Core 17

Part 1

1. Find

$$\int \left(2x^5 - \frac{1}{4x^3} - 5 \right) \mathrm{d}x$$

giving each term in its simplest form.

(Total 4 marks)

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2. Given

$$y = \sqrt{x} + \frac{4}{\sqrt{x}} + 4, \qquad x > 0$$

find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 8$, writing your answer in the form $a\sqrt{2}$, where a is a rational number.

(Total 5 marks)

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3. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 1$$

$$a_{n+1} = \frac{k(a_n + 1)}{a_n}, \qquad n \geqslant 1$$

where k is a positive constant.

(*a*) Write down expressions for a_2 and a_3 in terms of k, giving your answers in their simplest form.

(3)

Given that $\sum_{r=1}^{3} a_r = 10$

(*b*) find an exact value for k.

(3)

(Total 6 marks)

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4. A company, which is making 140 bicycles each week, plans to increase its production. The number of bicycles produced is to be increased by d each week, starting from 140 in week 1, to $140 + d$ in week 2, to $140 + 2d$ in week 3 and so on, until the company is producing 206 in week 12.

(*a*) Find the value of d.

(2)

After week 12 the company plans to continue making 206 bicycles each week.

(*b*) Find the total number of bicycles that would be made in the first 52 weeks starting from and including week 1.

(5)

(Total 7 marks)

5.

$$f(x) = x^2 - 8x + 19$$

(*a*) Express $f(x)$ in the form $(x + a)^2 + b$, where a and b are constants.

(2)

The curve C with equation $y = f(x)$ crosses the y-axis at the point P and has a minimum point at the point Q.

(*b*) Sketch the graph of C showing the coordinates of point P and the coordinates of point Q.

(3)

(*c*) Find the distance PQ, writing your answer as a simplified surd.

(3)

(Total 8 marks)

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6. (*a*) Given $y = 2^x$, show that

$$2^{2x+1} - 17(2^x) + 8 = 0$$

can be written in the form

$$2y^2 - 17y + 8 = 0$$

(2)

(*b*) Hence solve

$$2^{2x+1} - 17(2^x) + 8 = 0$$

(4)

(Total 6 marks)

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7. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, where

$$\mathrm{f}'(x) = 30 + \frac{6 - 5x^2}{\sqrt{x}}$$

Given that the point $P(4, -8)$ lies on C,

(a) find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

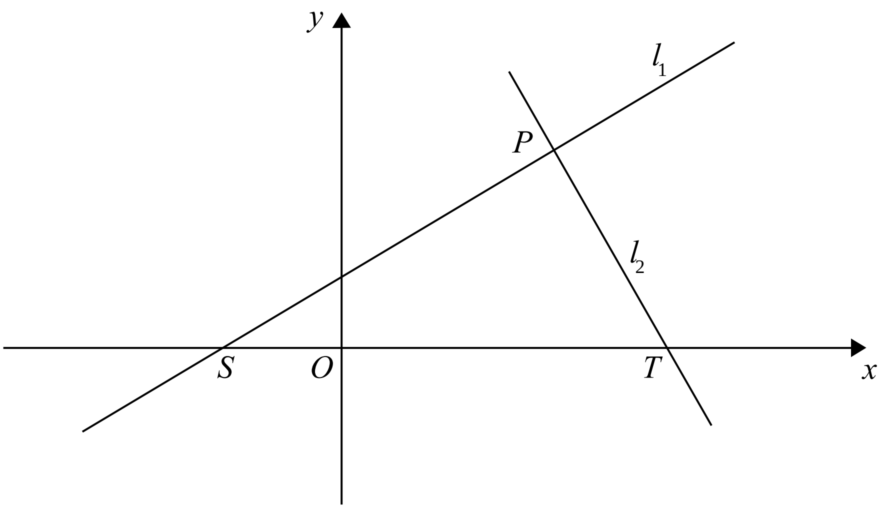
(4)

(b) Find $\mathrm{f}(x)$, giving each term in its simplest form.

(5)

(Total 9 marks)

8.



Not to scale

Figure 1

The straight line l_1 , shown in Figure 1, has equation $5y = 4x + 10$

The point P with x coordinate 5 lies on l_1

The straight line l_2 is perpendicular to l_1 and passes through P.

(a) Find an equation for l_2 , writing your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(4)

The lines l_1 and l_2 cut the x-axis at the points S and T respectively, as shown in Figure 1.

(b) Calculate the area of triangle SPT.

(4)

(Total 8 marks)

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9. (*a*) On separate axes sketch the graphs of

(i) $y = -3x + c$, where c is a positive constant,

(ii) $y = \frac{1}{x} + 5$

On each sketch show the coordinates of any point at which the graph crosses the y-axis and the equation of any horizontal asymptote.

(4)

Given that $y = -3x + c$, where c is a positive constant, meets the curve $y = \frac{1}{x} + 5$ at two distinct points,

(*b*) show that $(5 - c)^2 > 12$

(3)

(*c*) Hence find the range of possible values for c.

(4)

(Total 11 marks)

10.

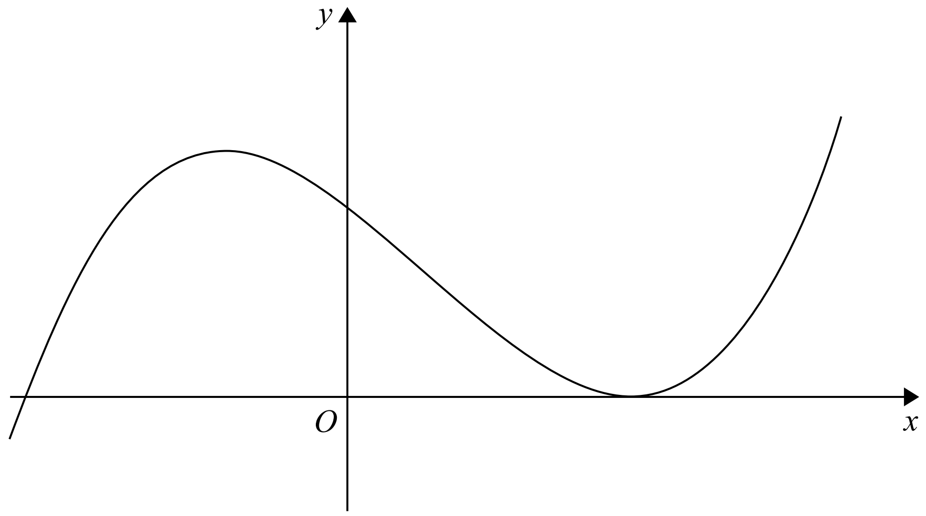

Figure 2

Figure 2 shows a sketch of part of the curve $y = f(x)$, $x \in \mathbb{R}$, where

$$f(x) = (2x - 5)^2 (x + 3)$$

(*a*) Given that

(i) the curve with equation $y = f(x) - k$, $x \in \mathbb{R}$, passes through the origin, find the value of the constant k,

(ii) the curve with equation $y = f(x + c)$, $x \in \mathbb{R}$, has a minimum point at the origin, find the value of the constant c.

(3)

(*b*) Show that $f'(x) = 12x^2 - 16x - 35$

(3)

Points A and B are distinct points that lie on the curve $y = f(x)$.

The gradient of the curve at A is equal to the gradient of the curve at B.

Given that point A has x coordinate 3

(*c*) find the x coordinate of point B.

(5)

(Total 11 marks)

1. Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{1}{3}x\right)^5$$

giving each term in its simplest form.

(Total 4 marks)

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3. (*a*) $y = 5^x + \log_2(x + 1), \quad 0 \leqslant x \leqslant 2$

Complete the table below, by giving the value of y when $x = 1$

x	0	0.5	1	1.5	2
y	1	2.821		12.502	26.585

(1)

(*b*) Use the trapezium rule, with all the values of y from the completed table, to find an approximate value for

$$\int_0^2 (5^x + \log_2(x+1))\,\mathrm{d}x$$

giving your answer to 2 decimal places.

(4)

(*c*) Use your answer to part (*b*) to find an approximate value for

$$\int_0^2 (5 + 5^x + \log_2(x+1))\,\mathrm{d}x$$

giving your answer to 2 decimal places.

(1)

(Total 6 marks)

4.

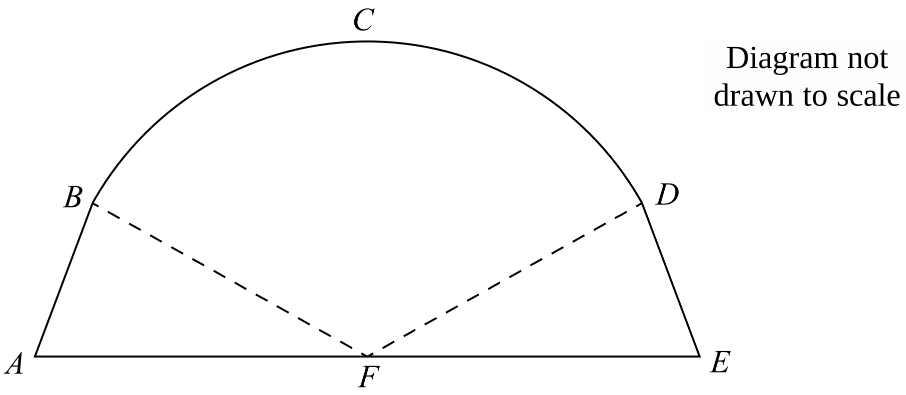

Figure 1

Figure 1 is a sketch representing the cross-section of a large tent *ABCDEF*.
AB and *DE* are line segments of equal length.
Angle *FAB* and angle *DEF* are equal.
F is the midpoint of the straight line *AE* and *FC* is perpendicular to *AE*.
BCD is an arc of a circle of radius 3.5 m with centre at *F*.
It is given that

$$AF = FE = 3.7 \text{ m}$$

$$BF = FD = 3.5 \text{ m}$$

$$\text{angle } BFD = 1.77 \text{ radians}$$

Find

(*a*) the length of the arc *BCD* in metres to 2 decimal places,

(2)

(*b*) the area of the sector *FBCD* in m^2 to 2 decimal places,

(2)

(*c*) the total area of the cross-section of the tent in m^2 to 2 decimal places.

(4)

(Total 8 marks)

5. The circle C has equation

$$x^2 + y^2 - 10x + 6y + 30 = 0$$

Find

(a) the coordinates of the centre of C,

(2)

(b) the radius of C,

(2)

(c) the y coordinates of the points where the circle C crosses the line with equation $x = 4$, giving your answers as simplified surds.

(3)

(Total 7 marks)