4.

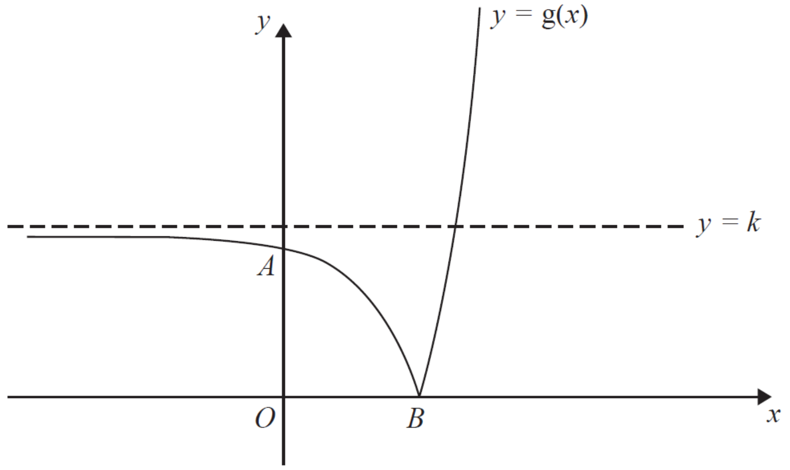

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = |4e^{2x} - 25|, \qquad x \in \mathbb{R}.$$

The curve cuts the y-axis at the point A and meets the x-axis at the point B. The curve has an asymptote $y = k$, where k is a constant, as shown in Figure 1.

(*a*) Find, giving each answer in its simplest form,

(i) the y coordinate of the point A,

(ii) the exact x coordinate of the point B,

(iii) the value of the constant k.

(5)

The equation $\mathrm{g}(x) = 2x + 43$ has a positive root at $x = \alpha$.

(*b*) Show that α is a solution of $x = \frac{1}{2}\ln\left(\frac{1}{2}x + 17\right)$.

(2)

The iteration formula

$$x_{n+1} = \frac{1}{2}\ln\left(\frac{1}{2}x_n + 17\right)$$

can be used to find an approximation for α.

(*c*) Taking $x_0 = 1.4$, find the values of x_1 and x_2. Give each answer to 4 decimal places.

(2)

(d) By choosing a suitable interval, show that $\alpha = 1.437$ to 3 decimal places.

(2)

(Total 11 marks)

5. (i) Find, using calculus, the x coordinate of the turning point of the curve with equation

$$y = e^{3x} \cos 4x, \quad \frac{\pi}{4} \leq x < \frac{\pi}{2}.$$

Give your answer to 4 decimal places.

(5)

(ii) Given $x = \sin^2 2y$, $0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of y.

Write your answer in the form

$$\frac{dy}{dx} = p \operatorname{cosec}(qy), \quad 0 < y < \frac{\pi}{4},$$

where p and q are constants to be determined.

(5)

(Total 10 marks)

---

6. $$f(x) = \frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6}, \qquad x > 2, \qquad x \in \mathbb{R}.$$

(*a*) Given that

$$\frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6} \equiv x^2 + A + \frac{B}{x - 2},$$

find the values of the constants A and B.

(4)

(*b*) Hence or otherwise, using calculus, find an equation of the normal to the curve with equation $y = f(x)$ at the point where $x = 3$.

(5)

(Total 9 marks)

---

7. (*a*) For $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, sketch the graph of $y = g(x)$ where

$$g(x) = \arcsin x, \qquad -1 \leq x \leq 1.$$

(2)

(*b*) Find the exact value of x for which

$$3g(x + 1) + \pi = 0.$$

(3)

(Total 5 marks)

---

8. (*a*) Prove that

$$2 \cot 2x + \tan x \equiv \cot x, \qquad x \neq \frac{n\pi}{2}, \qquad n \in \mathbb{Z}.$$

(4)

(*b*) Hence, or otherwise, solve, for $-\pi \leq x < \pi$,

$$6 \cot 2x + 3 \tan x = \text{cosec}^2 x - 2.$$

Give your answers to 3 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(Total 8 marks)

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9. The amount of an antibiotic in the bloodstream, from a given dose, is modelled by the formula

$$x = De^{-0.2t},$$

where x is the amount of the antibiotic in the bloodstream in milligrams, D is the dose given in milligrams and t is the time in hours after the antibiotic has been given.

A first dose of 15 mg of the antibiotic is given.

(*a*) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the dose is given. Give your answer in mg to 3 decimal places.

(2)

A second dose of 15 mg is given 5 hours after the first dose has been given. Using the same model for the second dose,

(*b*) show that the **total** amount of the antibiotic in the bloodstream 2 hours after the second dose is given is 13.754 mg to 3 decimal places.

(2)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the total amount of the antibiotic in the bloodstream is 7.5 mg.

(*c*) Show that $T = a \ln\left(b + \frac{b}{c}\right)$, where a and b are integers to be determined.

1. Use the binomial series to find the expansion of

$$\frac{1}{(2+5x)^3}, \quad |x| < \frac{2}{5},$$

in ascending powers of x, up to and including the term in x^3.
Give each coefficient as a fraction in its simplest form.

(Total 6 marks)

2.

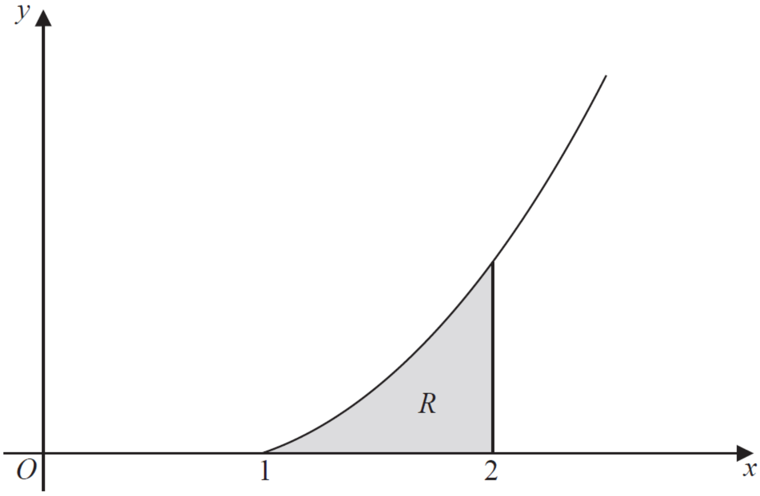

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = x^2 \ln x$, $x \geq 1$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$.

The table below shows corresponding values of x and y for $y = x^2 \ln x$.

x	1	1.2	1.4	1.6	1.8	2
y	0	0.2625		1.2032	1.9044	2.7726

(*a*) Complete the table above, giving the missing value of y to 4 decimal places.

(1)

(*b*) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of R, giving your answer to 3 decimal places.

(3)

(*c*) Use integration to find the exact value for the area of R.

(5)

(Total 9 marks)

3. The curve C has equation

$$2x^2y + 2x + 4y - \cos(\pi y) = 17.$$

(a) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P with coordinates $\left(3, \frac{1}{2}\right)$ lies on C.

The normal to C at P meets the x-axis at the point A.

(b) Find the x coordinate of A, giving your answer in the form $\frac{a\pi + b}{c\pi + d}$, where a, b, c and d are integers to be determined.

(4)

(Total 9 marks)

---

4. The rate of decay of the mass of a particular substance is modelled by the differential equation

$$\frac{dx}{dt} = -\frac{5}{2}x, \quad t \geq 0,$$

where x is the mass of the substance measured in grams and t is the time measured in days.

Given that $x = 60$ when $t = 0$,

(a) solve the differential equation, giving x in terms of t. You should show all steps in your working and give your answer in its simplest form.

(4)

(b) Find the time taken for the mass of the substance to decay from 60 grams to 20 grams.
Give your answer to the nearest minute.

(3)

(Total 7 marks)

5.

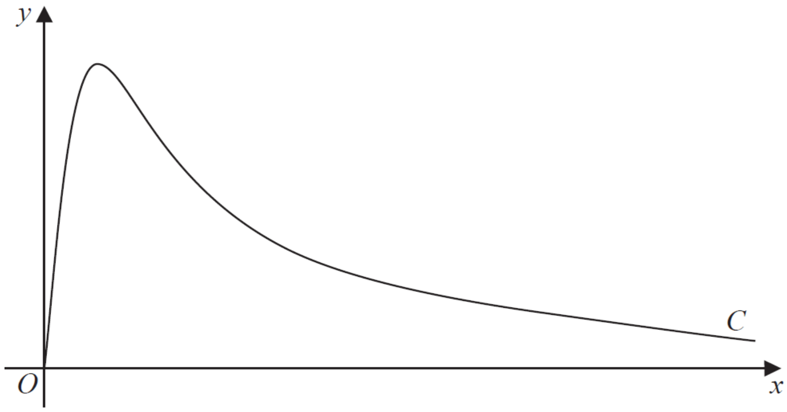

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\tan t, \qquad y = 5\sqrt{3}\sin 2t, \quad 0 \leq t < \frac{\pi}{2}.$$

The point P lies on C and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$.

(a) Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point P.

Give your answer as a simplified surd.

(4)

The point Q lies on the curve C, where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$.

(b) Find the exact coordinates of the point Q.

(2)

(Total 6 marks)

6. (i) Given that $y > 0$, find

$$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y\,.$$

(6)

(ii) (*a*) Use the substitution $x = 4\sin^2\theta$ to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x = \lambda\int_0^{\frac{\pi}{3}} \sin^2\theta\,\mathrm{d}\theta\,,$$

where λ is a constant to be determined.

(5)

(*b*) Hence use integration to find

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x\,,$$

giving your answer in the form $a\pi + b$, where a and b are exact constants.

(4)

(Total 15 marks)