Pearson Edexcel Level 3

GCE Mathematics

Advanced Level

Paper 2: Pure Mathematics

Practice Paper 15b **Time: 2 hours**	**Paper Reference(s)** **9MA0/02**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. (i) All the terms of a geometric series are positive. The sum of the first two terms is 34 and the sum to infinity is 162.

Find

(*a*) the common ratio,

(4)

(*b*) the first term.

(2)

(ii) A different geometric series has a first term of 42 and a common ratio of $\frac{6}{7}$.

Find the smallest value of n for which the sum of the first n terms of the series exceeds 290.

(4)

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2. Given that

$$f(x) = 2e^x - 5, \qquad x \in \mathbb{R},$$

(*a*) sketch, on separate diagrams, the curve with equation

(i) $y = f(x)$,

(ii) $y = |f(x)|$.

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes.

On each diagram state the equation of the asymptote.

(6)

(*b*) Deduce the set of values of x for which $f(x) = |f(x)|$.

(1)

(*c*) Find the exact solutions of the equation $|f(x)| = 2$.

(3)

3. (*a*) Find

$$\int 10x(x^{\frac{1}{2}} - 2) \, dx,$$

giving each term in its simplest form.

(4)

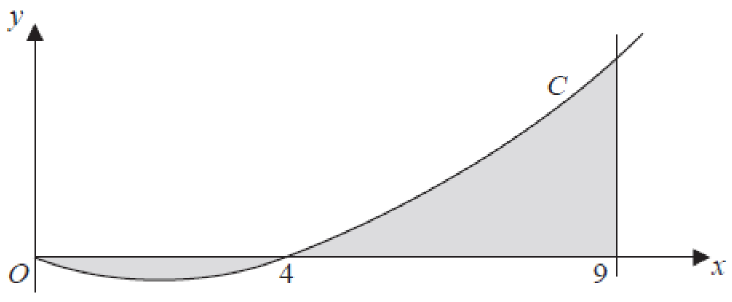

Figure 1

Figure 2 shows a sketch of part of the curve C with equation

$$y = 10x(x^{\frac{1}{2}} - 2), \qquad x \geq 0.$$

The curve C starts at the origin and crosses the x-axis at the point (4, 0).

The area, shown shaded in Figure 1, consists of two finite regions and is bounded by the curve C, the x-axis and the line $x = 9$.

(*b*) Use your answer from part (*a*) to find the total area of the shaded regions.

(5)

4. $$g(\theta) = 4\cos 2\theta + 2\sin 2\theta.$$

Given that $g(\theta) = R\cos(2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(*a*) find the exact value of R and the value of α to 2 decimal places.

(3)

(*b*) Hence solve, for $-90° < \theta < 90°$,

$$4\cos 2\theta + 2\sin 2\theta = 1,$$

giving your answers to one decimal place.

(5)

Given that k is a constant and the equation $g(\theta) = k$ has no solutions,

(*c*) state the range of possible values of k.

(2)

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5. Water is being heated in an electric kettle. The temperature, θ °C, of the water t seconds after the kettle is switched on, is modelled by the equation

$$\theta = 120 - 100e^{-\lambda t}, \qquad 0 \le t \le T.$$

(*a*) State the value of θ when $t = 0$.

(1)

Given that the temperature of the water in the kettle is 70 °C when $t = 40$,

(*b*) find the exact value of λ, giving your answer in the form $\frac{\ln a}{b}$, where a and b are integers.

(4)

When $t = T$, the temperature of the water reaches 100 °C and the kettle switches off.

(*c*) Calculate the value of T to the nearest whole number.

(2)

6.

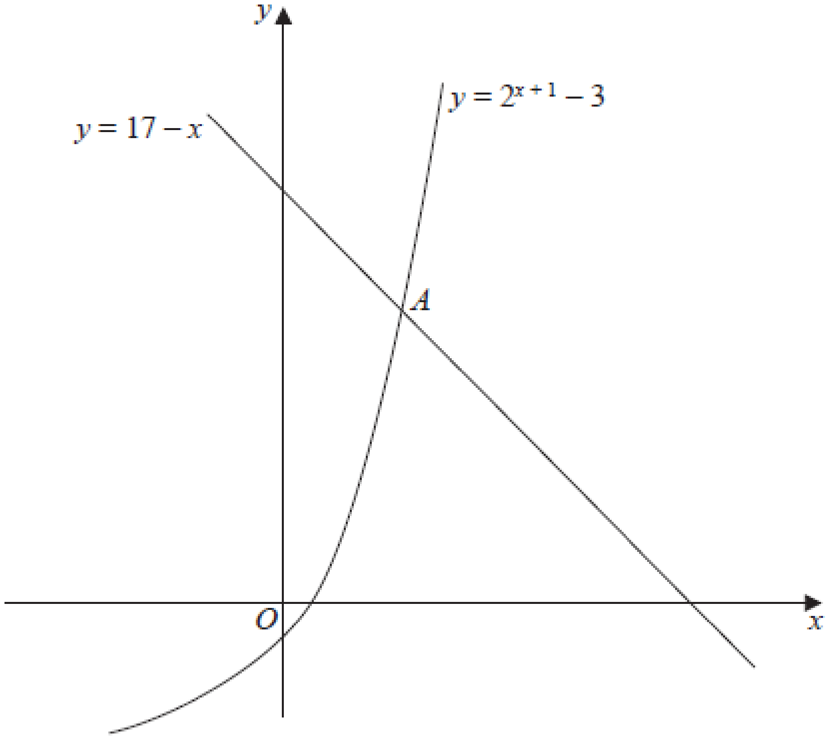

Figure 2

Figure 2 is a sketch showing part of the curve with equation $y = 2^{x+1} - 3$ and part of the line with equation $y = 17 - x$.

The curve and the line intersect at the point A.

(a) Show that the x-coordinate of A satisfies the equation

$$x = \frac{\ln(20 - x)}{\ln 2} - 1.$$

(3)

(b) Use the iterative formula

$$x_{n+1} = \frac{\ln(20 - x_n)}{\ln 2} - 1, \qquad x_0 = 3,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(c) Use your answer to part (b) to deduce the coordinates of the point A, giving your answers to one decimal place.

(2)

7.

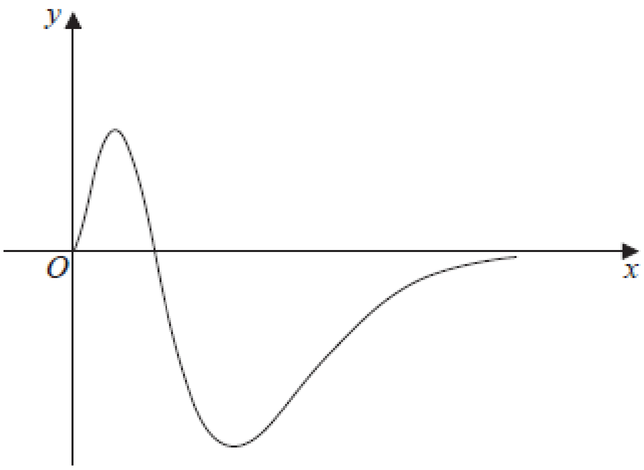

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$g(x) = x^2(1 - x)e^{-2x}, \qquad x \geq 0,$$

(*a*) Show that $g'(x) = f(x)e^{-2x}$, where f(*x*) is a cubic function to be found.

(3)

(*b*) Hence find the range of g.

(6)

(*c*) State a reason why the function $g^{-1}(x)$ does not exist.

(1)

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8. (*a*) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \qquad A \neq \frac{(2n+1)\pi}{4}, \quad n \in \mathbb{Z},$$

(5)

(*b*) Hence solve, for $0 \leq \theta < 2\pi$,

$$\sec 2\theta + \tan 2\theta = \frac{1}{2},$$

Give your answers to 3 decimal places.

(4)

9. Given that k is a **negative** constant and that the function f(x) is defined by

$$f(x) = 2 - \frac{(x-5k)(x-k)}{x^2 - 3kx + 2k^2}, \qquad x \geq 0,$$

show that $f(x) = \frac{x+k}{x-2k}$

(3)

(b) Hence find $f'(x)$, giving your answer in its simplest form.

(3)

(c) State, with a reason, whether f(x) is an increasing or a decreasing function.
Justify your answer.

(2)

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10. (a) Find the binomial expansion of

$$(4+5x)^{\frac{1}{2}}, \qquad |x| < \frac{4}{5},$$

in ascending powers of x, up to and including the term in x^2.
Give each coefficient in its simplest form.

(5)

(b) Find the exact value of $(4+5x)^{\frac{1}{2}}$ when $x = \frac{1}{10}$.

Give your answer in the form $k\sqrt{2}$, where k is a constant to be determined.

(1)

(c) Substitute $x = \frac{1}{10}$ into your binomial expansion from part (a)
and hence find an approximate value for $\sqrt{2}$.

Give your answer in the form $\frac{p}{q}$, where p and q are integers.

(2)

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11. The curve C has equation

$$x^2 - 3xy - 4y^2 + 64 = 0.$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Find the coordinates of the points on C where $\frac{dy}{dx} = 0$.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)