)(x+3) + B(x+3) + C(2x+1)^2$ $x = -3 \Rightarrow 25 = 25C \Rightarrow C = 1$ $x = -\frac{1}{2} \Rightarrow 13 - -2 = \frac{5}{2}B \Rightarrow 15 = 2.5B \Rightarrow B = 6$	Writes down a correct identity and attempts to find the value of either one of A or B or C	M1
	Either $x^2: 0 = 2A + 4C$, constant: $13 = 3A + 3B + C$, $x: -4 = 7A + B + 4C$ or $x = 0 \Rightarrow 13 = 3A + 3B + C$ leading to $A = -2$	Using a correct identity to find $A = -2$	A1
			[4]
(b)	$\int \frac{13-4x}{(2x+1)^2(x+3)}\,dx = \int \frac{-2}{(2x+1)} + \frac{6}{(2x+1)^2} + \frac{1}{(x+3)}\,dx$		
	$= \frac{(-2)}{2}\ln(2x+1) + \frac{6(2x+1)^{-1}}{(-1)(2)} + \ln(x+3)\ \{+c\}$ o.e. $\left\{= -\ln(2x+1) - 3(2x+1)^{-1} + \ln(x+3)\ \{+c\}\right\}$	See notes	M1
		At least two terms correctly integrated	A1ft
		Correct answer, o.e. Simplified or un-simplified. The correct answer must be stated on one line Ignore the absence of '+ c'	A1
			[3]
(ii)	$\left\{(e^x + 1)^3 =\right\} e^{3x} + 3e^{2x} + 3e^x + 1$	$e^{3x} + 3e^{2x} + 3e^x + 1$, simplified or un-simplified	B1
	$\left\{\int (e^x + 1)^3\,dx\right\} = \frac{1}{3}e^{3x} + \frac{3}{2}e^{2x} + 3e^x + x\ \{+c\}$	At least 3 examples (see notes) of correct ft integration	M1
		$\frac{1}{3}e^{3x} + \frac{3}{2}e^{2x} + 3e^x + x$, simplified or un-simplified with or without $+c$	A1
			[3]
(iii)	$\int \frac{1}{4x + 5x^{\frac{1}{3}}}\,dx,\ x > 0;\ u^3 = x$		
	$3u^2\frac{du}{dx} = 1$	$3u^2\frac{du}{dx} = 1$ or $\frac{dx}{du} = 3u^2$ or $\frac{du}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$	B1

		or $3u^2\mathrm{d}u = \mathrm{d}x$ o.e.	
	$= \int \frac{1}{4u^3 + 5u} .3u^2 \,\mathrm{d}u \;\left\{ = \int \frac{3u}{4u^2 + 5}\,\mathrm{d}u \right\}$	Expression of the form $\int \frac{\pm ku^2}{4u^3 \pm 5u}\{\mathrm{d}u\}$, $k \neq 0$ Does not have to include integral sign or $\mathrm{d}u$ Can be implied by later working	M1
	$= \frac{3}{8}\ln(4u^2 + 5)\ \{+ c\}$	**dependent on the previous M mark** $\pm\lambda \ln(4u^2 + 5)$; λ is a constant; $\lambda \neq 0$	dM1
	$= \frac{3}{8}\ln\left(4x^{\frac{2}{3}} + 5\right)\ \{+ c\}$	Correct answer in x with or without $+\ c$	A1
			[4]
			14

	Question 3 Notes		
3. (iii) **Alt 1**	**Alternative method 1 for part (iii)**		
	$\left\{\int \frac{1}{4x+5x^{\frac{1}{3}}}\,dx\right\} = \int \frac{x^{-\frac{1}{3}}}{4x^{\frac{2}{3}}+5}\,dx$	Attempts to multiply numerator and denominator by $x^{-\frac{1}{3}}$	M1
		Expression of the form $\int \frac{\pm kx^{-\frac{1}{3}}}{4x^{\frac{2}{3}} \pm 5}dx,\ k \neq 0$ Does not have to include integral sign or du Can be implied by later working	M1
	$= \frac{3}{8}\ln\left(4x^{\frac{2}{3}}+5\right)\ \{+c\}$	$\pm\lambda\ln(4x^{\frac{2}{3}}+5)$; λ is a constant; $\lambda \neq 0$	dM1
		Correct answer in x with or without $+c$	A1
			[4]

3. (i) (a)	**M1**	Writes down ***a correct identity*** (although this can be implied) and attempts ***to find the value of at least one*** of either A or B or C. This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients.
	Note	The correct partial fraction from no working scores B1B1M1A1
(i) (b)	**M1**	At least 2 of either $\pm\frac{P}{(2x+1)} \to \pm D\ln(2x+1)$ or $\pm D\ln(x+\frac{1}{2})$ or $\pm\frac{Q}{(2x+1)^2} \to \pm E(2x+1)^{-1}$ or $\pm\frac{R}{(x+3)} \to \pm F\ln(x+3)$ for their constants P, Q, R.
	A1ft	At least two terms from any of $\pm\frac{P}{(2x+1)}$ or $\pm\frac{Q}{(2x+1)^2}$ or $\pm\frac{R}{(x+3)}$ correctly integrated.
	Note	Can be un-simplified for the A1ft mark.
	A1	Correct answer of $\frac{(-2)}{2}\ln(2x+1) + \frac{6(2x+1)^{-1}}{(-1)(2)} + \ln(x+3)\ \{+c\}$ simplified or un-simplified. with or without '$+ c$'.
	Note	Allow final A1 for equivalent answers, e.g. $\ln\left(\frac{x+3}{2x+1}\right) - \frac{3}{2x+1}\ \{+c\}$ or $\ln\left(\frac{2x+6}{2x+1}\right) - \frac{3}{2x+1}\ \{+c\}$

	Note	**Beware that** $\int \frac{-2}{(2x+1)}\,dx = \int \frac{-1}{(x+\frac{1}{2})}\,dx = -\ln(x+\frac{1}{2})\ \{+\,c\}$ is correct integration
	Note	E.g. Allow M1 A1ft A1 for a correct un-simplified $\ln(x+3) - \ln(x+\frac{1}{2}) - \frac{3}{2}(x+\frac{1}{2})^{-1}\ \{+\,c\}$
	Note	Condone 1st A1ft for poor bracketing, but do not allow poor bracketing for the final A1 E.g. Give final A0 for $-\ln 2x+1 - 3(2x+1)^{-1} + \ln x+3\ \{+\,c\}$ unless recovered
(ii)	**Note**	Give B1 for an un-simplified $e^{3x} + 2e^{2x} + e^{2x} + 2e^{x} + e^{x} + 1$
	M1	At least 3 of either $\alpha e^{3x} \to \frac{\alpha}{3}e^{3x}$ **or** $\beta e^{2x} \to \frac{\beta}{2}e^{2x}$ **or** $\delta e^{x} \to \delta e^{x}$ **or** $\mu \to \mu x;\ \alpha, \beta, \delta, \mu \neq 0$
	Note	Give A1 for an un-simplified $\frac{1}{3}e^{3x} + e^{2x} + \frac{1}{2}e^{2x} + 2e^{x} + e^{x} + x$, with or without $+c$
(iii)	**Note**	1st M1 can be implied by $\int \frac{\pm ku}{4u^2 \pm 5}\{du\},\ k \neq 0$. Does not have to include integral sign or du
	Note	Condone 1st M1 for expressions of the form $\int \left(\frac{\pm 1}{4u^3 \pm 5u}.\frac{\pm k}{u^{-2}}\right)\{du\},\ k \neq 0$
	Note	Give 2nd M0 for $\frac{3u}{8u}\ln(4u^2+5)\ \{+\,c\}$ (u's not cancelled) unless recovered in later working
	Note	E.g. Give 2nd M0 for integration leading to $\frac{3}{4}u\ln(4u^2+5)$ as this is not in the form $\pm\lambda\ln(4u^2+5)$
	Note	Condone 2nd M1 for poor bracketing, but do not allow poor bracketing for the final A1 E.g. Give final A0 for $\frac{3}{8}\ln 4x^{\frac{2}{3}} + 5\ \{+c\}$ unless recovered

Question Number	Scheme	Notes	Marks
3. (ii) **Alt 1**	$\int (e^x+1)^3\,dx;\quad u = e^x + 1 \Rightarrow \frac{du}{dx} = e^x$		
	$\left\{= \int \frac{u^3}{(u-1)}\,du =\right\} \int \left(u^2 + u + 1 + \frac{1}{u-1}\right) du$	$\int \left(u^2 + u + 1 + \frac{1}{u-1}\right)\{du\}$ where $u = e^x + 1$	B1
	$= \frac{1}{3}u^3 + \frac{1}{2}u^2 + u + \ln(u-1)\ \{+\,c\}$	At least 3 of either $\alpha u^2 \to \frac{\alpha}{3}u^3$ **or** $\beta u \to \frac{\beta}{2}u^2$ **or** $\delta \to \delta u$ **or** $\frac{\lambda}{u-1} \to \lambda\ln(u-1);\ \alpha, \beta, \delta, \lambda \neq 0$	M1
	$= \frac{1}{3}(e^x+1)^3 + \frac{1}{2}(e^x+1)^2 + (e^x+1) + \ln(e^x+1-1)\ \{+\,c\}$		

	$= \frac{1}{3}(e^x+1)^3 + \frac{1}{2}(e^x+1)^2 + (e^x+1) + x \; \{+c\}$	$\frac{1}{3}(e^x+1)^3 + \frac{1}{2}(e^x+1)^2 + (e^x+1) + x$ or $\frac{1}{3}(e^x+1)^3 + \frac{1}{2}(e^x+1)^2 + e^x + x$ simplified or un-simplified with or without $+c$ **Note:** $\ln(e^x+1-1)$ needs to be simplified to x for this mark	A1
			[3]
3. (ii) **Alt 2**	$\int (e^x+1)^3 \, dx; \quad u = e^x \Rightarrow \frac{du}{dx} = e^x$		
	$\left\{= \int \frac{(u+1)^3}{u} du = \right\} \int \left(u^2 + 3u + 3 + \frac{1}{u}\right) du$	$\int \left(u^2 + 3u + 3 + \frac{1}{u}\right) \{du\}$ where $u = e^x$	B1
	$= \frac{1}{3}u^3 + \frac{3}{2}u^2 + 3u + \ln u \; \{+c\}$	At least 3 of either $\alpha u^2 \to \frac{\alpha}{3}u^3$ **or** $\beta u \to \frac{\beta}{2}u^2$ **or** $\delta \to \delta u$ **or** $\frac{\lambda}{u} \to \lambda \ln u$; $\alpha, \beta, \delta, \lambda \neq 0$	M1
	$= \frac{1}{3}e^{3x} + \frac{3}{2}e^{2x} + 3e^x + x \; \{+c\}$	$\frac{1}{3}e^{3x} + \frac{3}{2}e^{2x} + 3e^x + x$, simplified or un-simplified with or without $+c$ **Note:** $\ln(e^x)$ needs to be simplified to x for this mark	A1
			[3]

Question Number	Scheme	Notes	Marks
4. (a)	$\frac{r}{h}=\tan 30 \Rightarrow r = h\tan 30 \left\{\Rightarrow r=\frac{h}{\sqrt{3}} \text{ or } r=\frac{\sqrt{3}}{3}h\right\}$ **or** $\frac{h}{r}=\tan 60 \Rightarrow r=\frac{h}{\tan 60}\left\{\Rightarrow r=\frac{h}{\sqrt{3}} \text{ or } r=\frac{\sqrt{3}}{3}h\right\}$ **or** $\frac{r}{\sin 30}=\frac{h}{\sin 60}\Rightarrow r=\frac{h\sin 30}{\sin 60}\left\{\Rightarrow r=\frac{h}{\sqrt{3}} \text{ or } r=\frac{\sqrt{3}}{3}h\right\}$ **or** $h^2+r^2=(2r)^2 \Rightarrow r^2=\frac{1}{3}h^2$	Correct use of trigonometry to find r in terms of h **or** correct use of Pythagoras to find r^2 in terms of h^2	M1
	$\left\{V=\frac{1}{3}\pi r^2 h \Rightarrow\right\} V=\frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h \Rightarrow V=\frac{1}{9}\pi h^3$ *	Correct proof of $V=\frac{1}{9}\pi h^3$ or $V=\frac{1}{9}h^3\pi$ Or shows $\frac{1}{9}\pi h^3$ or $\frac{1}{9}h^3\pi$ with some reference to $V=$ in their solution	A1 *
			[2]
(b) **Way 1**	$\frac{dV}{dt}=200$		
	$\frac{dV}{dh}=\frac{1}{3}\pi h^2$	$\frac{1}{3}\pi h^2$ o.e.	B1
	Either • $\left\{\frac{dV}{dh}\times\frac{dh}{dt}=\frac{dV}{dt}\Rightarrow\right\}\left(\frac{1}{3}\pi h^2\right)\frac{dh}{dt}=200$ • $\left\{\frac{dh}{dt}=\frac{dV}{dt}\div\frac{dV}{dh}\Rightarrow\right\}\frac{dh}{dt}=200\times\frac{1}{\frac{1}{3}\pi h^2}$	**either** $\left(\text{their } \frac{dV}{dh}\right)\times\frac{dh}{dt}=200$ **or** $200\div\left(\text{their } \frac{dV}{dh}\right)$	M1
	When $h=15,\ \frac{dh}{dt}=200\times\frac{1}{\frac{1}{3}\pi(15)^2}\ \left\{=\frac{200}{75\pi}=\frac{600}{225\pi}\right\}$	**dependent on the previous M mark**	dM1
	$\frac{dh}{dt}=\frac{8}{3\pi}\ (\text{cm s}^{-1})$	$\frac{8}{3\pi}$	A1 **cao**
			[4]
			6
(b) **Way 2**	$\frac{dV}{dt}=200 \Rightarrow V=200t+c \Rightarrow \frac{1}{9}\pi h^3=200t+c$		

$\left(\frac{1}{3}\pi h^2\right)\frac{dh}{dt} = 200$	$\frac{1}{3}\pi h^2$ o.e.	B1
	as in Way 1	M1
When $h = 15,\ \frac{dh}{dt} = 200 \times \frac{1}{\frac{1}{3}\pi(15)^2}\quad \left\{= \frac{200}{75\pi} = \frac{600}{225\pi}\right\}$	**dependent on the previous M mark**	dM1
$\frac{dh}{dt} = \frac{8}{3\pi}\ (\text{cm s}^{-1})$	$\frac{8}{3\pi}$	A1 **cao**
		[4]

	Question 4 Notes	
4. (a)	**Note**	Allow M1 for writing down $r = h\tan 30$
	Note	Give M0 A0 for writing down $r = \frac{h\sqrt{3}}{3}$ or $r = \frac{h}{\sqrt{3}}$ with no evidence of using trigonometry on r and h or Pythagoras on r and h
	Note	Give M0 (unless recovered) for evidence of $\frac{1}{3}\pi r^2 h = \frac{1}{9}\pi h^3$ leading to either $r^2 = \frac{1}{3}h^2$ or $r = \frac{h\sqrt{3}}{3}$ or $r = \frac{h}{\sqrt{3}}$
(b)	**B1** **Note**	Correct simplified or un-simplified differentiation of V. E.g. $\frac{1}{3}\pi h^2$ or $\frac{3}{9}\pi h^2$ $\frac{dV}{dh}$ does not have to be explicitly stated, but it should be clear that they are differentiating their V
	M1	$\left(\text{their } \frac{dV}{dh}\right) \times \frac{dh}{dt} = 200$ or $200 \div \left(\text{their } \frac{dV}{dh}\right)$
	dM1	**dependent on the previous M mark** Substitutes $h = 15$ into an expression ***which is a result*** of either $200 \div \left(\text{their } \frac{dV}{dh}\right)$ or $200 \times \frac{1}{\left(\text{their } \frac{dV}{dh}\right)}$
	A1	$\frac{8}{3\pi}$ (units are not required)
	Note	Give final A0 for using $\frac{dV}{dt} = -200$ to give $\frac{dh}{dt} = -\frac{8}{3\pi}$, unless recovered to $\frac{dh}{dt} = \frac{8}{3\pi}$

Question Number	Scheme	Notes	Marks
5.	$x = 1 + t - 5\sin t,\ y = 2 - 4\cos t,\ -\pi \leq t \leq \pi;\ A(k, 2),\ k > 0$, lies on C		
(a)	$\{\text{When } y = 2,\}\ 2 = 2 - 4\cos t \Rightarrow t = -\frac{\pi}{2}, \frac{\pi}{2}$ $k\,(\text{or } x) = 1 + \frac{\pi}{2} - 5\sin\left(\frac{\pi}{2}\right)$ **or** $k\,(\text{or } x) = 1 - \frac{\pi}{2} - 5\sin\left(-\frac{\pi}{2}\right)$	Sets $y = 2$ to find t and some evidence of using their t to find $x = ...$	M1
	$\left\{\text{When } t = -\frac{\pi}{2}, k > 0,\right\}$ so $k = 6 - \frac{\pi}{2}$ or $\frac{12-\pi}{2}$	k (or x) $= 6 - \frac{\pi}{2}$ or $\frac{12-\pi}{2}$	A1
			[2]
(b)	$\frac{dx}{dt} = 1 - 5\cos t,\quad \frac{dy}{dt} = 4\sin t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct (Can be implied)	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct (Can be implied)	B1
	$\frac{dy}{dx} = \frac{4\sin t}{1 - 5\cos t}$ at $t = -\frac{\pi}{2},\ \frac{dy}{dx} = \frac{4\sin\left(-\frac{\pi}{2}\right)}{1 - 5\cos\left(-\frac{\pi}{2}\right)}\quad \{= -4\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes their t into their $\frac{dy}{dx}$ **Note:** their t can lie outside $-\pi \leq t \leq \pi$ for this mark	M1
	• $y - 2 = -4\left(x - \left(6 - \frac{\pi}{2}\right)\right)$ • $2 = (-4)\left(6 - \frac{\pi}{2}\right) + c \Rightarrow y = -4x + 2 + 4\left(6 - \frac{\pi}{2}\right)$	Correct straight line method for an equation of a tangent where $m_T\ (\neq m_N)$ is found using calculus **Note:** their k (or x) must be in terms of π and correct bracketing must be used or implied	M1
	$\{y - 2 = -4x + 24 - 2\pi \Rightarrow\}\quad y = -4x + 26 - 2\pi$	**dependent on all previous marks in part (b)** $y = -4x + 26 - 2\pi$	A1 **cso**
		$(p = -4,\ q = 26 - 2\pi)$	**[5]**
			7

Question 5 Notes

5. (a)	**Note**	M1 can be implied by either x or $k = 6 - \frac{\pi}{2}$ or awrt 4.43 or x or $k = \frac{\pi}{2} - 4$ or awrt -2.43
	Note	An answer of 4.429… without reference to a correct **exact** answer is A0
	Note	M1 can be earned in part (a) by working in degrees
	Note	Give M0 for not substituting their t back into x. E.g. $2 = 2 - 4\cos t \Rightarrow t = -\frac{\pi}{2} \Rightarrow k = -\frac{\pi}{2}$
	Note	If two values for k are found, they must identify the correct answer for A1

	Note	Condone M1 for $2 = 2 - 4\cos t \Rightarrow t = -\frac{\pi}{2}, \frac{\pi}{2} \Rightarrow x = 1 - \frac{\pi}{2} - 5\sin\left(\frac{\pi}{2}\right)$
(b)	**Note**	The 1st M mark may be implied by their value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ e.g. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4\sin t}{1 - 5\cos t}$, followed by an answer of -4 (from $t = -\frac{\pi}{2}$) or 4 (from $t = \frac{\pi}{2}$)
	Note	Give 1st M0 for **applying** their $\frac{\mathrm{d}x}{\mathrm{d}t}$ divided by their $\frac{\mathrm{d}y}{\mathrm{d}t}$ even if they state $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$
	2nd M1	• **applies** $y - 2 = (\text{their } m_T)(x - (\text{their } k))$, • **applies** $2 = (\text{their } m_T)(\text{their } k) + c$ **leading to** $y = (\text{their } m_T)x + (\text{their } c)$ where k must be in terms of π and m_T $(\neq m_N)$ is a numerical value found using calculus
	Note	Correct bracketing must be used for 2nd M1, but this mark can be implied by later working

	Question 5 Notes Continued	
5. (b)	**Note**	The final A mark is dependent on all previous marks in part (b) being scored. This is because the correct answer can follow from an incorrect $\frac{\mathrm{d}y}{\mathrm{d}x}$
	Note	The first 3 marks can be gained by using degrees in part (b)
	Note	Condone mixing a correct t with an incorrect x or an incorrect t with a correct x for the M marks
	Note	Allow final A1 for any answer in the form $y = px + q$ E.g. Allow final A1 for $y = -4x + 26 - 2\pi$, $y = -4x + 2 + 4\left(6 - \frac{\pi}{2}\right)$ or $y = -4x + \left(\frac{52 - 4\pi}{2}\right)$
	Note	Do not apply isw in part (b). So, an incorrect answer following from a correct answer is A0
	Note	Do not allow $y = 2(-2x + 13 - \pi)$ for A1
	Note	$y = -4x + 26 - 2\pi$ followed by $y = 2(-2x + 13 - \pi)$ is condoned for final A1

Question Number	Scheme	Notes	Marks
6.	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y^2}{3\cos^2 2x};\ -\dfrac{1}{2} < x < \dfrac{1}{2};\ y = 2 \text{ at } x = -\dfrac{\pi}{8}$		
	$\displaystyle\int \frac{1}{y^2}\,\mathrm{d}y = \int \frac{1}{3\cos^2 2x}\,\mathrm{d}x$	Separates variables as shown. Can be implied by a correct attempt at integration. Ignore the integral signs	B1
	$\displaystyle\int \frac{1}{y^2}\,\mathrm{d}y = \int \frac{1}{3}\sec^2 2x\,\mathrm{d}x$		
	$-\dfrac{1}{y} = \dfrac{1}{3}\left(\dfrac{\tan 2x}{2}\right)\ \{+c\}$	$\pm\dfrac{A}{y^2} \to \pm\dfrac{B}{y};\ A, B \neq 0$	M1
		$\pm\lambda\tan 2x$	M1
		$-\dfrac{1}{y} = \dfrac{1}{3}\left(\dfrac{\tan 2x}{2}\right)$	A1
	$-\dfrac{1}{2} = \dfrac{1}{6}\tan\left(2\left(-\dfrac{\pi}{8}\right)\right) + c$	Use of $x = -\dfrac{\pi}{8}$ and $y = 2$ in an integrated equation ***containing a constant of integration***, e.g. c	M1
	$-\dfrac{1}{2} = -\dfrac{1}{6} + c \Rightarrow c = -\dfrac{1}{3}$		
	$-\dfrac{1}{y} = \dfrac{1}{6}\tan 2x - \dfrac{1}{3} = \dfrac{\tan(2x) - 2}{6}$		
	$y = \dfrac{-1}{\frac{1}{6}\tan 2x - \frac{1}{3}}$ or $y = \dfrac{6}{2 - \tan 2x}$ or $y = \dfrac{6\cot 2x}{-1 + 2\cot 2x}$ $\left\{-\dfrac{1}{2} < x < \dfrac{1}{2}\right\}$		A1 o.e.
			[6]
			6

Question 6 Notes

6.	**B1**	Separates variables as shown. $\mathrm{d}y$ and $\mathrm{d}x$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. The number "3" may appear on either side. E.g. $\displaystyle\int \frac{1}{y^2}\,\mathrm{d}y = \int \frac{1}{3}\sec^2 2x\,\mathrm{d}x$ or $\displaystyle\int \frac{3}{y^2}\,\mathrm{d}y = \int \frac{1}{\cos^2 2x}\,\mathrm{d}x$ are fine for B1
	Note	Allow e.g. $\displaystyle\int \frac{1}{y^2}\frac{\mathrm{d}y}{\mathrm{d}x}\,\mathrm{d}x = \int \frac{1}{3}\sec^2 2x\,\mathrm{d}x$ for B1 or condone $\displaystyle\int \frac{1}{y^2} = \int \frac{1}{3}\sec^2 2x$ for B1
	Note	B1 can be implied by correct integration of both sides

	M1	$\pm\frac{A}{y^2} \to \pm\frac{B}{y}$; $A, B \neq 0$
	M1	$\frac{1}{\cos^2 2x}$ or $\sec^2 2x \to \pm\lambda\tan 2x$; $\lambda \neq 0$
	A1	$-\frac{1}{y} = \frac{1}{3}\left(\frac{\tan 2x}{2}\right)$ with or without $'+c'$. E.g. $-\frac{6}{y} = \tan 2x$
	M1	Evidence of using both $x = -\frac{\pi}{8}$ and $y = 2$ in an integrated or changed equation containing c
	Note	This mark can be implied by the correct value of c
	Note	You may need to use your calculator to check that they have satisfied the final M mark
	Note	Condone using $x = \frac{\pi}{8}$ instead of $x = -\frac{\pi}{8}$
	A1	$y = \frac{-1}{\frac{1}{6}\tan 2x - \frac{1}{3}}$ or $y = \frac{6}{2 - \tan 2x}$ **or any equivalent correct answer in the form** $y = \mathrm{f}(x)$
	Note	You can ignore subsequent working, which follows from a correct answer

	Question 6 Notes Continued	
6.	**Note**	Writing $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2}{3\cos^2 2x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3}y^2\sec^2 2x$ leading to e.g. • $y = \frac{1}{9}y^3\left(\frac{1}{2}\tan 2x\right)$ gets 2nd M0 for $\pm\lambda\tan 2x$ • $u = \frac{1}{3}y^2,\ \frac{\mathrm{d}v}{\mathrm{d}x} = \sec^2 2x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2}{3}y,\ v = \frac{1}{2}\tan 2x$ gets 2nd M0 for $\pm\lambda\tan 2x$ because the variables have not been separated

Question Number	Scheme	Notes	Marks
8. (a)	$\left\{\int x\cos 4x\,\mathrm{d}x\right\}$ $= \frac{1}{4}x\sin 4x - \int \frac{1}{4}\sin 4x\,\{\mathrm{d}x\}$	$\pm\alpha x\sin 4x \pm \beta\int \sin 4x\,\{\mathrm{d}x\}$, with or without $\mathrm{d}x$; $\alpha, \beta \neq 0$	M1
		$\frac{1}{4}x\sin 4x - \int \frac{1}{4}\sin 4x\,\{\mathrm{d}x\}$, with or without $\mathrm{d}x$ **Can be simplified or un-simplified**	A1
	$= \frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x\ \{+c\}$	$\frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x$ o.e. with or without $+c$ **Can be simplified or un-simplified**	A1
	Note: You can ignore subsequent working following on from a correct solution		**[3]**
	Question 8 Notes		

	Question 8 Notes Continued	
8. (a)	**SC**	Give ***Special Case*** M1A0A0 for writing down the correct "by parts" formula and using $u = x, \frac{\mathrm{d}v}{\mathrm{d}x} = \cos 4x,$ but making only one error in the application of the correct formula