2. In the triangle ABC, AB = 16 cm, AC = 13 cm, angle ABC = 50° and angle $BCA = x°$

Find the two possible values for x, giving your answers to one decimal place.

(Total 4 marks)

6. $f(x) = -6x^3 - 7x^2 + 40x + 21$

(*a*) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$

(2)

(*b*) Factorise $f(x)$ completely.

(4)

(*c*) Hence solve the equation

$$6(2^{3y}) + 7(2^{2y}) = 40(2^y) + 21$$

giving your answer to 2 decimal places.

(3)

(Total 9 marks)

---

7. (i) $2\log(x + a) = \log(16a^6)$, where a is a positive constant

Find x in terms of a, giving your answer in its simplest form.

(3)

(ii) $\log_3(9y + b) - \log_3(2y - b) = 2$, where b is a positive constant

Find y in terms of b, giving your answer in its simplest form.

(4)

(Total 7 marks)

---

8. (*a*) Show that the equation

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

can be written in the form

$$(3\sin x - 1)^2 = 2$$

(3)

(*b*) Hence solve, for $0 \leqslant x < 360°$,

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

giving your answers to 2 decimal places.

(5)

(Total 8 marks)

---

9. The first three terms of a geometric sequence are

$$7k - 5, \qquad 5k - 7, \qquad 2k + 10$$

where k is a constant.

(*a*) Show that $11k^2 - 130k + 99 = 0$

(4)

Given that k is not an integer,

(*b*) show that $k = \frac{9}{11}$

(2)

For this value of k,

(*c*) (i) evaluate the fourth term of the sequence, giving your answer as an exact fraction,

(ii) evaluate the sum of the first ten terms of the sequence.

(6)

(Total 12 marks)

10.

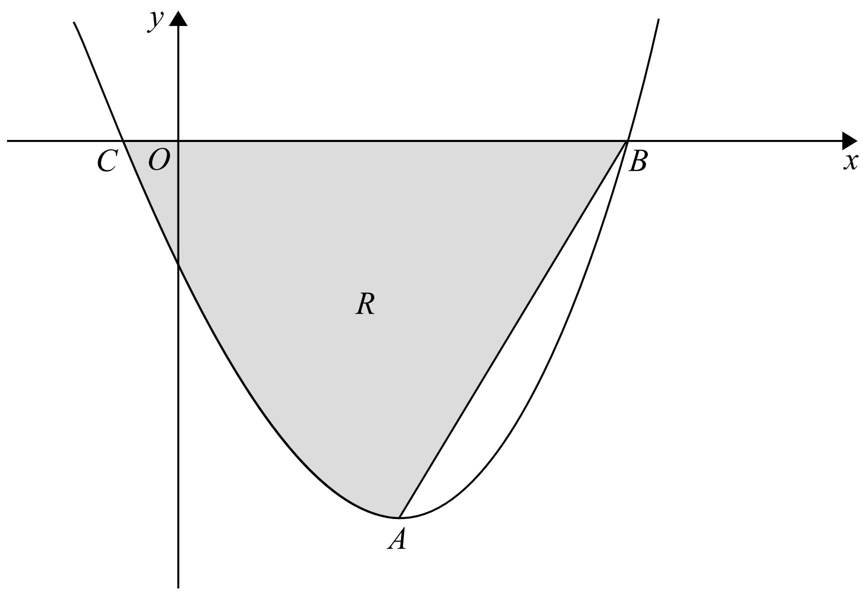

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 4x^3 + 9x^2 - 30x - 8, \qquad -0.5 \leqslant x \leqslant 2.2$$

The curve has a turning point at the point *A*.

(*a*) Using calculus, show that the *x* coordinate of *A* is 1

(3)

The curve crosses the *x*-axis at the points *B* (2, 0) and *C* $\left(-\frac{1}{4}, 0\right)$

The finite region *R*, shown shaded in Figure 2, is bounded by the curve, the line *AB*, and the *x*-axis.

(*b*) Use integration to find the area of the finite region *R*, giving your answer to 2 decimal places.

(7)

(Total 10 marks)

1. Express $\dfrac{4x}{x^2 - 9} - \dfrac{2}{x+3}$ as a single fraction in its simplest form.

(Total 4 marks)

---

2. Find the exact solutions, in their simplest form, to the equations

(*a*) $e^{3x-9} = 8$

(3)

(*b*) $\ln(2y + 5) = 2 + \ln(4 - y)$

(4)

(Total 7 marks)

3.

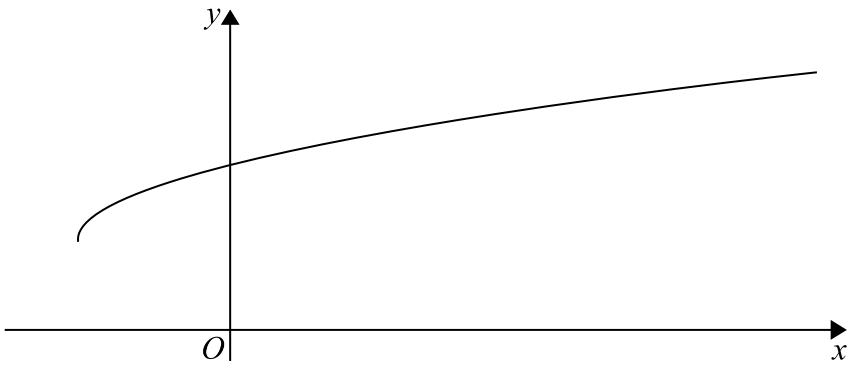

Figure 1

Figure 1 shows a sketch of part of the graph of $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = 3 + \sqrt{x+2}, \qquad x \geqslant -2$$

(*a*) State the range of g.

(1)

(*b*) Find $\mathrm{g}^{-1}(x)$ and state its domain.

(3)

(*c*) Find the exact value of x for which

$$\mathrm{g}(x) = x$$

(4)

(*d*) Hence state the value of a for which

$$\mathrm{g}(a) = \mathrm{g}^{-1}(a)$$

(1)

(Total 9 marks)

---

4. (*a*) Write $5 \cos \theta - 2 \sin \theta$ in the form $R \cos (\theta + \alpha)$, where R and α are constants,

$R > 0$ and $0 \leqslant \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

(*b*) Show that the equation

$$5 \cot 2x - 3 \operatorname{cosec} 2x = 2$$

can be rewritten in the form

$$5 \cos 2x - 2 \sin 2x = c$$

where c is a positive constant to be determined.

(2)

(*c*) Hence or otherwise, solve, for $0 \leqslant x < \pi$,

$$5 \cot 2x - 3 \operatorname{cosec} 2x = 2$$

giving your answers to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

(Total 9 marks)

5.

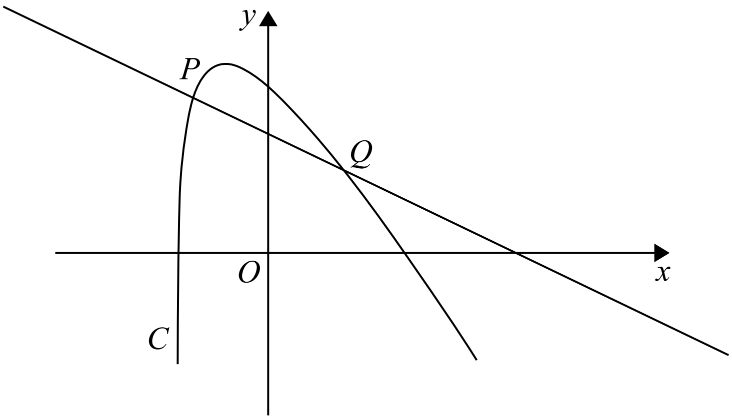

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 2\ln(2x + 5) - \frac{3x}{2}, \qquad x > -2.5$$

The point P with x coordinate -2 lies on C.

(a) Find an equation of the normal to C at P. Write your answer in the form $ax + by = c$, where a, b and c are integers.

(5)

The normal to C at P cuts the curve again at the point Q, as shown in Figure 2.

(b) Show that the x coordinate of Q is a solution of the equation

$$x = \frac{20}{11}\ln(2x + 5) - 2$$

(3)

The iteration formula

$$x_{n+1} = \frac{20}{11}\ln(2x_n + 5) - 2$$

can be used to find an approximation for the x coordinate of Q.

(c) Taking $x_1 = 2$, find the values of x_2 and x_3, giving each answer to 4 decimal places.

(2)

(Total 10 marks)

6. Given that a and b are positive constants,

(a) on separate diagrams, sketch the graph with equation

(i) $y = |2x - a|$

(ii) $y = |2x - a| + b$

Show, on each sketch, the coordinates of each point at which the graph crosses or meets the axes.

(4)

Given that the equation

$$|2x - a| + b = \frac{3}{2}x + 8$$

has a solution at $x = 0$ and a solution at $x = c$,

(b) find c in terms of a.

(4)

(Total 8 marks)