2. Express 9^{3x+1} in the form 3^y, giving y in the form $ax + b$, where a and b are constants.

(Total 2 marks)

4.

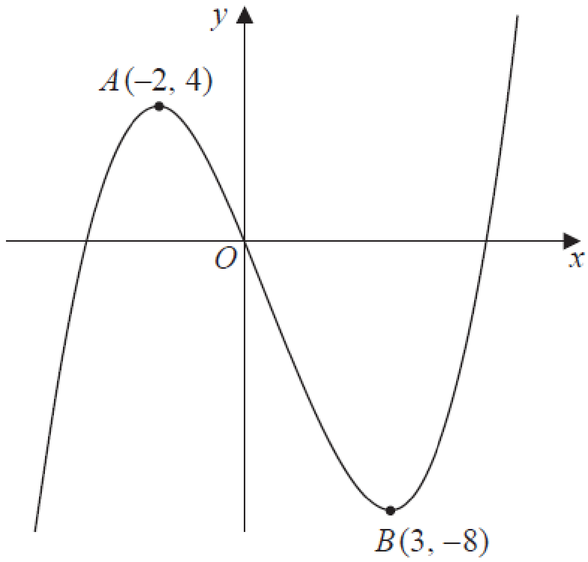

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$. The curve has a maximum point A at (–2, 4) and a minimum point B at (3, –8) and passes through the origin O.

On separate diagrams, sketch the curve with equation

(*a*) $y = 3\mathrm{f}(x)$,

(2)

(*b*) $y = \mathrm{f}(x) - 4$.

(3)

On each diagram, show clearly the coordinates of the maximum and the minimum points and the coordinates of the point where the curve crosses the y-axis.

(Total 5 marks)

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5. Solve the simultaneous equations

$$y + 4x + 1 = 0$$

$$y^2 + 5x^2 + 2x = 0$$

(Total 6 marks)

---

6. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4,$$

$$a_{n+1} = 5 - ka_n, \quad n \geq 1,$$

where k is a constant.

(a) Write down expressions for a_2 and a_3 in terms of k.

(2)

Find

(b) $\sum_{r=1}^{3}(1+a_r)$ in terms of k, giving your answer in its simplest form,

(3)

(c) $\sum_{r=1}^{100}(a_{r+1}+ka_r)$.

(1)

(Total 6 marks)

---

7. Given that

$$y = 3x^2 + 6x^{\frac{1}{3}} + \frac{2x^3 - 7}{3\sqrt{x}}, \quad x > 0,$$

find $\frac{dy}{dx}$. Give each term in your answer in its simplified form.

(Total 6 marks)

9. On John's 10th birthday he received the first of an annual birthday gift of money from his uncle. This first gift was £60 and on each subsequent birthday the gift was £15 more than the year before. The amounts of these gifts form an arithmetic sequence.

(a) Show that, immediately after his 12th birthday, the total of these gifts was £225.

(1)

(b) Find the amount that John received from his uncle as a birthday gift on his 18th birthday.

(2)

(c) Find the total of these birthday gifts that John had received from his uncle up to and including his 21st birthday.

(3)

When John had received n of these birthday gifts, the total money that he had received from these gifts was £3375.

(d) Show that $n^2 + 7n = 25 \times 18$.

(3)

(e) Find the value of n, when he had received £3375 in total, and so determine John's age at this time.

(2)

(Total 11 marks)

1. A geometric series has first term a and common ratio $r = \frac{3}{4}$.

The sum of the first 4 terms of this series is 175.

(a) Show that $a = 64$.

(2)

(b) Find the sum to infinity of the series.

(2)

(c) Find the difference between the 9th and 10th terms of the series.
Give your answer to 3 decimal places.

(3)

(Total 7 marks)

3.

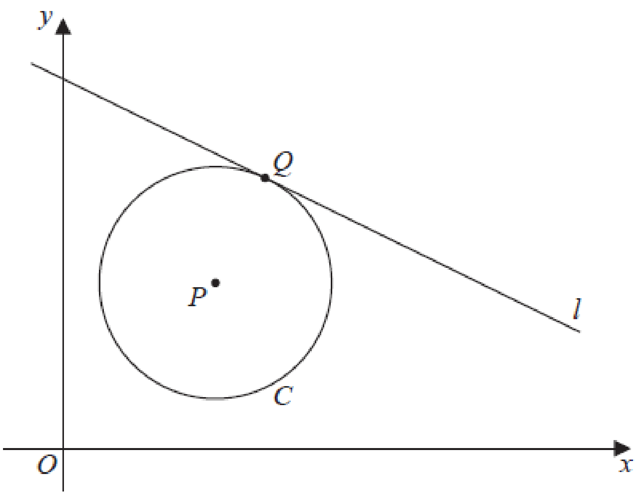

Diagram not drawn to scale

Figure 2

The circle C has centre $P(7, 8)$ and passes through the point $Q(10, 13)$, as shown in Figure 2.

(a) Find the length PQ, giving your answer as an exact value.

(2)

(b) Hence write down an equation for C.

(2)

The line l is a tangent to C at the point Q, as shown in Figure 2.

(c) Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

(Total 8 marks)

5. (*a*) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 9x)^4,$$

giving each term in its simplest form.

(4)

$$f(x) = (1 + kx)(2 - 9x)^4, \quad \text{where } k \text{ is a constant.}$$

The expansion, in ascending powers of x, of $f(x)$ up to and including the term in x^2 is

$$A - 232x + Bx^2,$$

where A and B are constants.

(*b*) Write down the value of A.

(1)

(*c*) Find the value of k.

(2)

(*d*) Hence find the value of B.

(2)

(Total 9 marks)

---

6. (i) Solve, for $-\pi < \theta \leq \pi$,

$$1 - 2\cos\left(\theta - \frac{\pi}{5}\right) = 0,$$

giving your answers in terms of π.

(3)

(ii) Solve, for $0 \leq x < 360°$,

$$4\cos^2 x + 7\sin x - 2 = 0,$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(Total 9 marks)

8 (i) Given that

$$\log_3(3b + 1) - \log_3(a - 2) = -1, \qquad a > 2,$$

express b in terms of a.

(3)

(ii) Solve the equation

$$2^{2x + 5} - 7(2^x) = 0,$$

giving your answer to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(Total 7 marks)

9.

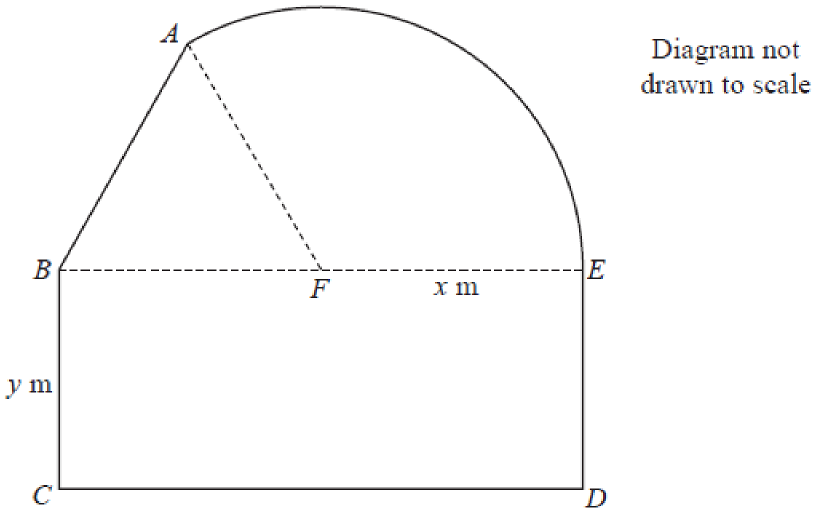

Figure 4

Figure 4 shows a plan view of a sheep enclosure.

The enclosure *ABCDEA*, as shown in Figure 4, consists of a rectangle *BCDE* joined to an equilateral triangle *BFA* and a sector *FEA* of a circle with radius *x* metres and centre *F*.

The points *B*, *F* and *E* lie on a straight line with $FE = x$ metres and $10 \leq x \leq 25$.

(*a*) Find, in m^2, the exact area of the sector *FEA*, giving your answer in terms of *x*, in its simplest form.

(2)

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is $1000\ m^2$,

(*b*) show that

$$y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right).$$

(3)

(*c*) Hence show that the perimeter *P* metres of the enclosure is given by

$$P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right).$$

(3)

(*d*) Use calculus to find the minimum value of *P*, giving your answer to the nearest metre.

(5)

(*e*) Justify, by further differentiation, that the value of *P* you have found is a minimum.

(2)

(Total 15 marks)

1. The functions f and g are defined by

$$f : x \to 7x - 1, \qquad x \in \mathbb{R},$$

$$g : x \to \frac{4}{x-2}, \qquad x \neq 2, x \in \mathbb{R},$$

(a) Solve the equation fg(x) = x.

(4)

(b) Hence, or otherwise, find the largest value of a such that g(a) = $f^{-1}(a)$.

(1)

(Total 5 marks)

---

3. (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$
Give the exact value of R and give the value of α to 2 decimal places.

(3)

(b) Hence solve, for $0 \leq \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15.$$

Give your answers to one decimal place.

(5)

(c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of θ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15.$$

Give your answer to one decimal place.

(2)

(Total 10 marks)
