

Question Number	Scheme	Notes	Marks
2	$9^{3x+1} = \text{for example}$ $3^{2(3x+1)} \text{ or } (3^2)^{3x+1} \text{ or } (3^{3x+1})^2 \text{ or } 3^{3x+1} \times 3^{3x+1}$ $\text{or } (3 \times 3)^{3x+1} \text{ or } 3^2 \times (3^2)^{3x} \text{ or } (9^{\frac{1}{2}})^y \text{ or } 9^{\frac{1}{2}y}$ $\text{or } y = 2(3x+1)$ $= 3^{6x+2} \text{ or } y = 6x + 2 \text{ or } a = 6, b = 2$	Expresses 9^{3x+1} correctly as a power of 3 or expresses 3^y correctly as a power of 9 or expresses y correctly in terms of x (This mark is <u>not</u> for just $3^2 = 9$) Cao (isw if necessary)	M1
	Providing there is no incorrect work, allow sight of $6x + 2$ to score both marks		
	Correct answer only implies both marks		
	Special case: 3^{6x+1} only scores M1A0		
			[2]
	Alternative using logs		
	$9^{3x+1} = 3^y \Rightarrow \log 9^{3x+1} = \log 3^y$		
	$(3x+1)\log 9 = y\log 3$	Use power law correctly on both sides	M1
	$y = \frac{\log 9}{\log 3}(3x+1)$		
	$y = 6x + 2$	cao	A1
			2 marks

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	Note original points are $A(-2, 4)$ and $B(3, -8)$		
4.(a)	<p>The graph shows a cubic curve passing through the origin (0,0). It has a local maximum at the point (-2, 12) and a local minimum at the point (3, -24). The curve is symmetric about the point (0.5, -4).</p>	Similar shape to given figure passing through the origin. A cubic shape with a maximum in the second quadrant and a minimum in the 4 th quadrant. There must be evidence of a change in at least one of the y -coordinates (inconsistent changes in the y -coordinates are acceptable) but not the x-coordinates .	B1
		Maximum at $(-2, 12)$ and minimum at $(3, -24)$ with coordinates written the right way round. Condone missing brackets. The coordinates may appear on the sketch, or separately in the text (not necessarily referenced as A and B). If they are on the sketch, the x and y coordinates can be positioned correctly on the axes rather than given as coordinate pairs. In cases of ambiguity, the sketch has precedence. The origin does not need to be labelled. Nor do the x and y axes.	B1

			[2]
(b)	<p>The graph shows a cubic curve that does not pass through the origin. It has a local maximum at the point (-2, 0) and a local minimum at the point (3, -12). The curve crosses the y-axis at (0, -4).</p>	A positive cubic which does not pass through the origin with a maximum to the left of the y -axis and a minimum to the right of the y -axis.	M1
		Maximum at $(-2, 0)$ and minimum at $(3, -12)$. Condone missing brackets. For the max allow just -2 or $(0, -2)$ if marked in the correct place. If the coordinates are in the text, they must appear as $(-2, 0)$ and must not contradict the sketch. The curve must <u>touch</u> the x -axis at $(-2, 0)$. For the min allow coordinates as shown or 3 and -12 to be marked in the correct places on the axes. In cases of ambiguity, the sketch has precedence.	A1
		Crosses y -axis at $(0, -4)$. Allow just -4 (not +4) and allow $(-4, 0)$ if marked in the correct place. If the coordinates are in the text, they must appear as $(0, -4)$ and must not contradict the sketch. In cases of ambiguity, the sketch has precedence.	A1
			[3]
			5 marks

Question Number	Scheme	Notes	Marks
WAY 1			
5.	$y = -4x - 1$ $\Rightarrow (-4x - 1)^2 + 5x^2 + 2x = 0$	Attempts to make y the subject of the linear equation and substitutes into the other equation. Allow slips e.g. substituting $y = -4x + 1$ etc.	M1
	$21x^2 + 10x + 1 = 0$	Correct 3 term quadratic (terms do not need to be all on the same side). The “= 0” may be implied by subsequent work.	A1
	$(7x+1)(3x+1) = 0 \Rightarrow (x =) -\frac{1}{7}, -\frac{1}{3}$	dM1: Solves a 3 term quadratic by the usual rules (see general guidance) to give at least one value for x . Dependent on the first method mark. A1: $(x =) -\frac{1}{7}, -\frac{1}{3}$ (two separate correct exact answers). Allow exact equivalents e.g. $(x =) -\frac{6}{42}, -\frac{14}{42}$	dM1 A1
	$y = -\frac{3}{7}, -\frac{1}{3}$	M1: Substitutes to find at least one y value (Allow substitution into their rearranged equation above but not into an equation that has not been seen earlier). You may need to check here if there is no working and x values are incorrect. A1: $y = -\frac{3}{7}, \frac{1}{3}$ (two correct exact answers) Allow exact equivalents e.g. $y = -\frac{18}{42}, \frac{14}{42}$	M1 A1

Question Number	Scheme	Notes	Marks
$a_1 = 4, a_{n+1} = 5 - ka_n, n \dots 1$			
6. (a)	$a_2 = 5 - ka_1 = 5 - 4k$ $a_3 = 5 - ka_2 = 5 - k(5 - 4k)$	M1: Uses the recurrence relation correctly at least once. This may be implied by $a_2 = 5 - 4k$ or by the use of $a_3 = 5 - k(a_2)$ A1: Two correct expressions – need not be simplified but must be seen in (a). Allow $a_2 = 5 - 4k$ and $a_3 = 5 - 5k + 4k^2$ Is w if necessary for a_3 .	M1A1 [2]
(b)	$\sum_{r=1}^3 (l) = 1+1+1$	Finds 1+1+1 or 3 somewhere in their solution (may be implied by e.g. $5 + 6 - 4k + 6 - 5k + 4k^2$). Note that $5 + 6 - 4k + 6 - 5k + 4k^2$ would score B1 and the M1 below.	B1
	$\sum_{r=1}^3 a_r = 4 + "5-4k" + "5-5k+4k^2"$	Adds 4 to their a_2 and their a_3 where a_2 and a_3 are functions of k . The statement as shown is sufficient.	M1
	$\sum_{r=1}^3 (l+a_r) = 17 - 9k + 4k^2$	Cao but condone ‘= 0’ after the expression	A1
Allow full marks in (b) for correct answer only			

Number	Scheme	Notes	Marks
7.	$y = 3x^2 + 6x^{\frac{1}{3}} + \frac{2x^3 - 7}{3\sqrt{x}}$		
	$\frac{2x^3 - 7}{3\sqrt{x}} = \frac{2x^3}{3\sqrt{x}} - \frac{7}{3\sqrt{x}} = \frac{2}{3}x^{\frac{5}{2}} - \frac{7}{3}x^{-\frac{1}{2}}$	Attempts to split the fraction into 2 terms and obtains either $\alpha x^{\frac{5}{2}}$ or $\beta x^{-\frac{1}{2}}$. This may be implied by a correct power of x in their differentiation of one of these terms. But beware of $\beta x^{-\frac{1}{2}}$ coming from $\frac{2x^3 - 7}{3\sqrt{x}} = 2x^3 - 7 + 3x^{-\frac{1}{2}}$	M1
	$x^n \rightarrow x^{n-1}$	Differentiates by reducing power by one for any of their powers of x	M1
	$\left(\frac{dy}{dx} = \right) 6x + 2x^{-\frac{2}{3}} + \frac{5}{3}x^{\frac{2}{3}} + \frac{7}{6}x^{-\frac{5}{2}}$	A1: $6x$. Do not accept $6x^1$. Depends on second M mark only. Award when first seen and isw. A1: $2x^{-\frac{2}{3}}$. Must be simplified so do not accept e.g. $\frac{2}{1}x^{-\frac{2}{3}}$ but allow $\frac{2}{\sqrt[3]{x^2}}$. Depends on second M mark only. Award when first seen and isw. A1: $\frac{5}{3}x^{\frac{2}{3}}$. Must be simplified but allow e.g. $1\frac{2}{3}x^{1.5}$ or e.g. $\frac{5}{3}\sqrt{x^3}$. Award when first seen and isw. A1: $\frac{7}{6}x^{-\frac{5}{2}}$. Must be simplified but allow e.g. $1\frac{1}{6}x^{-\frac{11}{2}}$ or e.g. $\frac{7}{6\sqrt{x^3}}$. Award when first seen and isw.	A1A1A1A1

In an otherwise fully correct solution, penalise the presence of + c by deducting the final

Question Number	Scheme	Notes	Marks
9.(a)	John; arithmetic series, $a = 60$, $d = 15$. $60 + 75 + 90 = 225^*$ or $S_3 = \frac{3}{2}(120 + (3-1)(15)) = 225^*$	Finds and adds the first 3 terms or uses sum of 3 terms of an AP and obtains the printed answer, with no errors.	B1 *
	<u>Beware:</u> The 12 th term of the sequence is 225 also so look out for $60 + (12-1)\times 15 = 225$. This is B0.		
(b)	$t_9 = 60 + (n-1)15 = (\text{£})180$	M1: Uses $60 + (n-1)15$ with $n = 8$ or 9 A1: (£)180	M1 A1 [1]
	<u>Listing:</u> M1: Uses $a = 60$ and $d = 15$ to select the 8 th or 9 th term (allow arithmetic slips) A1: (£)180 (Special case (£)165 only scores M1A0)		
(c)	$S_n = \frac{n}{2}(120 + (n-1)(15))$ or $S_n = \frac{n}{2}(60 + 60 + (n-1)(15))$	Uses correct formula for sum of n terms with $a = 60$ and $d = 15$ (must be a correct formula but ignore the value they use for n or could be in terms of n)	M1
	$S_n = \frac{12}{2}(120 + (12-1)(15))$ = (£)1710	Correct numerical expression cao	A1 A1
	<u>Listing:</u> M1: Uses $a = 60$ and $d = 15$ and finds the sum of at least 12 terms (allow arithmetic slips) A2: (£)1710		

[3]

	$S_n = \frac{12}{2}(120 + (12-1)(15))$ = £1710	Correct numerical expression cao	A1 A1
Listing: M1: Uses $a = 60$ and $d = 15$ and finds the sum of at least 12 terms (allow arithmetic slips) A2: £1710			
(d)	$3375 = \frac{n}{2}(120 + (n-1)(15))$	Uses correct formula for sum of n terms with $a = 60$, $d = 15$ and puts = 3375	M1
	$6750 = 15n(8 + (n-1)) \Rightarrow 15n^2 + 105n = 6750$	Correct three term quadratic. E.g. $6750 = 105n + 15n^2$, $3375 = \frac{15}{2}n^2 + \frac{105}{2}n$ This may be implied by equations such as $6750 = 15n(n+7)$ or $3375 = \frac{15}{2}(n^2 + 7n)$	A1
	$n^2 + 7n = 25 \times 18^*$	Achieves the printed answer with no errors but must see the 450 or 450 in factorised form or e.g. 6750, 3375 in factorised form i.e. an intermediate step.	A1*
(e)	$n = 18 \Rightarrow$ Aged 27	M1: Attempts to solve the given quadratic or states $n = 18$ A1: Age = 27 or just 27	M1 A1
	Age = 27 only scores both marks (i.e. $n = 18$ need not be seen)		
	Note that (e) is not hence so allow valid attempts to solve the given equation for M1		
			[2]
			11 marks

Question Number	Scheme	Marks
1.	$r = \frac{3}{4}$, $S_4 = 175$	
(a) Way 1	$\frac{a(1 - (\frac{3}{4})^4)}{1 - \frac{3}{4}}$ or $\frac{a(1 - \frac{3}{4}^4)}{1 - \frac{3}{4}}$ or $\frac{a(1 - 0.75^4)}{1 - 0.75}$ Substituting $r = \frac{3}{4}$ or 0.75 and $n = 4$ into the formula for S_n	M1
	$175 = \frac{a(1 - (\frac{3}{4})^4)}{1 - \frac{3}{4}} \Rightarrow a = \frac{175(1 - \frac{3}{4})}{(1 - (\frac{3}{4})^4)} \left\{ \Rightarrow a = \frac{\frac{175}{4}}{\frac{175}{256}} \right\} a = 64^*$ Correct proof	A1*
(a) Way 2	$a + a(\frac{3}{4}) + a(\frac{3}{4})^2 + a(\frac{3}{4})^3$ $\frac{175}{64}a = 175 \left(\Rightarrow a = \frac{175}{(\frac{175}{64})} \right) \Rightarrow a = 64^*$ or $2.734375a = 175 \Rightarrow a = 64$	M1 Correct proof A1*
(a) Way 3	$\{S_4 =\} \frac{64(1 - (\frac{3}{4})^4)}{1 - \frac{3}{4}}$ or $\frac{64(1 - \frac{3}{4}^4)}{1 - \frac{3}{4}}$ or $\frac{64(1 - 0.75^4)}{1 - 0.75}$ $= 175$ so $a = 64^*$ Applying the formula for S_n with $r = \frac{3}{4}$, $n = 4$ and a as 64.	M1 [2] Obtains 175 with no errors seen and concludes $a = 64^*$ A1*
		[2]

Question Number	Scheme	Marks
3.	$P(7, 8)$ and $Q(10, 13)$ (a) $\{PQ\} = \sqrt{(7-10)^2 + (8-13)^2}$ or $\sqrt{(10-7)^2 + (13-8)^2}$ $\{PQ\} = \sqrt{34}$	Applies distance formula. Can be implied. $\sqrt{34}$ or $\sqrt{17}\sqrt{2}$ [2]
(b) Way 1	$(x-7)^2 + (y-8)^2 = 34$ (or $(\sqrt{34})^2$)	$(x \pm 7)^2 + (y \pm 8)^2 = k$, where k is a positive value. $(x-7)^2 + (y-8)^2 = 34$ [2]
(b) Way 2	$x^2 + y^2 - 14x - 16y + 79 = 0$	$x^2 + y^2 \pm 14x \pm 16y + c = 0$, where c is any value < 113. $x^2 + y^2 - 14x - 16y + 79 = 0$ [2]
(c) Way 1	{Gradient of radius} = $\frac{13-8}{10-7}$ or $\frac{5}{3}$ Gradient of tangent = $-\frac{1}{m} = -\frac{3}{5}$ $y - 13 = -\frac{3}{5}(x - 10)$ $3x + 5y - 95 = 0$	This must be seen or implied in part (c). Using a perpendicular gradient method on their gradient. So Gradient of tangent = $-\frac{1}{\text{gradient of radius}}$ $y - 13 = (\text{their changed gradient})(x - 10)$ $3x + 5y - 95 = 0$ o.e. [4]
(c) Way 2	$2(x-7) + 2(y-8) \frac{dy}{dx} = 0$ $2(10-7) + 2(13-8) \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{3}{5}$	Correct differentiation (or equivalent). Seen or implied Substituting both $x = 10$ and $y = 13$ into a valid differentiation to find a value for $\frac{dy}{dx}$ [4]

Question Number	Scheme	Marks
5.	$(a) (2-9x)^4 = 2^4 + {}^4C_1 2^3(-9x) + {}^4C_2 2^2(-9x)^2$, (b) $f(x) = (1+kx)(2-9x)^4 = A - 232x + Bx^2$	
(a) Way 1	First term of 16 in their final series At least one of $({}^4C_1 \times \dots \times x)$ or $({}^4C_2 \times \dots \times x^2)$ $= (16) - 288x + 1944x^2$	B1 M1 At least one of $-288x$ or $+1944x^2$ Both $-288x$ and $+1944x^2$ [4]
(a) Way 2	$(2-9x)^4 = (4-36x+81x^2)(4-36x+81x^2)$ $= 16 - 144x + 324x^2 - 144x + 1296x^2 + 324x^2$ $= (16) - 288x + 1944x^2$	B1 Attempts to multiply a 3 term quadratic by the same 3 term quadratic to achieve either 2 terms in x^2 or at least 2 terms in x^2 . At least one of $-288x$ or $+1944x^2$ Both $-288x$ and $+1944x^2$ [4]
(a) Way 3	$\{(2-9x)^4\} = 2^4 \left(1 - \frac{9}{2}x\right)^4$ $= 2^4 \left(1 + 4\left(-\frac{9}{2}x\right) + \frac{4(3)}{2} \left(-\frac{9}{2}x\right)^2 + \dots\right)$ $= (16) - 288x + 1944x^2$	First term of 16 in final series At least one of $(4 \times \dots \times x)$ or $\left(\frac{4(3)}{2} \times \dots \times x^2\right)$ At least one of $-288x$ or $+1944x^2$ Both $-288x$ and $+1944x^2$ [4]
(b)	$A = "16"$	Follow through their value from (a) B1ft [1]
(c)	$\{(1+kx)(2-9x)^4\} = (1+kx)(16 - 288x + \{1944x^2 + \dots\})$ x terms: $-288x + 16kx = -232x$ giving, $16k = 56 \Rightarrow k = \frac{7}{2}$	May be seen in part (b) or (d) and can be implied by work in parts (c) or (d). $k = \frac{7}{2}$ A1 [2]
(d)	x^2 terms: $1944x^2 - 288kx^2$ So, $B = 1944 - 288\left(\frac{7}{2}\right)$; $= 1944 - 1008 = 936$	See notes 936 M1 A1 [2]

Question Number	Scheme	Marks
6.	$1 - 2 \cos\left(\theta - \frac{\pi}{5}\right) = 0 ; -\pi < \theta , \pi$	
(i)	$\cos\left(\theta - \frac{\pi}{5}\right) = \frac{1}{2}$ Rearranges to give $\cos\left(\theta - \frac{\pi}{5}\right) = \frac{1}{2}$ or $-\frac{1}{2}$	M1
	$\theta = \left\{-\frac{2\pi}{15}, \frac{8\pi}{15}\right\}$ At least one of $-\frac{2\pi}{15}$ or $\frac{8\pi}{15}$ or -24° or 96° or awrt 1.68 or awrt -0.419	A1
		Both $-\frac{2\pi}{15}$ and $\frac{8\pi}{15}$
		[3]
NB Misread	Misreading $\frac{\pi}{5}$ as $\frac{\pi}{6}$ or $\frac{\pi}{3}$ (or anything else) – treat as misread so M1 A0 A0 is maximum mark	
	$4\cos^2 x + 7\sin x - 2 = 0, 0 < x < 360^\circ$	
(ii)	$4(1 - \sin^2 x) + 7\sin x - 2 = 0$ Applies $\cos^2 x = 1 - \sin^2 x$	M1
	$4 - 4\sin^2 x + 7\sin x - 2 = 0$	
	$4\sin^2 x - 7\sin x + 2 = 0$ Correct 3 term, $4\sin^2 x - 7\sin x - 2 \{ = 0\}$	A1 oe
	$(4\sin x + 1)(\sin x - 2) \{ = 0\} , \sin x = \dots$ Valid attempt at solving and $\sin x = \dots$	M1
	$\sin x = -\frac{1}{4}, \{ \sin x = 2\}$ $\sin x = -\frac{1}{4}$ (See notes.)	A1 cso
	$x = \text{awrt}\{194.5, 345.5\}$ At least one of awrt 194.5 or awrt 345.5 or awrt 3.4 or awrt 6.0	A1ft
		awrt 194.5 and awrt 345.5
		A1
		[6]
		9
NB Misread	Writing equation as $4\cos^2 x - 7\sin x - 2 = 0$ with a sign error should be marked by applying the scheme as it simplifies the solution (do not treat as misread) Max mark is 3/6	
	$4(1 - \sin^2 x) - 7\sin x - 2 = 0$	M1
	$4\sin^2 x + 7\sin x - 2 = 0$	A0
	$(4\sin x - 1)(\sin x + 2) \{ = 0\} , \sin x = \dots$ Valid attempt at solving and $\sin x = \dots$	M1

Question Number	Scheme	Marks
8(i)	Two Ways of answering the question are given in part (i)	
Way 1	$\log_3\left(\frac{3b+1}{a-2}\right) = -1$ or $\log_3\left(\frac{a-2}{3b+1}\right) = 1$ Applying the subtraction law of logarithms	M1
	$\frac{3b+1}{a-2} = 3^{-1} \left\{ = \frac{1}{3}\right\}$ or $\left(\frac{a-2}{3b+1}\right) = 3$ Making a correct connection between log base 3 and 3 to a power.	M1
	$\{9b+3 = a-2 \Rightarrow\} b = \frac{1}{9}a - \frac{5}{9}$ $b = \frac{1}{9}a - \frac{5}{9}$ or $b = \frac{a-5}{9}$	A1 oe
		[3]
	In Way 2 a correct connection between log base 3 and “3 to a power” is used before applying the subtraction or addition law of logs	
(i) Way 2	Either $\log_3(3b+1) - \log_3(a-2) = -\log_3 3$ or $\log_3(3b+1) + \log_3 3 = \log_3(a-2)$	2 nd M1
	$\log_3(3b+1) = \log_3(a-2) - \log_3 3 = \log_3\left(\frac{a-2}{3}\right)$ or $\log_3 3(3b+1) = \log_3(a-2)$	1 st M1
	$\{3b+1 = \frac{a-2}{3}\} b = \frac{1}{9}a - \frac{5}{9}$	A1
		[3]
	Five Ways of answering the question are given in part (ii)	
(ii)	$32(2^{2x}) - 7(2^x) = 0$ Deals with power 5 correctly giving $\times 32$	M1
Way 1 See also common approach below in notes	So, $2^x = \frac{7}{32}$ $2^x = \frac{7}{32}$ or $y = \frac{7}{32}$ or awrt 0.219	A1 oe dM1
	$x \log 2 = \log\left(\frac{7}{32}\right)$ or $x = \frac{\log\left(\frac{7}{32}\right)}{\log 2}$ or $x = \log_2\left(\frac{7}{32}\right)$ A valid method for solving $2^x = \frac{7}{32}$	
		Or $2^x = k$ to achieve $x = \dots$
	$x = -2.192645\dots$ awrt -2.19	A1
		[4]

Question Number	Scheme	Marks
9. (a)	$\text{Area}(FEA) = \frac{1}{2}x^2 \left(\frac{2\pi}{3}\right); = \frac{\pi x^2}{3}$ $\frac{1}{2}x^2 \times \left(\frac{2\pi}{3}\right)$ or $\frac{120}{360} \times \pi x^2$ simplified or unsimplified $\frac{\pi x^2}{3}$	M1 A1 [2]
(b)	$\{A = \} \frac{1}{2}x^2 \sin 60^\circ + \frac{1}{3}\pi x^2 + 2xy$ $1000 = \frac{\sqrt{3}x^2}{4} + \frac{\pi x^2}{3} + 2xy \Rightarrow y = \frac{500}{x} - \frac{\sqrt{3}x}{8} - \frac{\pi x}{6}$ $\Rightarrow y = \frac{500}{x} - \frac{x}{24}(4\pi + 3\sqrt{3})$ *	Attempt to sum 3 areas (at least one correct) Correct expression for at least two terms of A Correct proof. A1 * [3]
(c)	$\{P = \} x + x\theta + y + 2x + y \quad \left\{ = 3x + \frac{2\pi x}{3} + 2y \right\}$ $\dots 2y = + 2\left(\frac{500}{x} - \frac{x}{24}(4\pi + 3\sqrt{3})\right)$ $P = 3x + \frac{2\pi x}{3} + \frac{1000}{x} - \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x \Rightarrow P = \frac{1000}{x} + 3x + \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x$ $\Rightarrow P = \frac{1000}{x} + \frac{x}{12}(4\pi + 36 - 3\sqrt{3})$ *	Correct expression in x and y for their θ measured in rads Substitutes expression from (b) into y term. Correct proof. A1 * [3]
	$P = 3x + \frac{2\pi x}{3} + \frac{1000}{x} - \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x \Rightarrow P = \frac{1000}{x} + 3x + \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x$ $\Rightarrow P = \frac{1000}{x} + \frac{x}{12}(4\pi + 36 - 3\sqrt{3})$ *	Correct proof. A1 * [3]
(d)	$\frac{dP}{dx} = -1000x^{-2} + \frac{4\pi + 36 - 3\sqrt{3}}{12}; = 0$ $\Rightarrow x = \sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}} (= 16.63392808\dots)$ $\left\{ P = \frac{1000}{(16.63\dots)} + \frac{(16.63\dots)}{12}(4\pi + 36 - 3\sqrt{3}) \right\} \Rightarrow P = 120.236.. (\text{m})$	$\frac{1000}{x} \rightarrow \frac{\pm \lambda}{x^2}$ M1 Correct differentiation (need not be simplified). A1; Their $P' = 0$ M1 A1 [5]
(e)	$\frac{d^2P}{dx^2} = \frac{2000}{x^3} > 0 \Rightarrow \text{Minimum}$ $\frac{2000}{x^3}$ (need not be simplified) and > 0 and conclusion. Only follow through on a correct P'' and x in range $10 < x < 25$.	Finds P'' and considers sign. A1ft [2]

Question	Scheme	Marks
1(a)	$fg(x) = \frac{28}{x-2} - 1 \quad \left(= \frac{30-x}{x-2} \right)$ <p>Sets $fg(x) = x \Rightarrow \frac{28}{x-2} - 1 = x$ $\Rightarrow 28 = (x+1)(x-2)$ $\Rightarrow x^2 - x - 30 = 0$ $\Rightarrow (x-6)(x+5) = 0$ $\Rightarrow x = 6, x = -5$</p>	M1 M1 dM1 A1 (4)
(b)	$a = 6$	B1 ft (1) 5 marks
Alt 1(a)	$fg(x) = x \Rightarrow g(x) = f^{-1}(x)$ $\frac{4}{x-2} = \frac{x+1}{7}$ $\Rightarrow x^2 - x - 30 = 0$ $\Rightarrow (x-6)(x+5) = 0$ $\Rightarrow x = 6, x = -5$	M1 M1 dM1 A1 4 marks

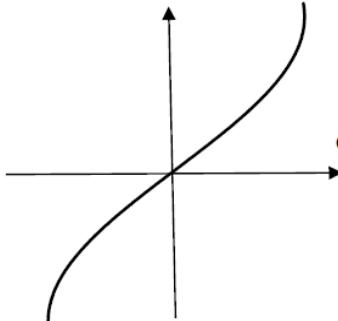
Question	Scheme	Marks
2(a)	$y = \frac{4x}{(x^2 + 5)} \Rightarrow \left(\frac{dy}{dx} \right) = \frac{4(x^2 + 5) - 4x \times 2x}{(x^2 + 5)^2}$ $\Rightarrow \left(\frac{dy}{dx} \right) = \frac{20 - 4x^2}{(x^2 + 5)^2}$	M1A1 M1A1 (4)
(b)	$\frac{20 - 4x^2}{(x^2 + 5)^2} < 0 \Rightarrow x^2 > \frac{20}{4}$ Critical values of $\pm\sqrt{5}$ $x < -\sqrt{5}, x > \sqrt{5}$ or equivalent	M1 dM1A1 (3) 7 marks

Question	Scheme	Marks
3.(a)	$R = \sqrt{5}$ $\tan \alpha = \frac{1}{2} \Rightarrow \alpha = 26.57^\circ$	B1 M1A1 (3)
(b)	$\frac{2}{2 \cos \theta - \sin \theta - 1} = 15 \Rightarrow \frac{2}{\sqrt{5} \cos(\theta + 26.6^\circ) - 1} = 15$ $\Rightarrow \cos(\theta + 26.6^\circ) = \frac{17}{15\sqrt{5}} = (\text{awrt } 0.507)$ $\theta + 26.57^\circ = 59.54^\circ$ $\Rightarrow \theta = \text{awrt } 33.0^\circ \text{ or awrt } 273.9^\circ$ $\theta + 26.6^\circ = 360^\circ - \text{their } 59.5^\circ$ $\Rightarrow \theta = \text{awrt } 273.9^\circ \text{ and awrt } 33.0^\circ$	M1A1 A1 dM1 A1 (5)
(c)	$\theta - \text{their } 26.57^\circ = \text{their } 59.54^\circ \Rightarrow \theta = \dots$ $\theta = \text{awrt } 86.1^\circ$	M1 A1 (2) (10 marks)

Question	Scheme	Marks
4.(a)	(i) 21 (ii) $4e^{2x} - 25 = 0 \Rightarrow e^{2x} = \frac{25}{4} \Rightarrow 2x = \ln\left(\frac{25}{4}\right) \Rightarrow x = \frac{1}{2} \ln\left(\frac{25}{4}\right), \Rightarrow x = \ln\left(\frac{5}{2}\right)$ (iii) 25	B1 M1A1, A1 B1 (5)
(b)	$4e^{2x} - 25 = 2x + 43 \Rightarrow e^{2x} = \frac{1}{2}x + 17$ $\Rightarrow 2x = \ln\left(\frac{1}{2}x + 17\right) \Rightarrow x = \frac{1}{2} \ln\left(\frac{1}{2}x + 17\right)$	M1 A1* (2)
(c)	$x_1 = \frac{1}{2} \ln\left(\frac{1}{2} \times 1.4 + 17\right) = \text{awrt } 1.44$ awrt $x_1 = 1.4368, x_2 = 1.4373$	M1 A1 (2)
(d)	Defines a suitable interval 1.4365 and 1.4375 ...and substitutes into a suitable function Eg $4e^{2x} - 2x - 68$, obtains correct values with both a reason and conclusion	M1 A1 (2) (11 marks)

Question	Scheme	Marks
5 (i)	$y = e^{3x} \cos 4x \Rightarrow \left(\frac{dy}{dx}\right) = \cos 4x \times 3e^{3x} + e^{3x} \times -4 \sin 4x$ Sets $\cos 4x \times 3e^{3x} + e^{3x} \times -4 \sin 4x = 0 \Rightarrow 3 \cos 4x - 4 \sin 4x = 0$ $\Rightarrow x = \frac{1}{4} \arctan \frac{3}{4}$ $\Rightarrow x = \text{awrt } 0.9463 \quad 4\text{dp}$	M1A1 M1 M1 A1 (5)
(ii)	$x = \sin^2 2y \Rightarrow \frac{dx}{dy} = 2 \sin 2y \times 2 \cos 2y$ Uses $\sin 4y = 2 \sin 2y \cos 2y$ in their expression $\frac{dx}{dy} = 2 \sin 4y \Rightarrow \frac{dy}{dx} = \frac{1}{2 \sin 4y} = \frac{1}{2} \operatorname{cosec} 4y$	M1A1 M1 M1A1 (5) (10 marks)

Question	Scheme	Marks
6(a)	$\begin{array}{r} x^2 + 3 \\ \hline x^2 + x - 6 \end{array}$ $\begin{array}{r} x^4 + x^3 - 6x^2 \\ \hline 3x^2 + 7x - 6 \end{array}$ $\begin{array}{r} 3x^2 + 3x - 18 \\ \hline 4x + 12 \end{array}$	M1 A1
(b)	$\frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6} \equiv x^2 + 3 + \frac{4(x+3)}{(x+3)(x-2)}$ $\equiv x^2 + 3 + \frac{4}{(x-2)}$ $f'(x) = 2x - \frac{4}{(x-2)^2}$ <p>Subs $x = 3$ into $f'(x = 3) = 2 \times 3 - \frac{4}{(3-2)^2} = (2)$</p> <p>Uses $m = -\frac{1}{f'(3)} = \left(-\frac{1}{2}\right)$ with $(3, f(3)) = (3, 16)$ to form eqn of normal</p> $y - 16 = -\frac{1}{2}(x - 3) \text{ or equivalent cso}$	M1 A1 (4) M1A1ft M1 M1A1 (5) (9 marks)

Question	Scheme	Marks
7(a)	 <p>Correct position or curvature Correct position and curvature</p>	M1 A1 (2)
(b)	$3 \arcsin(x+1) + \pi = 0 \Rightarrow \arcsin(x+1) = -\frac{\pi}{3}$ $\Rightarrow (x+1) = \sin\left(-\frac{\pi}{3}\right)$ $\Rightarrow x = -1 - \frac{\sqrt{3}}{2}$	M1 dM1A1 (3) (5 marks)

- (a) Ignore any scales that appear on the axes
M1 Accept for the method mark
Either one of the two sections with correct curvature passing through (0,0),
Or both sections condoning dubious curvature passing through (0,0) (but do not accept any negative gradients)
Or a curve with a different range or an "extended range"
See the next page for a useful guide for clarification of this mark.

Question	Scheme	Marks
8 (a)	$\begin{aligned} 2 \cot 2x + \tan x &\equiv \frac{2}{\tan 2x} + \tan x \\ &\equiv \frac{(1 - \tan^2 x)}{\tan x} + \frac{\tan^2 x}{\tan x} \\ &\equiv \frac{1}{\tan x} \\ &\equiv \cot x \end{aligned}$	B1 M1 M1 A1* (4)
(b)	$\begin{aligned} 6 \cot 2x + 3 \tan x &= \operatorname{cosec}^2 x - 2 \Rightarrow 3 \cot x = \operatorname{cosec}^2 x - 2 \\ &\Rightarrow 3 \cot x = 1 + \cot^2 x - 2 \\ &\Rightarrow 0 = \cot^2 x - 3 \cot x - 1 \\ &\Rightarrow \cot x = \frac{3 \pm \sqrt{13}}{2} \\ &\Rightarrow \tan x = \frac{2}{3 \pm \sqrt{13}} \Rightarrow x = \dots \\ &\Rightarrow x = 0.294, -2.848, -1.277, 1.865 \end{aligned}$	M1 A1 M1 M1 A2,1,0 (6) (10 marks)

Question	Scheme	Marks
9(a)	Subs $D = 15$ and $t = 4$ $x = 15e^{-0.2 \times 4} = 6.740$ (mg)	M1A1
(b)	$15e^{-0.2 \times 7} + 15e^{-0.2 \times 2} = 13.754$ (mg)	M1A1* (2)
(c)	$\begin{aligned} 15e^{-0.2 \times T} + 15e^{-0.2 \times (T+5)} &= 7.5 \\ 15e^{-0.2 \times T} + 15e^{-0.2 \times T} e^{-1} &= 7.5 \\ 15e^{-0.2 \times T} (1 + e^{-1}) &= 7.5 \Rightarrow e^{-0.2 \times T} = \frac{7.5}{15(1 + e^{-1})} \\ T &= -5 \ln \left(\frac{7.5}{15(1 + e^{-1})} \right) = 5 \ln \left(2 + \frac{2}{e} \right) \end{aligned}$	M1 dM1 A1, A1 (4) (8 marks)

Question Number	Scheme	Notes	Marks
1. Way 1	$\left\{ \frac{1}{(2+5x)^3} \right\} (2+5x)^{-3}$	Writes down $(2+5x)^{-3}$ or uses power of -3	M1
	$= (2)^{-3} \left(1 + \frac{5x}{2} \right)^{-3} = \frac{1}{8} \left(1 + \frac{5x}{2} \right)^{-3}$	$\underline{2^{-3}}$ or $\underline{\frac{1}{8}}$	<u>B1</u>
	$= \left\{ \frac{1}{8} \right\} \left[1 + (-3)(kx) + \frac{(-3)(-4)}{2!} (kx)^2 + \frac{(-3)(-4)(-5)}{3!} (kx)^3 + \dots \right]$	see notes	M1 A1
	$= \left\{ \frac{1}{8} \right\} \left[1 + (-3) \left(\frac{5x}{2} \right) + \frac{(-3)(-4)}{2!} \left(\frac{5x}{2} \right)^2 + \frac{(-3)(-4)(-5)}{3!} \left(\frac{5x}{2} \right)^3 + \dots \right]$		
	$= \frac{1}{8} \left[1 - \frac{15}{2}x + \frac{75}{2}x^2 - \frac{625}{4}x^3 + \dots \right]$		
	$= \frac{1}{8} \left[1 - 7.5x + 37.5x^2 - 156.25x^3 + \dots \right]$		
	$= \frac{1}{8} - \frac{15}{16}x + \frac{75}{16}x^2 - \frac{625}{32}x^3 + \dots$		
	or $\frac{1}{8} - \frac{15}{16}x + 4\frac{11}{16}x^2 - 19\frac{17}{32}x^3 + \dots$		A1; A1

2.	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;">x</td><td style="width: 10%;">1</td><td style="width: 10%;">1.2</td><td style="width: 10%;">1.4</td><td style="width: 10%;">1.6</td><td style="width: 10%;">1.8</td><td style="width: 10%;">2</td><td style="width: 10%;"></td></tr> <tr> <td>y</td><td>0</td><td>0.2625</td><td>0.659485...</td><td>1.2032</td><td>1.9044</td><td>2.7726</td><td></td></tr> </table>	x	1	1.2	1.4	1.6	1.8	2		y	0	0.2625	0.659485...	1.2032	1.9044	2.7726		$y = x^2 \ln x$	
x	1	1.2	1.4	1.6	1.8	2													
y	0	0.2625	0.659485...	1.2032	1.9044	2.7726													
(a)	{At $x=1.4$,} $y = 0.6595$ (4 dp)		0.6595	B1 cao															
				[1]															
(b)	$\frac{1}{2} \times (0.2) \times [0 + 2.7726 + 2(0.2625 + \text{their } 0.6595 + 1.2032 + 1.9044)]$ [Note: The "0" does not have to be included in [.....]]	Outside brackets $\frac{1}{2} \times (0.2)$ or $\frac{1}{10}$ For structure of [.....]	B1 o.e. M1																
	$\left\{ = \frac{1}{10}(10.8318) \right\} = 1.08318 = 1.083$ (3 dp)	anything that rounds to 1.083	A1																
				[3]															
(c) Way 1	$I = \int x^2 \ln x dx$, $\begin{cases} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^2 \Rightarrow v = \frac{1}{3}x^3 \end{cases}$																		
	$= \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \left(\frac{1}{x}\right) \{dx\}$	Either $x^2 \ln x \rightarrow \pm \lambda x^3 \ln x - \int \mu x^3 \left(\frac{1}{x}\right) \{dx\}$ or $\pm \lambda x^3 \ln x - \int \mu x^2 \{dx\}$, where $\lambda, \mu > 0$	M1																
		$x^2 \ln x \rightarrow \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \left(\frac{1}{x}\right) \{dx\}$, simplified or un-simplified	A1																
	$= \frac{x^3}{3} \ln x - \frac{x^3}{9}$	$\frac{x^3}{3} \ln x - \frac{x^3}{9}$, simplified or un-simplified	A1																
	$\text{Area } (R) = \left[\frac{x^3}{3} \ln x - \frac{x^3}{9} \right]_1^2 = \left(\frac{8}{3} \ln 2 - \frac{8}{9} \right) - \left(0 - \frac{1}{9} \right)$	dependent on the previous M mark. Applies limits of 2 and 1 and subtracts the correct way round	dM1																
	$= \frac{8}{3} \ln 2 - \frac{7}{9}$	$\frac{8}{3} \ln 2 - \frac{7}{9}$ or $\frac{1}{9}(24 \ln 2 - 7)$	A1 oe cso																
			[5]																

Question Number	Scheme	Notes	Marks
3.	$2x^2y + 2x + 4y - \cos(\pi y) = 17$		
(a) Way 1	$\left\{ \frac{\cancel{dx}}{\cancel{dx}} \times \right\} \left(4xy + 2x^2 \frac{dy}{dx} \right) + 2 + 4 \frac{dy}{dx} + \pi \sin(\pi y) \frac{dy}{dx} = 0$		M1 <u>A1</u> <u>B1</u>
	$\frac{dy}{dx} (2x^2 + 4 + \pi \sin(\pi y)) + 4xy + 2 = 0$		dM1
	$\left\{ \frac{dy}{dx} = \right\} \frac{-4xy - 2}{2x^2 + 4 + \pi \sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi \sin(\pi y)}$	Correct answer or equivalent	A1 cso
			[5]
(b)	At $(3, \frac{1}{2})$, $m_T = \frac{dy}{dx} = \frac{-4(3)(\frac{1}{2}) - 2}{2(3)^2 + 4 + \pi \sin(\frac{1}{2}\pi)} \left\{ = \frac{-8}{22 + \pi} \right\}$	Substituting $x = 3$ & $y = \frac{1}{2}$ into an equation involving $\frac{dy}{dx}$	M1
	$m_N = \frac{22 + \pi}{8}$	Applying $m_N = \frac{-1}{m_T}$ to find a numerical m_N Can be implied by later working	M1
	$\bullet \quad y - \frac{1}{2} = \left(\frac{22 + \pi}{8} \right)(x - 3)$ $\bullet \quad \frac{1}{2} = \left(\frac{22 + \pi}{8} \right)(3) + c \Rightarrow c = \frac{1}{2} - \frac{66 + 3\pi}{8}$ $\Rightarrow y = \left(\frac{22 + \pi}{8} \right)x + \frac{1}{2} - \frac{66 + 3\pi}{8}$ Cuts x -axis $\Rightarrow y = 0$ $\Rightarrow -\frac{1}{2} = \left(\frac{22 + \pi}{8} \right)(x - 3)$	$y - \frac{1}{2} = m_N(x - 3)$ or $y = m_Nx + c$ where $\frac{1}{2} = (\text{their } m_N)3 + c$ with a numerical m_N ($\neq m_T$) where m_N is in terms of π and sets $y = 0$ in their normal equation.	dM1
	So, $\left\{ x = \frac{-4}{22 + \pi} + 3 \Rightarrow \right\} x = \frac{3\pi + 62}{\pi + 22}$	$\frac{3\pi + 62}{\pi + 22}$ or $\frac{6\pi + 124}{2\pi + 44}$ or $\frac{62 + 3\pi}{22 + \pi}$	A1 o.e.
			[4]
			9

Question Number	Scheme	Notes	Marks
4.	$\frac{dx}{dt} = -\frac{5}{2}x, \quad x \in \mathbb{R}, x \geq 0$		
(a) Way 1	$\int \frac{1}{x} dx = \int -\frac{5}{2} dt$	Separates variables as shown. dx and dt should not be in the wrong positions, though this mark can be implied by later working. Ignore the integral signs.	B1
	$\ln x = -\frac{5}{2}t + c$	Integrates both sides to give either $\pm \frac{\alpha}{x} \rightarrow \pm \alpha \ln x$ or $\pm k \rightarrow \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$\ln x = -\frac{5}{2}t + c$, including "+c"	A1
	$\{t=0, x=60 \Rightarrow \ln 60 = c\}$ $\ln x = -\frac{5}{2}t + \ln 60 \Rightarrow x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with no incorrect working seen	A1 cso
(a) Way 2	$\frac{dt}{dx} = -\frac{2}{5x}$ or $t = \int -\frac{2}{5x} dx$	Either $\frac{dt}{dx} = -\frac{2}{5x}$ or $t = \int -\frac{2}{5x} dx$	B1
	$t = -\frac{2}{5} \ln x + c$	Integrates both sides to give either $t = \dots$ or $\pm \alpha \ln px; \alpha \neq 0, p > 0$	M1
		$t = -\frac{2}{5} \ln x + c$, including "+c"	A1
	$\{t=0, x=60 \Rightarrow c = \frac{2}{5} \ln 60 \Rightarrow t = -\frac{2}{5} \ln x + \frac{2}{5} \ln 60\}$ $\Rightarrow -\frac{5}{2}t = \ln x - \ln 60 \Rightarrow x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$	Finds their c and uses correct algebra to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with no incorrect working seen	A1 cso
[4]			
(a) Way 3	$\int_{60}^x \frac{1}{x} dx = \int_0^t -\frac{5}{2} dt$	to achieve $x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$ with no incorrect working seen	A1 cso
	$[\ln x]_{60}^x = \left[-\frac{5}{2}t \right]_0^t$	Ignore limits	B1
		Integrates both sides to give either $\pm \frac{\alpha}{x} \rightarrow \pm \alpha \ln x$ or $\pm k \rightarrow \pm kt$ (with respect to t); $k, \alpha \neq 0$	M1
		$[\ln x]_{60}^x = \left[-\frac{5}{2}t \right]_0^t$ including the correct limits	A1
(b)	$\ln x - \ln 60 = -\frac{5}{2}t \Rightarrow x = 60e^{-\frac{5}{2}t}$ or $x = \frac{60}{e^{\frac{5}{2}t}}$	Correct algebra leading to a correct result	A1 cso
			[4]
	$20 = 60e^{-\frac{5}{2}t}$ or $\ln 20 = -\frac{5}{2}t + \ln 60$	Substitutes $x = 20$ into an equation in the form of either $x = \pm \lambda e^{\pm \mu t} \pm \beta$ or $x = \pm \lambda e^{\pm \mu t \pm \alpha \ln \delta x}$ or $\pm \alpha \ln \delta x = \pm \mu t \pm \beta$ or $t = \pm \lambda \ln \delta x \pm \beta$; $\alpha, \lambda, \mu, \delta \neq 0$ and β can be 0	M1
	$t = -\frac{2}{5} \ln \left(\frac{20}{60} \right)$ $\{= 0.4394449\dots \text{ (days)}\}$ Note: t must be greater than 0 $\Rightarrow t = 632.8006\dots = 633$ (to the nearest minute)	dependent on the previous M mark Uses correct algebra to achieve an equation of the form of either $t = A \ln \left(\frac{60}{20} \right)$ or $A \ln \left(\frac{20}{60} \right)$ or $A \ln 3$ or $A \ln \left(\frac{1}{3} \right)$ o.e. or $t = A(\ln 20 - \ln 60)$ or $A(\ln 60 - \ln 20)$ o.e. ($A \in \mathbb{Q}, t > 0$)	dM1
		Note: dM1 can be implied by $t = \text{awrt } 0.44$ from no incorrect working.	A1 cso
7			

5.	$x = 4 \tan t, \quad y = 5\sqrt{3} \sin 2t, \quad 0 \leq t < \frac{\pi}{2}$		
(a) Way 1	$\frac{dx}{dt} = 4 \sec^2 t, \quad \frac{dy}{dt} = 10\sqrt{3} \cos 2t$ $\Rightarrow \frac{dy}{dx} = \frac{10\sqrt{3} \cos 2t}{4 \sec^2 t} \quad \left\{ = \frac{5}{2}\sqrt{3} \cos 2t \cos^2 t \right\}$ $\left\{ \text{At } P\left(4\sqrt{3}, \frac{15}{2}\right), \quad t = \frac{\pi}{3}\right\}$	Either both x and y are differentiated correctly with respect to t or their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or applies $\frac{dy}{dt}$ multiplied by their $\frac{dx}{dt}$	M1
		Correct $\frac{dy}{dx}$ (Can be implied)	A1 oe
	$\frac{dy}{dx} = \frac{10\sqrt{3} \cos(\frac{2\pi}{3})}{4 \sec^2(\frac{\pi}{3})}$	dependent on the previous M mark <i>Some evidence</i> of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ from a correct solution only	A1 cso
			[4]
(b)	$\left\{ 10\sqrt{3} \cos 2t = 0 \Rightarrow t = \frac{\pi}{4} \right\}$		
	So $x = 4 \tan\left(\frac{\pi}{4}\right), \quad y = 5\sqrt{3} \sin\left(2\left(\frac{\pi}{4}\right)\right)$	At least one of either $x = 4 \tan\left(\frac{\pi}{4}\right)$ or $y = 5\sqrt{3} \sin\left(2\left(\frac{\pi}{4}\right)\right)$ or $x = 4$ or $y = 5\sqrt{3}$ or $y = \text{awrt } 8.7$	M1
	Coordinates are $(4, 5\sqrt{3})$	$(4, 5\sqrt{3})$ or $x = 4, y = 5\sqrt{3}$	A1
			[2]
			6

Question Number	Scheme	Notes	Marks
6.	(i) $\int \frac{3y-4}{y(3y+2)} dy, \quad y > 0$, (ii) $\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)} dx, \quad x = 4 \sin^2 \theta$		
(i) Way 1	$\frac{3y-4}{y(3y+2)} = \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y-4 = A(3y+2) + By$ $y=0 \Rightarrow -4=2A \Rightarrow A=-2$ $y=-\frac{2}{3} \Rightarrow -6=-\frac{2}{3}B \Rightarrow B=9$	See notes At least one of their $A = -2$ or their $B = 9$	M1 A1
		Both their $A = -2$ and their $B = 9$	A1
	$\int \frac{3y-4}{y(3y+2)} dy = \int -\frac{2}{y} + \frac{9}{(3y+2)} dy$ $= -2 \ln y + 3 \ln(3y+2) \{+c\}$	Integrates to give at least one of either $\frac{A}{y} \rightarrow \pm \lambda \ln y$ or $\frac{B}{(3y+2)} \rightarrow \pm \mu \ln(3y+2)$ $A \neq 0, B \neq 0$	M1
		At least one term correctly followed through from their A or from their B $-2 \ln y + 3 \ln(3y+2)$ or $-2 \ln y + 3 \ln(y + \frac{2}{3})$ with correct bracketing, simplified or un-simplified. Can apply isw.	A1 ft A1 cao
			[6]
(ii) (a) Way 1	$\{x = 4 \sin^2 \theta \Rightarrow\} \frac{dx}{d\theta} = 8 \sin \theta \cos \theta \quad \text{or} \quad \frac{dx}{d\theta} = 4 \sin 2\theta \quad \text{or} \quad dx = 8 \sin \theta \cos \theta d\theta$		B1
	$\int \sqrt{\frac{4 \sin^2 \theta}{4 - 4 \sin^2 \theta}} \cdot 8 \sin \theta \cos \theta \{d\theta\} \quad \text{or} \quad \int \sqrt{\frac{4 \sin^2 \theta}{4 - 4 \sin^2 \theta}} \cdot 4 \sin 2\theta \{d\theta\}$		M1
	$= \int \underline{\tan \theta} \cdot 8 \sin \theta \cos \theta \{d\theta\} \quad \text{or} \quad \int \underline{\tan \theta} \cdot 4 \sin 2\theta \{d\theta\}$	$\sqrt{\left(\frac{x}{4-x}\right)} \rightarrow \pm K \tan \theta \text{ or } \pm K \left(\frac{\sin \theta}{\cos \theta}\right)$	<u>M1</u>
	$= \int 8 \sin^2 \theta d\theta$	$\int 8 \sin^2 \theta d\theta$ including $d\theta$	A1
	$3 = 4 \sin^2 \theta \quad \text{or} \quad \frac{3}{4} = \sin^2 \theta \quad \text{or} \quad \sin \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$ $\{x = 0 \rightarrow \theta = 0\}$	Writes down a correct equation involving $x = 3$ leading to $\theta = \frac{\pi}{3}$ and no incorrect work seen regarding limits	B1
			[5]

(ii) (b)	$= \{8\} \int \left(\frac{1 - \cos 2\theta}{2} \right) d\theta \quad \left\{ = \int (4 - 4\cos 2\theta) d\theta \right\}$	Applies $\cos 2\theta = 1 - 2\sin^2 \theta$ to their integral. (See notes)	M1
	$= \{8\} \left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \right) \quad \{= 4\theta - 2\sin 2\theta\}$	For $\pm \alpha\theta \pm \beta\sin 2\theta, \alpha, \beta \neq 0$	M1
		$\sin^2 \theta \rightarrow \left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \right)$	A1
	$\left\{ \int_0^{\frac{\pi}{3}} 8\sin^2 \theta d\theta = 8 \left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \right]_0^{\frac{\pi}{3}} \right\} = 8 \left[\left(\frac{\pi}{6} - \frac{1}{4} \left(\frac{\sqrt{3}}{2} \right) \right) - (0 + 0) \right]$		
	$= \frac{4}{3}\pi - \sqrt{3}$	“two term” exact answer of e.g. $\frac{4}{3}\pi - \sqrt{3}$ or $\frac{1}{3}(4\pi - 3\sqrt{3})$	A1 o.e.
			[4]
			15