Core 17 Mark Scheme

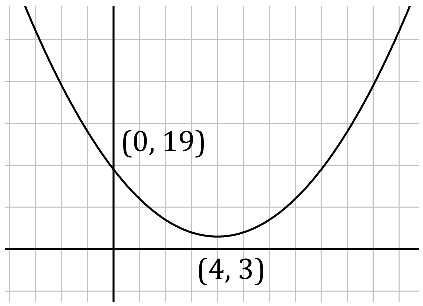
Part 1

Question Number	Scheme	Marks
1.	$\int\left(2x^5-\frac{1}{4}x^{-3}-5\right)dx$	
	$x^n \to x^{n+1}$	M1
	$2\times\frac{x^{5+1}}{6}$ or $-\frac{1}{4}\times\frac{x^{-3+1}}{-2}$	A1
	Two of: $\frac{1}{3}x^6$, $\frac{1}{8}x^{-2}$, $-5x$	A1
	$\frac{1}{3}x^6+\frac{1}{8}x^{-2}-5x+c$	A1
		(4 marks)

Question Number	Scheme	Marks
2.	$y=\sqrt{x}+\frac{4}{\sqrt{x}}+4=x^{\frac{1}{2}}+4x^{-\frac{1}{2}}+4$	
	$x^n \to x^{n-1}$	M1
	$\left(\frac{dy}{dx}=\right)\frac{1}{2}x^{-\frac{1}{2}}+4\times-\frac{1}{2}x^{-\frac{3}{2}}$ $\left(=\frac{1}{2}x^{-\frac{1}{2}}-2x^{-\frac{3}{2}}\right)$	A1
	$x=8\Rightarrow\frac{dy}{dx}=\frac{1}{2}8^{-\frac{1}{2}}+4\times-\frac{1}{2}8^{-\frac{3}{2}}$	M1
	$=\frac{1}{2\sqrt{8}}-\frac{2}{\left(\sqrt{8}\right)^3}=\frac{1}{2\sqrt{8}}-\frac{2}{8\sqrt{8}}=\frac{1}{8\sqrt{2}}=\frac{1}{16}\sqrt{2}$	B1A1
		(5 marks)

Question Number	Scheme	Marks
3.(a)	$(a_2 =)2k$	B1
	$(a_3 =)\dfrac{k("2k"+1)}{"2k"}$	M1
	$(a_3 =)\dfrac{2k+1}{2}$	A1
		(3)
	Note that there are <u>no</u> marks in (b) for using an AP (or GP) sum formula unless their terms do form an AP (or GP).	
(b)	$\sum_{r=1}^{3} a_r = 10 \Rightarrow 1+"2k"+"\dfrac{2k+1}{2}"=10$	M1
	$\Rightarrow 2+4k+2k+1=20 \Rightarrow k=\ldots$ or e.g. $\Rightarrow 6k^2-17k=0 \Rightarrow k=\ldots$	M1
	$(k=)\dfrac{17}{6}$	A1
		(3)
		(6 marks)

Question Number	Scheme	Marks
4. (a)	$206=140+(12-1)\times d \Rightarrow d=\ldots$	M1
	$(d=)6$	A1
		(2)
(b)	$S_{12}=\dfrac{12}{2}(140+206)$ or $S_{12}=\dfrac{12}{2}(2\times 140+(12-1)\times "6")$ or $S_{11}=\dfrac{11}{2}(140+206-"6")$ or $S_{11}=\dfrac{11}{2}(2\times 140+(11-1)\times "6")$	M1
	$S=2076$ **WAY1** or $S=1870$ **WAY 2**	A1
	$(52-12)\times 206=\ldots$ or $(52-11)\times 206=\ldots$	M1
	Total $="2076"+"8240"=\ldots$ **(WAY 1)** or Total $="1870"+"8446"=\ldots$ **(WAY 2)**	**dd**M1
	10316	A1
		(5)
		(7 marks)

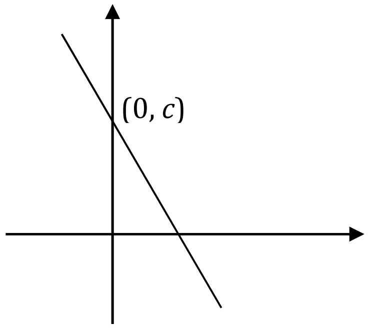
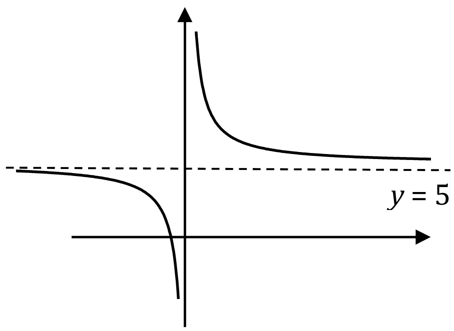
Question Number	Scheme	Marks
5.(a)	$f(x) = (x-4)^2 + 3$	M1A1
		(2)
(b)		B1 B1 B1
		(3)
(c)	$PQ^2 = (0-4)^2 + (19-3)^2$	M1
	$PQ = \sqrt{4^2 + 16^2}$	A1
	$PQ = 4\sqrt{17}$	A1
		(3)
		(8 marks)

Question Number	Scheme	Marks
6.(a)	Replaces 2^{2x+1} with $2^{2x}\times 2$ or states $2^{2x+1}=2^{2x}\times 2$ or states $\left(2^x\right)^2=2^{2x}$	M1
	$2^{2x+1}-17\times 2^x+8=0$ $\Rightarrow 2y^2-17y+8=0$*	A1*
		(2)
(b)	$2y^2-17y+8=0\Rightarrow(2y-1)(y-8)(=0)\Rightarrow y=...$ or $2\left(2^x\right)^2-17\left(2^x\right)+8=0\Rightarrow\left(2\left(2^x\right)-1\right)\left(\left(2^x\right)-8\right)(=0)\Rightarrow 2^x=...$	M1
	$(y=)\frac{1}{2},8$ or $(2^x=)\frac{1}{2},8$	A1
	$\Rightarrow 2^x=\frac{1}{2},8\Rightarrow x=-1,3$	M1 A1
		(4)
		(6 marks)

Question Number	Scheme	Marks
7.(a)	$\mathrm{f}'(4)=30+\dfrac{6-5\times 4^2}{\sqrt{4}}$	M1
	$\mathrm{f}'(4)=-7$	A1
	$y-(-8)="-7"\times(x-4)$ or $y="-7"x+c\Rightarrow -8="-7"\times 4+c$ $\Rightarrow c=...$	M1
	$y=-7x+20$	A1
		(4)
(b)	**Allow the marks in (b) to score in (a) i.e. <u>mark (a) and (b) together</u>**	
	$\Rightarrow \mathrm{f}(x)=30x+6\dfrac{x^{\frac{1}{2}}}{0.5}-5\dfrac{x^{\frac{5}{2}}}{2.5}(+c)$	M1A1A1
	$x=4,\mathrm{f}(x)=-8\Rightarrow$ $-8=120+24-64+c\Rightarrow c=...$	M1
	$\Rightarrow(\mathrm{f}(x)=)30x+12x^{\frac{1}{2}}-2x^{\frac{5}{2}}-88$	A1
		(5)
		(9 marks)

Question Number	Scheme		Marks
8.(a)	Gradient of $l_1 = \frac{4}{5}$ oe		B1
	Point P = (5, 6)		B1
	$-\frac{5}{4} = \frac{y-"6"}{x-5}$ or $y-"6" = -\frac{5}{4}(x-5)$ or $"6" = -\frac{5}{4}(5)+c \Rightarrow c = \ldots$		M1
	$5x+4y-49=0$		A1
			(4)
8(b)	$y=0 \Rightarrow 5x+4(0)-49=0 \Rightarrow x=\ldots$ **or** $y=0 \Rightarrow 5(0)=4x+10 \Rightarrow x=\ldots$		M1
	$y=0 \Rightarrow 5x+4(0)-49=0 \Rightarrow x=\ldots$ **and** $y=0 \Rightarrow 5(0)=4x+10 \Rightarrow x=\ldots$		M1
	<u>Method 1:</u> $\frac{1}{2}ST \times "6"$ $\frac{1}{2}\times('9.8'-'-2.5')\times'6' = \ldots$		**dd**M1
	<u>Method 2:</u> $\frac{1}{2}SP \times PT$ $\frac{1}{2}\times\sqrt{(5-'-2.5')^2+('6')^2}\times\sqrt{('9.8'-5)^2+('6')^2} = \ldots$ $\left(=\frac{1}{2}\times\frac{3\sqrt{41}}{2}\times\frac{6\sqrt{41}}{5}\right)$		
	<u>Method 3:</u> 2 Triangles $\frac{1}{2}\times(5+'2.5')\times'6'+\frac{1}{2}\times('9.8'-5)\times'6' = \ldots$		
	<u>Method 4:</u> Shoelace method $\frac{1}{2}\begin{vmatrix} 5 & 9.8 & -2.5 & 5 \\ 6 & 0 & 0 & 6 \end{vmatrix} = \frac{1}{2}\left\|(0+0-15)-(58.8+0+0)\right\| = \frac{1}{2}\left\|-73.8\right\| = \ldots$		
	<u>Method 5:</u> Trapezium + 2 triangles $\frac{1}{2}\times('2.5')\times'2'+\frac{1}{2}("2"+"6")\times 5+\frac{1}{2}\times("9.8"-5')\times'6' = \ldots$		
	$=36.9$		A1
			(4)
			(8 marks)

Question Number	Scheme	Marks

Question Number	Scheme	Marks
9.(a)(i)		B1 B1
(a)(ii)		B1 B1
		(4)
(b)	$\frac{1}{x}+5=-3x+c \Rightarrow 1+5x=-3x^2+cx$ $\Rightarrow 3x^2+5x-cx+1=0$	M1
	$b^2-4ac=(5-c)^2-4\times1\times3$	M1
	$(5-c)^2>12$ *	A1*
		(3)
(c)	$(5-c)^2=12 \Rightarrow (c=)5\pm\sqrt{12}$ or $(5-c)^2=12 \Rightarrow c^2-10c+13=0$ $\Rightarrow (c=)\frac{-{}^{-}10\pm\sqrt{(-10)^2-4\times13}}{2}$	M1A1
	$c<$ "$5-\sqrt{12}$", $c>$ "$5+\sqrt{12}$"	M1
	$0<c<5-\sqrt{12}$, $c>5+\sqrt{12}$	A1
		(4)
		(11 marks)

Question Number	Scheme	Marks

Question Number	Scheme	Marks
10.(a)(i)	$k=\left(-5\right)^2\times 3=75$	M1A1
(ii)	$c=\frac{5}{2}$ only	B1
		(3)
(b)	$\text{f}(x)=(2x-5)^2(x+3)=\left(4x^2-20x+25\right)(x+3)=4x^3-8x^2-35x+75$	M1
	$\left(\text{f}'(x)=\right)12x^2-16x-35$ *	M1A1*
		(3)
(c)	$\text{f}'(3)=12\times 3^2-16\times 3-35$	M1
	$12x^2-16x-35='25'$	**d**M1
	$12x^2-16x-60=0$	A1 cso
	$(x-3)(12x+20)=0\Rightarrow x=\ldots$	**dd**M1
	$x=-\frac{5}{3}$	A1 cso
		(5)
		(11 marks)

TOTAL FOR PART 1: 75 MARKS

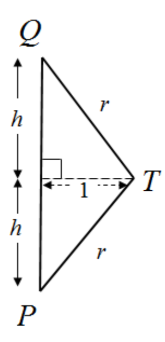
Part 2

Question Number	Scheme	Marks
1.	$\left(3-\frac{1}{3}x\right)^5$ -	
	$3^5 + {}^5C_1 3^4(-\frac{1}{3}x) + {}^5C_2 3^3(-\frac{1}{3}x)^2 + {}^5C_3 3^2(-\frac{1}{3}x)^3 ...$	
	First term of 243	B1
	$\left({}^5C_1 \times ... \times x\right) + \left({}^5C_2 \times ... \times x^2\right) + \left({}^5C_3 \times ... \times x^3\right) ...$	M1
	$=(243 ..) - \frac{405}{3}x + \frac{270}{9}x^2 - \frac{90}{27}x^3 ...$	A1
	$=(243 ..) - 135x + 30x^2 - \frac{10}{3}x^3 ..$	A1
		(4)
		[4]

Question Number	Scheme	Marks
2.	$\frac{\sin x}{16} = \frac{\sin 50^0}{13}$	M1
	$(\sin x) = \frac{16 \times \sin 50}{13}$ (= 0.943 but accept 0.94)	A1
		dM1 A1
	x = awrt 70.5(3) **and** 109.5 **or** 70.6 and 109.4	**(4)**
		[4]

Question Number	Scheme	Marks
3.	<table><tr><td>x</td><td>0</td><td>0.5</td><td>1</td><td>1.5</td><td>2</td></tr><tr><td>y</td><td>1</td><td>2.821</td><td>**6**</td><td>12.502</td><td>26.585</td></tr></table>	
(a)	$\{\text{At } x=1,\}\ y=6$ (allow 6.000 or even 6.00)	B1 cao
		(1)
(b)	$\frac{1}{2}\times 0.5$;	B1 oe
	$\underline{\{1+26.585+2(2.821+\text{their }6+12.502)\}}$ <u>For structure of {................}</u>;	M1<u>A1ft</u>
	$\frac{1}{2}\times 0.5\ \underline{\{1+26.585+2(2.821+\ 6+12.502)\}}\ \{=\ \frac{1}{4}(70.231)=17.557..\}$ = awrt 17.56	A1
		(4)
(c)	10 + "17.56" = "27.56"	B1ft
		(1)
		[6]

Question Number	Scheme	Marks
4. (a)	**Usually answered in radians: Uses** $BCD=3.5\times(angle)$, $=3.5\times 1.77=6.195$ (m) (accept awrt 6.20)	M1 A1
		(2)
(b)	Area $=\frac{1}{2}(3.5)^2\times 1.77=10.84$ (m^2)	M1 A1
		(2)
(c)	Area of triangle = $\frac{1}{2}\times 3.7\times 3.5\times\sin(\text{angle})$, $=\frac{1}{2}\times 3.7\times 3.5\times\sin(\frac{\pi}{2}-\frac{1.77}{2})$ (=awrt 4.1)	M1, A1
	Total area = $"10.84"+2\times"4.101"$	M1
	= 19.04	A1cao
		(4)
		[8]

Question number	Scheme	Marks
5	$x^2 + y^2 - 10x + 6y + 30 = 0$	
(a)	Uses any appropriate method to find the coordinates of the centre, e.g achieves $\underline{(x \pm 5)^2} + \underline{\underline{(y \pm 3)^2}} = \ldots$. Accept (±5,±3) as indication of this.	M1
	Centre is $(5, -3)$.	A1 **(2)**
(b) Way 1	Uses $\underline{(x \pm "5")^2 - "5^2"} + \underline{\underline{(y \pm "3")^2 - "3^2"}} + 30 = 0$ to give $r = \sqrt{"25"+"9"-30}$ or $r^2 = "25"+"9"-30$ (not $30 - 25 - 9$)	M1
	$r = 2$	A1cao **(2)**
Or Way 2	Using $\sqrt{g^2 + f^2 - c}$ from $x^2 + y^2 + 2gx + 2fy + c = 0$ (Needs formula stated or correct working)	M1
	$r = 2$	A1 **(2)**
(c) Way 1	Use $x = 4$ in *an* equation of circle and obtain equation in y only	M1
	e.g $(4-5)^2 + (y+3)^2 = 4$ or $4^2 + y^2 - 10 \times 4 + 6y + 30 = 0$	
	Solve their quadratic in y and obtain **two** solutions for y	dM1
	e.g. $(y+3)^2 = 3$ or $y^2 + 6y + 6 = 0$ so $y = -3 \pm \sqrt{3}$	A1 **(3)**
Or Way 2	 Divide triangle PTQ and use Pythagoras with $"r"^2 - ("5" - 4)^2 = h^2$,	M1
	Find h and evaluate $"-3" \pm h$. May recognise (1,√3, 2) triangle.	dM1
	So $y = -3 \pm \sqrt{3}$	A1 **(3)** **[7]**

Question Number	Scheme	Marks
6. (a)	Attempt $f(3)$ or $f(-3)$ Use of long division is M0A0 as factor theorem was required.	M1
	$f(-3) = 162 - 63 - 120 + 21 = 0$ so $(x + 3)$ is a factor	A1 **(2)**
(b)	Either (Way 1): $f(x) = (x + 3)(-6x^2 + 11x + 7)$	M1A1
	$= (x + 3)(-3x + 7)(2x + 1)$ or $-(x + 3)(3x - 7)(2x + 1)$	M1A1 **(4)**
	Or (Way 2) Uses trial or factor theorem to obtain $x = -1/2$ **or** $x = 7/3$	M1
	Uses trial or factor theorem to obtain both $x = -1/2$ **and** $x = 7/3$	A1
	Puts three factors together (see notes below)	M1
	Correct factorisation : $(x + 3)(7 - 3x)(2x + 1)$ or $-(x + 3)(3x - 7)(2x + 1)$ oe	A1 **(4)**
	Or (Way 3) No working three factors $(x + 3)(-3x + 7)(2x + 1)$ otherwise need working	M1A1M1A1 (4)
(c)	$2^y = \frac{7}{3}, \rightarrow \log\left(2^y\right) = \log\left(\frac{7}{3}\right)$ or $y = \log_2\left(\frac{7}{3}\right)$ or $\frac{\log(7/3)}{\log 2}$	B1, M1
	$\{y = 1.222392421...\} \Rightarrow y =$ awrt 1.22	A1 **(3)** **[9]**

Question Number	Scheme	Marks
7. (i)	Use of power rule so $\log(x+a)^2 = \log 16a^6$ or $2\log(x+a) = 2\log 4a^3$ or $\log(x+a) = \log(16a^6)^{\frac{1}{2}}$	M1
	Removes logs and square roots, **or** halves then removes logs to give $(x+a) = 4a^3$ Or $x^2 + 2ax + a^2 - 16a^6 = 0$ followed by factorisation or formula to give $x = \sqrt{16a^6} - a$	M1
	$(x =)$ $4a^3 - a$ (depends on previous M's and must be this expression or equivalent)	A1cao **(3)**
(ii) Way 1	$\log_3 \frac{(9y+b)}{(2y-b)} = 2$ Applies quotient law of logarithms	M1
	$\frac{(9y+b)}{(2y-b)} = 3^2$ Uses $\log_3 3^2 = 2$	M1
	$(9y+b) = 9(2y-b) \Rightarrow y =$ Multiplies across and makes y the subject	M1
	$y = \frac{10}{9}b$	A1cso **(4)**
Way 2	Or : $\log_3(9y+b) = \log_3 9 + \log_3(2y-b)$ 2nd M mark	M1
	$\log_3(9y+b) = \log_3 9(2y-b)$ 1st M mark	M1
	$(9y+b) = 9(2y-b) \Rightarrow y = \frac{10}{9}b$ Multiplies across and makes y the subject	M1 A1cso **(4)** **[7]**

Question Number	Scheme		Marks
8. (a)	Way 1	Way 2	
	$1-\sin^2 x = 8\sin^2 x - 6\sin x$	$2=(3\sin x-1)^2$ gives $9\sin^2 x - 6\sin x + 1 = 2$ so $\sin^2 x + 8\sin^2 x - 6\sin x + 1 = 2$	B1
	E.g. $9\sin^2 x - 6\sin x = 1$ or $9\sin^2 x - 6\sin x - 1 = 0$ or $9\sin^2 x - 6\sin x + 1 = 2$	so $8\sin^2 x - 6\sin x = 1-\sin^2 x$	M1
	So $9\sin^2 x - 6\sin x + 1 = 2$ or $(3\sin x - 1)^2 - 2 = 0$ so $(3\sin x - 1)^2 = 2$ or $2=(3\sin x-1)^2$ *	$8\sin^2 x - 6\sin x = \cos^2 x$ *	A1cso*
			(3)
(b)	Way 1: $(3\sin x - 1) = (\pm)\sqrt{2}$	Way 2: Expands $(3\sin x-1)^2 = 2$ and uses quadratic formula on 3TQ	M1
	$\sin x = \dfrac{1\pm\sqrt{2}}{3}$ or awrt 0.8047 **and** awrt − 0.1381		A1
	x = 53.58 , 126.42 (or 126.41), 352.06, 187.94		dM1A1 A1
			(5) **[8]**

Question Number	Scheme	Marks
9.(a)	$a = 7k-5,\ ar = 5k-7$ and $ar^2 = 2k+10$	B1
	(So $r=$) $\dfrac{5k-7}{7k-5} = \dfrac{2k+10}{5k-7}$ or $(7k-5)(2k+10) = (5k-7)^2$ or equivalent	M1
	See $(5k-7)^2 = 25k^2 - 70k + 49$	M1
	$14k^2 + 60k - 50 = 25k^2 - 70k + 49 \rightarrow 11k^2 - 130k + 99 = 0$ *	A1cso *
		(4)
(b)	$(k-11)(11k-9)$ so $k=$	M1
	$k = 9/11$ only* (after rejecting 11)	A1*
	N.B. Special case $k = 9/11$ can be verified in (b) (1 mark only)	
	$11\times\left(\dfrac{9}{11}\right)^2 - 130\times\left(\dfrac{9}{11}\right) + 99 = \dfrac{81}{11} - \dfrac{1170}{11} + \dfrac{1089}{11} = 0$ M1A0	
		(2)
(c)	$a = \dfrac{8}{11}$	B1
	$\dfrac{5\times\frac{9}{11}-7}{7\times\frac{9}{11}-5}$ *or* $\dfrac{2\times\frac{9}{11}+10}{5\times\frac{9}{11}-7}$ so $r=-4$	B1
	(i) Fourth term = $ar^3 = -\dfrac{512}{11}$	M1A1
	(ii) $S_{10} = \dfrac{a(1-r^{10})}{(1-r)} = \dfrac{\frac{8}{11}(1-(-4)^{10})}{(1-(-4))} = -152520$	M1A1

		(6) **[12]**

Question Number	Scheme	Marks
10. (a)	$\frac{dy}{dx} = 12x^2 + 18x - 30$	M1
	Either: Substitute $x = 1$ to give $\frac{dy}{dx} = 12 + 18 - 30 = 0$ / Or: Solve $\frac{dy}{dx} = 12x^2 + 18x - 30 = 0$ to give $x =$	A1
	Either: So turning point (all correct work so far) / Or: Deduce $x = 1$ from correct work	A1cso (3)
(b) Way 1	When $x = 1$, $y = 4 + 9 - 30 - 8 = -25$	B1
	Area of triangle $ABP = \frac{1}{2} \times 1 \times 25 = 12.5$ (Where P is at (1, 0))	B1
	Way 1: $\int (4x^3 + 9x^2 - 30x - 8)\,dx = x^4 + \frac{9}{3}x^3 - \frac{30x^2}{2} - 8x \{+c\}$ *or* $x^4 + 3x^3 - 15x^2 - 8x \{+c\}$	M1A1
	$\left[x^4 + 3x^3 - 15x^2 - 8x\right]_{-\frac{1}{4}}^{1} = (1 + 3 - 15 - 8) - \left(\left(-\frac{1}{4}\right)^4 + 3\left(-\frac{1}{4}\right)^3 - 15\left(-\frac{1}{4}\right)^2 - 8\left(-\frac{1}{4}\right)\right)$ $= (-19) - \frac{261}{256}$ or $-19 - 1.02$	dM1
	So Area = "*their* 12.5" + "*their* $20\frac{5}{256}$" or "12.5" + "20.02" or "12.5" + "*their* $\frac{5125}{256}$"	ddM1
	$= 32.52$ (NOT -32.52)	A1 (7) **[10]**

TOTAL FOR PART 2: 75 MARKS

Part 3

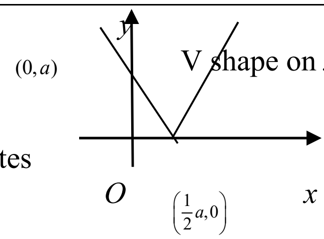
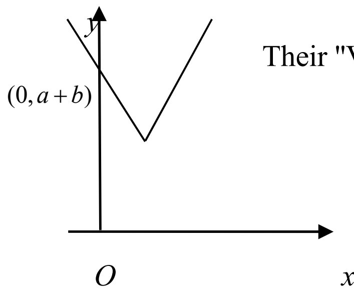
Question Number	Scheme	Marks
1.	$x^2-9=(x+3)(x-3)$	B1
	$\frac{4x}{x^2-9}-\frac{2}{(x+3)}=\frac{4x-2(x-3)}{(x+3)(x-3)}$	M1
	$=\frac{2x+6}{(x+3)(x-3)}$	A1
	$=\frac{2\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$	
	$=\frac{2}{(x-3)}$	A1
		(4)

Question Number	Scheme	Marks
2.(a)	$e^{3x-9}=8\Rightarrow 3x-9=\ln 8$	M1
	$\Rightarrow x=\frac{\ln 8+9}{3},=\ln 2+3$	A1, A1
		(3)
(b)	$\ln(2y+5)=2+\ln(4-y)$	
	$\ln\left(\frac{2y+5}{4-y}\right)=2$	M1
	$\left(\frac{2y+5}{4-y}\right)=e^2$	M1
	$2y+5=e^2(4-y)\Rightarrow 2y+e^2y=4e^2-5\Rightarrow y=\frac{4e^2-5}{2+e^2}$	dM1, A1
		(4)
		7 marks

Question Number	Scheme	Marks
3.(a)	$y \geqslant 3$	B1
		(1)
(b)	$y=3+\sqrt{x+2} \Rightarrow y-3=\sqrt{x+2} \Rightarrow x=(y-3)^2-2$	M1 A1
	$\Rightarrow g^{-1}(x)=(x-3)^2-2, \quad \text{with } x \geqslant 3$	A1
		(3)
(c)	$g(x)=x \Rightarrow 3+\sqrt{x+2}=x$	
	$\Rightarrow x+2=(x-3)^2 \Rightarrow x^2-7x+7=0$	M1, A1
	$\Rightarrow x=\dfrac{7\pm\sqrt{21}}{2} \Rightarrow x=\dfrac{7+\sqrt{21}}{2}$ only	M1, A1
		(4)
(d)	$a=\dfrac{7+\sqrt{21}}{2}$	B1 ft
		(1)
		9 marks

Question Number	Scheme	Marks
4.(a)	$R=\sqrt{29}$	B1
	$\tan\alpha=\dfrac{2}{5} \Rightarrow \alpha = \text{awrt } 0.381$	M1A1
		(3)
(b)	$5\cot 2x-3\operatorname{cosec} 2x=2 \Rightarrow 5\dfrac{\cos 2x}{\sin 2x}-\dfrac{3}{\sin 2x}=2$	M1
	$\Rightarrow 5\cos 2x-2\sin 2x=3$	A1
		(2)
(c)	$5\cos 2x-2\sin 2x=3 \Rightarrow \cos(2x+0.381)=\dfrac{3}{\sqrt{29}}$	M1
	$2x+0.381=\arccos\left(\dfrac{3}{\sqrt{29}}\right) \Rightarrow x=...$	dM1
	$x = \text{awrt } 0.30, 2.46$	A1A1
		(4)
		(9 marks)

Question Number	Scheme	Marks
5. (a)	At P $x=-2 \Rightarrow y=3$	B1
	$\frac{dy}{dx}=\frac{4}{2x+5}-\frac{3}{2}$	M1, A1
	$\left.\frac{dy}{dx}\right\|_{x=-2}=\frac{5}{2} \Rightarrow$ Equation of normal is $y-'3'=-\frac{2}{5}\left(x-(-2)\right)$	M1
	$\Rightarrow 2x+5y=11$	A1
		(5)
(b)	Combines $5y+2x=11$ and $y=2\ln(2x+5)-\frac{3x}{2}$ to form equation in x	
	$5\left(2\ln(2x+5)-\frac{3x}{2}\right)+2x=11$	M1
	$\Rightarrow x=\frac{20}{11}\ln(2x+5)-2$	dM1 A1*
		(3)
(c)	Substitutes $x_1=2 \Rightarrow x_2=\frac{20}{11}\ln 9-2$	M1
	Awrt $x_2=1.9950$ and $x_3=1.9929$.	A1
		(2)
		(10 marks)

Question Number	Scheme	Marks
6.(a)(i)	 V shape on x - axis **or** coordinates $\left(\frac{1}{2}a, 0\right)$ **and** $(0, a)$ Correct shape, position and coordinates	B1 B1
(ii)	 Their "V" shape translated up or $(0, a+b)$ Correct shape, position and $(0, a+b)$	B1ft B1 **(4)**
(b)	States or uses $a+b=8$ Attempts to solve $\lvert 2x-a\rvert + b = \frac{3}{2}x+8$ in either x or with $x = c$ $2c - a + b = \frac{3}{2}c + 8 \Rightarrow kc = \mathrm{f}(a,b)$ Combines $kc = \mathrm{f}(a,b)$ with $a+b=8 \quad \Rightarrow c = 4a$	B1 M1 dM1 A1 (4) **(8 marks)**

Question Number	Scheme	Marks
7(i) (a)	$y = 2x\left(x^2-1\right)^5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \left(x^2-1\right)^5 \times 2 + 2x \times 10x\left(x^2-1\right)^4$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \left(x^2-1\right)^4\left(2x^2-2+20x^2\right) = \left(x^2-1\right)^4\left(22x^2-2\right)$	M1A1 M1 A1 (4)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} \geqslant 0 \Rightarrow \left(22x^2-2\right) \geqslant 0 \Rightarrow$ critical values of $\pm\frac{1}{\sqrt{11}}$ $x \geqslant \frac{1}{\sqrt{11}} \quad x \leqslant -\frac{1}{\sqrt{11}}$	M1 A1 (2)
(ii)	$x = \ln(\sec 2y) \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{\sec 2y} \times 2\sec 2y \tan 2y$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2\tan 2y} = \frac{1}{2\sqrt{\sec^2 2y - 1}} = \frac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$	B1 M1 M1 A1 (4) **10 marks**

Question Number	Scheme	Marks
8 (a)	$P_0 = \dfrac{100}{1+3} + 40 = 65$	B1
		(1)
	$\dfrac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{kt} = C\mathrm{e}^{kt}$	M1
(b)	$\dfrac{\mathrm{d}P}{\mathrm{d}t} = \dfrac{\left(1+3\mathrm{e}^{-0.9t}\right)\times -10\mathrm{e}^{-0.1t} - 100\mathrm{e}^{-0.1t}\times -2.7\mathrm{e}^{-0.9t}}{\left(1+3\mathrm{e}^{-0.9t}\right)^2}$	M1 A1
		(3)
(c)(i)	At maximum $-10\mathrm{e}^{-0.1t} - 30\mathrm{e}^{-0.1t}\times\mathrm{e}^{-0.9t} + 270\mathrm{e}^{-0.1t}\times\mathrm{e}^{-0.9t} = 0$ $\mathrm{e}^{-0.1t}\left(-10+240\mathrm{e}^{-0.9t}\right) = 0$ $\mathrm{e}^{-0.9t} = \dfrac{10}{240}$ oe	M1
	$\mathrm{e}^{0.9t} = 24$	
	$-0.9t = \ln\left(\dfrac{1}{24}\right) \Rightarrow t = \dfrac{10}{9}\ln(24) = 3.53$	M1, A1
(c) (ii)	Sub $t = 3.53 \Rightarrow P_T = 102$	A1
		(4)
(d)	40	B1
		(1)
		9 marks

Question Number	Scheme	Marks
9(a)	$\sin 2x - \tan x = 2\sin x\cos x - \tan x$	M1
	$= \dfrac{2\sin x\cos^2 x}{\cos x} - \dfrac{\sin x}{\cos x}$	M1
	$= \dfrac{\sin x}{\cos x}\times(2\cos^2 x - 1)$	
	$= \tan x\cos 2x$	dM1 A1*
		(4)
(b)	$\tan x\cos 2x = 3\tan x\sin x \Rightarrow \tan x(\cos 2x - 3\sin x) = 0$	
	$\cos 2x - 3\sin x = 0$	M1
	$\Rightarrow 1 - 2\sin^2 x - 3\sin x = 0$	M1
	$\Rightarrow 2\sin^2 x + 3\sin x - 1 = 0 \Rightarrow \sin x = \dfrac{-3\pm\sqrt{17}}{4} \Rightarrow x = \ldots$	M1
	Two of $x = 16.3°, 163.7°, 0, 180°$	A1
	All four of $x = 16.3°, 163.7°, 0, 180°$	A1
		(5)
		(9 marks)

TOTAL FOR PART 3: 75 MARKS

Part 4

Question Number	Scheme	Marks
1.	$x = 3t - 4,\ \ y = 5 - \dfrac{6}{t},\quad t > 0$	
(a)	$\dfrac{dx}{dt} = 3,\quad \dfrac{dy}{dt} = 6t^{-2}$	
	$\dfrac{dy}{dx} = \dfrac{6t^{-2}}{3}\ \left\{= \dfrac{6}{3t^2} = 2t^{-2} = \dfrac{2}{t^2}\right\}$	M1 A1 isw
		[2]
(b)	$\left\{t = \dfrac{1}{2} \Rightarrow\right\} P\left(-\dfrac{5}{2}, -7\right)$	B1
	$\dfrac{dy}{dx} = \dfrac{2}{\left(\frac{1}{2}\right)^2}$ **and** either • $y - "-7" = "8"\left(x - "-\frac{5}{2}"\right)$ • $"-7" = ("8")("-\frac{5}{2}") + c$ So, $y = (\text{their } m_T)x + "c"$	M1
	T: $y = 8x + 13$	A1 **cso**
		[3]
(c)	$\left\{t = \dfrac{x+4}{3} \Rightarrow\right\}\ y = 5 - \dfrac{6}{\left(\dfrac{x+4}{3}\right)}$	M1 A1 o.e.
	$\Rightarrow\ y = 5 - \dfrac{18}{x+4}\quad \Rightarrow\quad y = \dfrac{5(x+4) - 18}{x+4}$	
	So, $y = \dfrac{5x+2}{x+4},\quad \{x > -4\}$	A1 **cso**
		[3]
		8

Question Number	Scheme	Marks
2.	$\left\{(2+kx)^{-3} = 2^{-3}\left(1+\frac{kx}{2}\right)^{-3} = \frac{1}{8}\left(1+(-3)\left(\frac{kx}{2}\right)+\frac{(-3)(-3-1)}{2!}\left(\frac{kx}{2}\right)^2+\ldots\right)\right\}, k>0$	
(a)	$\{A=\}\ \frac{1}{8}$	B1 **cao**
		[1]
(b)	$\left(\frac{1}{8}\right)\frac{(-3)(-4)}{2!}\left(\frac{k}{2}\right)^2$	M1 M1 o.e.
	$\left\{\text{So, } \left(\frac{1}{8}\right)\frac{(-3)(-4)}{2!}\left(\frac{k}{2}\right)^2 = \frac{243}{16} \Rightarrow \frac{3}{16}k^2 = \frac{243}{16} \Rightarrow k^2 = 81\right\}$	
	So, $k=9$	A1 **cso**
		[3]
(c)	$"\left(\frac{1}{8}\right)"(-3)\left(\frac{k}{2}\right)$	M1
	$\left\{\text{So, } B = \left(\frac{1}{8}\right)(-3)\left(\frac{9}{2}\right) \Rightarrow\right\}\ \underline{B = -\frac{27}{16}}$	A1 **cso**
		[2]
		6

Question Number	Scheme	Marks
3.	x: 0, 0.2, 0.4, 0.6, 0.8, 1; y: 2, ***1.8625426...***, 1.71830, 1.56981, 1.41994, 1.27165	
(a)	$\{\text{At } x = 0.2,\}\ y = 1.86254$ (5 dp)	B1 **cao**
		[1]
(b)	$\frac{1}{2}(0.2)\left[2 + 1.27165 + 2\left(\text{their } 1.86254 + 1.71830 + 1.56981 + 1.41994\right)\right]$	B1 o.e. M1
	$\left\{= \frac{1}{10}(16.41283)\right\} = 1.641283 = 1.6413$ (4 dp)	A1
		[3]
(c)	$\left\{u = \mathrm{e}^x \text{ or } x = \ln u \Rightarrow\right\}$	
	$\frac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x$ **or** $\frac{\mathrm{d}u}{\mathrm{d}x} = u$ **or** $\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{1}{u}$ **or** $\mathrm{d}u = u\,\mathrm{d}x$ etc., **and** $\int \frac{6}{(\mathrm{e}^x + 2)}\mathrm{d}x = \int \frac{6}{(u+2)u}\,\mathrm{d}u$	B1 *
	$\{x = 0\} \Rightarrow a = \mathrm{e}^0 \Rightarrow \underline{a = 1}$ $\{x = 1\} \Rightarrow b = \mathrm{e}^1 \Rightarrow \underline{b = \mathrm{e}}$	B1
		[2]
(d)	$\frac{6}{u(u+2)} \equiv \frac{A}{u} + \frac{B}{(u+2)}$	M1
	$\Rightarrow 6 \equiv A(u+2) + Bu$ $u = 0 \Rightarrow A = 3$ $u = -2 \Rightarrow B = -3$	A1
	$\int \frac{6}{u(u+2)}\,\mathrm{d}u = \int \left(\frac{3}{u} - \frac{3}{(u+2)}\right)\mathrm{d}u$	M1
	$= 3\ln u - 3\ln(u+2)$ or $= 3\ln 2u - 3\ln(2u+4)$	A1 ft
	$\left\{\text{So } \left[3\ln u - 3\ln(u+2)\right]_1^{\mathrm{e}}\right\}$ $= \left(3\ln(\mathrm{e}) - 3\ln(\mathrm{e}+2)\right) - \left(3\ln 1 - 3\ln 3\right)$	dM1
	$= 3 - 3\ln(\mathrm{e}+2) + 3\ln 3$ **or** $3(1 - \ln(\mathrm{e}+2) + \ln 3)$ **or** $3 + 3\ln\left(\frac{3}{\mathrm{e}+2}\right)$ **or** $3\ln\left(\frac{\mathrm{e}}{\mathrm{e}+2}\right) - 3\ln\left(\frac{1}{3}\right)$ **or** $3 - 3\ln\left(\frac{\mathrm{e}+2}{3}\right)$ **or** $3\ln\left(\frac{3\mathrm{e}}{\mathrm{e}+2}\right)$ **or** $\ln\left(\frac{27\mathrm{e}^3}{(\mathrm{e}+2)^3}\right)$	A1 **cso**
		[6]
		12

Question Number	Scheme	Marks
4.	$4x^2 - y^3 - 4xy + 2^y = 0$	
(a)	$\left\{\frac{\cancel{dy}}{\cancel{dx}} = \right\} \underline{8x - 3y^2\frac{dy}{dx}}\ \underline{\underline{-4y - 4x\frac{dy}{dx}}} + \overline{\overline{2^y \ln 2\frac{dy}{dx}}} \underline{= 0}$	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$ $\overline{\overline{\text{B1}}}$
	$8(-2) - 3(4)^2\frac{dy}{dx} - 4(4) - 4(-2)\frac{dy}{dx} + 2^4 \ln 2\frac{dy}{dx} = 0$	dM1
	$-16 - 48\frac{dy}{dx} - 16 + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} = 0$	
	$\frac{dy}{dx} = \frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$ or exact equivalent	A1 **cso**
		[6]
(b)	e.g. $m_N = \frac{-40+16\ln 2}{-32}$ or $\frac{40-16\ln 2}{32}$	M1
	• $y - 4 = \left(\frac{40-16\ln 2}{32}\right)(x - -2)$ Cuts y-axis $\Rightarrow x = 0 \Rightarrow y - 4 = \left(\frac{40-16\ln 2}{32}\right)(2)$ $4 = \left(\frac{40-16\ln 2}{32}\right)(-2) + c$	M1
	$\left\{\Rightarrow c = 4 + \frac{40-16\ln 2}{16}, \text{ so } y = \frac{104-16\ln 2}{16} \Rightarrow\right\}$	
	y (or c) $= \frac{13}{2} - \ln 2$	A1 **cso isw**
		[3]
		9

Question Number	Scheme	Marks
7.	$\frac{dh}{dt} = k\sqrt{(h-9)}, \quad 9 < h \le 200; \quad h = 130, \frac{dh}{dt} = -1.1$	
(a)	$-1.1 = k\sqrt{(130-9)} \Rightarrow k = ...$	M1
	so, $k = -\frac{1}{10}$ or -0.1	A1
		[2]
(b)	$\int \frac{dh}{\sqrt{(h-9)}} = \int k\,dt$	B1
	$\int (h-9)^{-\frac{1}{2}}\,dh = \int k\,dt$	
	$\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt\,(+c)$	M1 A1
	$\{t = 0, h = 200 \Rightarrow\}\ 2\sqrt{(200-9)} = k(0) + c$	M1
	$\Rightarrow c = 2\sqrt{191} \Rightarrow 2(h-9)^{\frac{1}{2}} = -0.1t + 2\sqrt{191}$ $\{h = 50 \Rightarrow\}\ 2\sqrt{(50-9)} = -0.1t + 2\sqrt{191}$ $t = ...$	dM1
	$t = 20\sqrt{191} - 20\sqrt{41}$ or $t = 148.3430145... = 148$ (minutes) (nearest minute)	A1 cso
		[6]
		8

Question Number	Scheme	Marks
8.	$x = 3\theta\sin\theta,\ \ y = \sec^3\theta,\quad 0 \le \theta < \frac{\pi}{2}$	
(a)	$\{\text{When } y = 8,\}\ 8 = \sec^3\theta \Rightarrow \cos^3\theta = \frac{1}{8} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ $k \text{ (or } x) = 3\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{3}\right)$	M1
	so $k \text{ (or } x) = \frac{\sqrt{3}\pi}{2}$	A1
		[2]
(b)	$\frac{dx}{d\theta} = 3\sin\theta + 3\theta\cos\theta$	B1
	$\left\{\int y\frac{dx}{d\theta}\{d\theta\}\right\} = \int(\sec^3\theta)(3\sin\theta + 3\theta\cos\theta)\{d\theta\}$	M1
	$= 3\int \theta\sec^2\theta + \tan\theta\sec^2\theta\ d\theta$	A1 *
	$x = 0$ and $x = k \Rightarrow \underline{\alpha = 0}$ and $\underline{\beta = \frac{\pi}{3}}$	B1
		[4]
(c)	$\left\{\int \theta\sec^2\theta\, d\theta\right\} = \theta\tan\theta - \int\tan\theta\{d\theta\}$	M1 dM1
	$= \theta\tan\theta - \ln(\sec\theta)$ **or** $= \theta\tan\theta + \ln(\cos\theta)$	A1
	$\left\{\int \tan\theta\sec^2\theta\, d\theta\right\} = \frac{1}{2}\tan^2\theta$ or $\frac{1}{2}\sec^2\theta$ or $\frac{1}{2u^2}$ where $u = \cos\theta$ or $\frac{1}{2}u^2$ where $u = \tan\theta$	M1 A1
	$\{\text{Area}(R)\} = \left[3\theta\tan\theta - 3\ln(\sec\theta) + \frac{3}{2}\tan^2\theta\right]_0^{\frac{\pi}{3}}$ **or** $\left[3\theta\tan\theta - 3\ln(\sec\theta) + \frac{3}{2}\sec^2\theta\right]_0^{\frac{\pi}{3}}$	
	$= \left(3\left(\frac{\pi}{3}\right)\sqrt{3} - 3\ln 2 + \frac{3}{2}(3)\right) - (0)$ **or** $\left(3\left(\frac{\pi}{3}\right)\sqrt{3} - 3\ln 2 + \frac{3}{2}(4)\right) - \left(\frac{3}{2}\right)$	
	$= \frac{9}{2} + \sqrt{3}\pi - 3\ln 2$ **or** $\frac{9}{2} + \sqrt{3}\pi + 3\ln\left(\frac{1}{2}\right)$ **or** $\frac{9}{2} + \sqrt{3}\pi - \ln 8$ **or** $\ln\left(\frac{1}{8}e^{\frac{9}{2}+\sqrt{3}\pi}\right)$	A1 o.e.
		[6]
		12

TOTAL FOR PART 4: 55 MARKS

Total Marks: 275