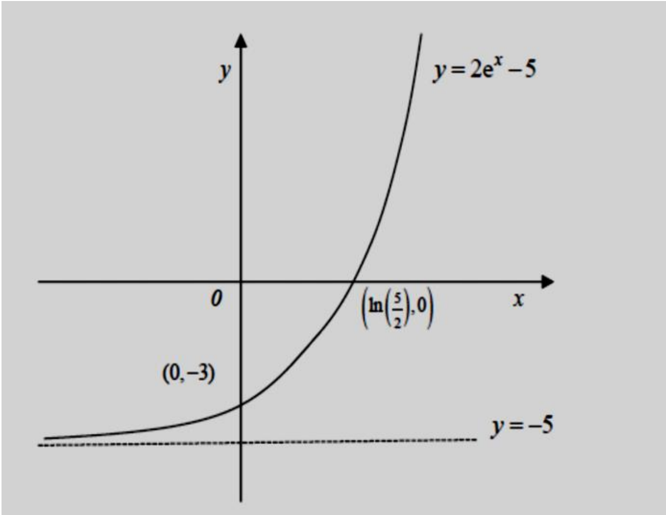
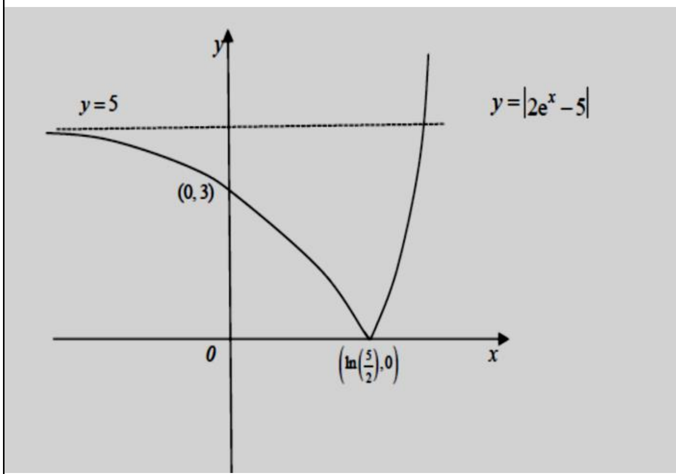
GCE A level Mathematics (9MA0)
Pure Mathematics 15b mark scheme

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

1(i)		
(a)	$a + ar = 34$ or $\frac{a(1-r^2)}{(1-r)} = 34$ or $\frac{a(r^2-1)}{(r-1)} = 34$; $\frac{a}{1-r} = 162$	**B1; B1**
	Eliminate a to give $(1+r)(1-r) = \frac{17}{81}$ or $1 - r^2 = \frac{34}{162}$ .. (not a cubic)	**aM1**
	(and so $r^2 = \frac{64}{81}$ and) $r = \frac{8}{9}$ only	**aA1** **(4)**
(b)	Substitute their $r = \frac{8}{9}$ (0 < r < 1) to give a = a = 18	**bM1** **bA1** **(2)**
(ii)	$\frac{42(1-\frac{6}{7}^n)}{1-\frac{6}{7}} > 290$ (For trial and improvement approach see notes below)	**M1**
	to obtain So $(\frac{6}{7})^n < (\frac{4}{294})$ or equivalent e.g. $(\frac{7}{6})^n > (\frac{294}{4})$ or $(\frac{6}{7})^n < (\frac{2}{147})$	**A1**
	So n > $\frac{\log"(\frac{4}{294})"}{\log(\frac{6}{7})}$ or $\log_{\frac{6}{7}}"(\frac{4}{294})"$ or equivalent but must be log of positive quantity	**M1**
	(i.e. n > 27.9) so n = 28	**A1** **(4)**
	May mark (a) and (b) together	

Question Number	Scheme	Marks
2.(ai)		
	Shape	B1
	$\left(\ln\left(\frac{5}{2}\right),0\right)$ and $(0,-3)$	B1
	$y=-5$	B1
		(3)
(aii)		
	Shape inc cusp	B1ft
	$\left(\ln\left(\frac{5}{2}\right),0\right)$ *and* $(0,3)$	B1ft
	$y=5$	B1ft
		(3)
(b)	$x \geq \ln\left(\frac{5}{2}\right)$	B1 ft **(1)**
(c)	$2e^x-5=-2 \Rightarrow (x)=\ln\left(\frac{3}{2}\right)$	M1A1
	$(x)=\ln\left(\frac{7}{2}\right)$	B1
		(3)
		(10 marks)

3. (a)	Expands to give $10x^{\frac{3}{2}} - 20x$	**B1**
	Integrates to give $\frac{10}{"\frac{5}{2}"} x^{"\frac{5}{2}"} + \frac{-"20" x^2}{2}$ (+ c)	**M1 A1ft**
	Simplifies to $4x^{\frac{5}{2}} - 10x^2$ (+ c)	**A1cao (4)**
(b)	Use limits 0 and 4 either way round on their integrated function (may only see 4 substituted)	**M1**
	Use limits 4 and 9 either way round on their integrated function	**dM1**
	Obtains either $\pm$–32 or $\pm$ 194 needs at least one of the previous M marks for this to be awarded	**A1**
	(So area = $\left\|\int_0^4 y dx\right\| + \int_4^9 y dx$) i.e. 32 + 194, = 226	**ddM1,A1 (5) [9]**
	$8^{2x+1} = 24$	

Question Number	Scheme	Marks
4(a)	$4\cos 2\theta + 2\sin 2\theta = R\cos(2\theta - \alpha)$	
	$R = \sqrt{4^2 + 2^2} = \sqrt{20} = \left(2\sqrt{5}\right)$	B1
	$\alpha = \arctan\left(\frac{1}{2}\right) = 26.565°\ldots =$ awrt $26.57°$	M1A1
		(3)
(b)	$\sqrt{20}\cos(2\theta - 26.6) = 1 \Rightarrow \cos(2\theta - 26.57) = \frac{1}{\sqrt{20}}$	M1
	$\Rightarrow (2\theta - 26.57) = +77.1.. \Rightarrow \theta = \ldots$	dM1
	$\theta =$ awrt $51.8°$	A1
	$2\theta - 26.57 = '-77.1...' \Rightarrow \theta = -$ awrt $25.3°$	ddM1A1
		(5)
(c)	$k < -\sqrt{20}, k > \sqrt{20}$	B1ft either B1ft both
		(2)
		(10 marks)

5(a)	$(\theta =)20$	B1 **(1)**
(b)	Sub $t = 40, \theta = 70 \Rightarrow 70 = 120 - 100e^{-40\lambda}$	
	$\Rightarrow e^{-40\lambda} = 0.5$	M1A1
	$\Rightarrow \lambda = \frac{\ln 2}{40}$	M1A1
		(4)
(c)	$\theta = 100 \Rightarrow T = \frac{\ln 0.2}{-\text{their}'\lambda'}$	M1
	$T =$ awrt 93	A1
		(2)
		(7 marks)

6.(a)	$2^{x+1} - 3 = 17 - x \Rightarrow 2^{x+1} = 20 - x$	M1
	$(x+1)\ln 2 = \ln(20 - x) \Rightarrow x = ...$	dM1
	$x = \dfrac{\ln(20-x)}{\ln 2} - 1$	A1*
		(3)
(b)	Sub $x_0 = 3$ into $x_{n+1} = \dfrac{\ln(20 - x_n)}{\ln 2} - 1, \Rightarrow$ $x_1 = 3.087$ (awrt)	M1A1
	$x_2 = 3.080, x_3 = 3.081$ (awrt)	A1
		(3)
(c)	$A = (3.1, 13.9)$ cao	M1,A1
		(2)
		(8 marks)

Question Number	Scheme	Marks
7.(a)	Applies $vu'+uv'$ to $\left(x^2-x^3\right)\mathrm{e}^{-2x}$	
	$\mathrm{g}'(x)=\left(x^2-x^3\right)\times\text{-}2\mathrm{e}^{-2x}+\left(2x-3x^2\right)\times\mathrm{e}^{-2x}$	M1 A1
	$\mathrm{g}'(x)=\left(2x^3-5x^2+2x\right)\mathrm{e}^{-2x}$	A1
		(3)
(b)	Sets $\left(2x^3-5x^2+2x\right)\mathrm{e}^{-2x}=0\Rightarrow 2x^3-5x^2+2x=0$	M1
	$x\left(2x^2-5x+2\right)=0\Rightarrow x=(0),\frac{1}{2},2$	M1,A1
	Sub $x=\frac{1}{2},2$ into $g(x)=\left(x^2-x^3\right)\mathrm{e}^{-2x}\Rightarrow g\left(\frac{1}{2}\right)=\frac{1}{8\mathrm{e}},\quad g(2)=-\frac{4}{\mathrm{e}^4}$	dM1,A1
	Range $-\frac{4}{\mathrm{e}^4}\leqslant g(x)\leqslant\frac{1}{8\mathrm{e}}$	A1 **(6)**
(c)	Accept $\mathrm{g}(x)$ is NOT a ONE to ONE function	
	Accept $\mathrm{g}(x)$ is a MANY to ONE function	B1
	Accept $\mathrm{g}^{-1}(x)$ would be ONE to MANY	**(1)**
		(10 marks)

Question Number	Scheme	Marks
8(a)	$\sec 2A + \tan 2A = \dfrac{1}{\cos 2A} + \dfrac{\sin 2A}{\cos 2A}$	B1
	$= \dfrac{1 + \sin 2A}{\cos 2A}$	M1
	$= \dfrac{1 + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$	M1
	$= \dfrac{\cos^2 A + \sin^2 A + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$	
	$= \dfrac{(\cos A + \sin A)(\cos A + \sin A)}{(\cos A + \sin A)(\cos A - \sin A)}$	M1
	$= \dfrac{\cos A + \sin A}{\cos A - \sin A}$	A1*
		(5)
(b)	$\sec 2\theta + \tan 2\theta = \dfrac{1}{2} \Rightarrow \dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \dfrac{1}{2}$	
	$\Rightarrow 2\cos\theta + 2\sin\theta = \cos\theta - \sin\theta$	
	$\Rightarrow \tan\theta = -\dfrac{1}{3}$	M1 A1
	$\Rightarrow \theta = awrt\ 2.820, 5.961$	dM1A1
		(4)
		(9 marks)

Question Number	Scheme	Marks
9.(a)	$x^2-3kx+2k^2=(x-2k)(x-k)$	B1
	$2-\dfrac{(x-5k)(x-k)}{(x-2k)(x-k)}=2-\dfrac{(x-5k)}{(x-2k)}=\dfrac{2(x-2k)-(x-5k)}{(x-2k)}$	M1
	$=\dfrac{x+k}{(x-2k)}$	A1*
		(3)
(b)	Applies $\dfrac{vu'-uv'}{v^2}$ to $y=\dfrac{x+k}{x-2k}$ with $u=x+k$ and $v=x-2k$	
	$\Rightarrow \mathrm{f}'(x)=\dfrac{(x-2k)\times 1-(x+k)\times 1}{(x-2k)^2}$	M1, A1
	$\Rightarrow \mathrm{f}'(x)=\dfrac{-3k}{(x-2k)^2}$	A1
		(3)
(c)	If $\mathrm{f}'(x)=\dfrac{-Ck}{(x-2k)^2}\Rightarrow \mathrm{f}(x)$ is an increasing function as $\mathrm{f}'(x)>0$,	M1
	$\mathrm{f}'(x)=\dfrac{-3k}{(x-2k)^2}>0$ for all values of x as $\dfrac{\text{negative}\times\text{negative}}{\text{positive}}=\text{positive}$	A1
		(2)
		(8 marks)

10.

Question Number	Scheme		Marks
1. (a)	$(4+5x)^{\frac{1}{2}} = \underline{(4)^{\frac{1}{2}}}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}} = \underline{2}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}}$	$\underline{(4)^{\frac{1}{2}}}$ or $\underline{2}$	B1
	$=\{2\}\left[1+\left(\frac{1}{2}\right)(kx)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2+\ldots\right]$	see notes	M1 A1ft
	$=\{2\}\left[1+\left(\frac{1}{2}\right)\left(\frac{5x}{4}\right)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}\left(\frac{5x}{4}\right)^2+\ldots\right]$		
	$= 2\left[1+\frac{5}{8}x-\frac{25}{128}x^2+\ldots\right]$	See notes below!	
	$= 2+\frac{5}{4}x;\ -\frac{25}{64}x^2+\ldots$	isw	A1; A1
			[5]
(b)	$\left\{x=\frac{1}{10} \Rightarrow (4+5(0.1))^{\frac{1}{2}} = \sqrt{4.5} = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}\right\}$		
	$=\frac{3}{2}\sqrt{2}$	$\frac{3}{2}\sqrt{2}$ or $k=\frac{3}{2}$ or 1.5 o.e.	B1
			[1]
(c)	$\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\frac{3}{\sqrt{2}} = 2+\frac{5}{4}\left(\frac{1}{10}\right)-\frac{25}{64}\left(\frac{1}{10}\right)^2+\ldots\ \{=2.121\ldots\}$	See notes	M1
	So, $\frac{3}{2}\sqrt{2}=\frac{543}{256}$ or $\frac{3}{\sqrt{2}}=\frac{543}{256}$		
	yields, $\sqrt{2}=\frac{181}{128}$ or $\sqrt{2}=\frac{256}{181}$	$\frac{181}{128}$ or $\frac{362}{256}$ or $\frac{543}{384}$ **or** $\frac{256}{181}$ etc.	A1 oe
			[2] **8**

11.

Question Number	Scheme		Marks
2.	$x^2 - 3xy - 4y^2 + 64 = 0$		
(a)	$\left\{\frac{dy}{dx} = \right\}$ $2x - \left(3y + 3x\frac{dy}{dx}\right) - 8y\frac{dy}{dx} = 0$		M1A1 M1
	$2x - 3y + (-3x - 8y)\frac{dy}{dx} = 0$		dM1
	$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$ **o.e.**		A1 **cso**
			[5]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow \right\}$ $2x - 3y = 0$		M1
	$y = \frac{2}{3}x$	$x = \frac{3}{2}y$	A1ft
	$x^2 - 3x\left(\frac{2}{3}x\right) - 4\left(\frac{2}{3}x\right)^2 + 64 = 0$	$\left(\frac{3}{2}y\right)^2 - 3\left(\frac{3}{2}y\right)y - 4y^2 + 64 = 0$	dM1
	$x^2 - 2x^2 - \frac{16}{9}x^2 + 64 = 0 \Rightarrow -\frac{25}{9}x^2 + 64 = 0$	$\frac{9}{4}y^2 - \frac{9}{2}y^2 - 4y^2 + 64 = 0 \Rightarrow -\frac{25}{4}y^2 + 64 = 0$	
	$\left\{\Rightarrow x^2 = \frac{576}{25} \Rightarrow\right\}$ $x = \frac{24}{5}$ or $-\frac{24}{5}$	$\left\{\Rightarrow y^2 = \frac{256}{25} \Rightarrow\right\}$ $y = \frac{16}{5}$ or $-\frac{16}{5}$	A1 **cso**
	When $x = \pm\frac{24}{5}$, $y = \frac{2}{3}\left(\frac{24}{5}\right)$ and $-\frac{2}{3}\left(\frac{24}{5}\right)$	When $y = \pm\frac{16}{5}$, $x = \frac{3}{2}\left(\frac{16}{5}\right)$ and $-\frac{3}{2}\left(\frac{16}{5}\right)$	
	$\left(\frac{24}{5}, \frac{16}{5}\right)$ **and** $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ **or** $x = \frac{24}{5}$, $y = \frac{16}{5}$ **and** $x = -\frac{24}{5}$, $y = -\frac{16}{5}$ **cso**		ddM1 A1
			[6] **11**