GCE A level Mathematics (9MA0)
Pure Mathematics 15a mark scheme

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

1.	$y = 2x + 4 \Rightarrow 4x^2 + (2x+4)^2 + 20x = 0$ **or** $2x = y - 4$ or $x = \frac{y-4}{2}$ $\Rightarrow (y-4)^2 + y^2 + 10(y-4) = 0$	M1
	$8x^2 + 36x + 16 = 0$ or $2y^2 + 2y - 24 = 0$	M1 A1
	$(4)(2x+1)(x+4) = 0 \Rightarrow x = ...$ or $(2)(y+4)(y-3) = 0 \Rightarrow y = ...$	M1
	$x = -0.5, x = -4$ or $y = -4, y = 3$	A1 cso
	Sub into $y = 2x + 4$ or Sub into $x = \frac{y-4}{2}$	M1
	$y = 3,\ \ y = -4$ and $x = -4,\ \ x = -0.5$	A1
		(7 marks)

2. (i)(a)	$U_3 = 4$	B1 (1)
(b)	$\sum_{n=1}^{n=20} U_n = 4 + 4 + 4 .... + 4$ or 20×4	M1
	$= 80$	A1 (2)
(ii)(a)	$V_3 = 3k, \quad V_4 = 4k$	B1, B1 (2)
(b)	$\sum_{n=1}^{n=5} V_n = k + 2k + 3k + 4k + 5k = 165$ or $\frac{1}{2} \times 5(2 \times k + 4 \times k) = 165$ or $\frac{1}{2} \times 5(k + 5k) = 165$	M1
	$15k = 165 \Rightarrow k = ..$	M1
	$k = 11$ cao and cso	A1 (3)
		(8 marks)

3. (a)	$b^2-4ac<0 \Rightarrow$ e.g. $4^2-4(p-1)(p-5)<0$ or $0>4^2-4(p-1)(p-5)$ or $4^2<4(p-1)(p-5)$ or $4(p-1)(p-5)>4^2$	M1 A1
	$4<p^2-6p+5$	
	$p^2-6p+1>0$	A1*
		(3)
(b)	$p^2-6p+1=0 \Rightarrow p=...$	M1
	$p=3\pm\sqrt{8}$	A1
	$p<3-\sqrt{8}$ or $p>3+\sqrt{8}$	M1A1
		(4) **(7 marks)**

4. (a)	$(x^2+4)(x-3)=x^3-3x^2+4x-12$	M1
	$\frac{x^3-3x^2+4x-12}{2x}=\frac{x^2}{2}-\frac{3}{2}x+2-6x^{-1}$	M1 A1
	$\frac{dy}{dx}=x-\frac{3}{2}+\frac{6}{x^2}$ oe e.g. $\frac{2x^3-3x^2+12}{2x^2}$	ddM1 A1
		(5)
b)	**At** $x=-1,\ y=10$	B1
	$\left(\frac{dy}{dx}=\right)-1-\frac{3}{2}+\frac{6}{1}=3.5$	M1A1
	$y-'10'='3.5'(x--1)$	M1
	$2y-7x-27=0$	A1
		(5)
		(10 marks)

5. (a)	$\left(4^x =\right) y^2$ **Allow** y^2 or $y \times y$ **or "y squared"** $"4^x ="$ **not required**	B1
		(1)
5(b)	$8y^2 - 9y + 1 = (8y-1)(y-1) = 0 \Rightarrow y = \ldots$ **or** $(8(2^x)-1)((2^x)-1) = 0 \Rightarrow 2^x = \ldots$	M1
	$2^x (\text{or } y) = \frac{1}{8},\ 1$	A1
	$x = -3 \quad x = 0$	M1 A1
		(4)
		(5 marks)

6. (a)	$32000 = 17000 + (k-1)\times 1500 \Rightarrow k = ...$	M1
	(k =) 11	A1
		(2)
(b)	$S = \frac{k}{2}(2\times 17000 + (k-1)\times 1500)$ or $\frac{k}{2}(17000 + 32000)$ $S = \frac{k-1}{2}(2\times 17000 + (k-2)\times 1500)$ or $\frac{k-1}{2}(17000 + 30500)$	M1
	$S = \frac{11}{2}(2\times 17000 + 10\times 1500)$ or $\frac{11}{2}(17000 + 32000)$ $S = \frac{10}{2}(2\times 17000 + 9\times 1500)$ or $\frac{10}{2}(17000 + 30500)$ **(= 269 500 or 237 500)**	A1
	$32000\times\alpha$	M1
	288 000 + 269 500 = 557 500 **or** **320 000 + 237 500 = 557 500**	ddM1A1
		(5)
		(7 marks)

7. (a)	$(x \mp 2)^2 + (y \pm 1)^2 = k$, $k > 0$	M1	
	Attempts to use $r^2 = (4-2)^2 + (-5+1)^2$	M1	
	Obtains $(x-2)^2 + (y+1)^2 = 20$	A1	**(3)**
(b)	Gradient of radius from centre to (4, –5) = –2 (must be correct)	B1	
	Tangent gradient = $-\dfrac{1}{\text{their numerical gradient of radius}}$	M1	
	Equation of tangent is $(y+5) = '\tfrac{1}{2}'(x-4)$	M1	
	So equation is $x - 2y - 14 = 0$	A1	**(4)**
			[7]

8.(a)	In triangle OCD complete method used to find angle COD so: Either $\cos COD = \frac{8^2+8^2-7^2}{2\times8\times8}$ or uses $\angle COD = 2\times\arcsin\frac{3.5}{8}$ oe so $\angle COD =$	**M1**
	($\angle COD = 0.9056(331894)$) = 0.906 (3sf) * accept awrt 0.906	**A1 *** **(2)**
(b)	Uses s = 8θ for any θ in radians or $\frac{\theta}{360}\times2\pi\times8$ for any θ in degrees	**M1**
	$\theta = \frac{\pi - "COD"}{2}$ $(= awrt\ 1.12)$ or $2\theta(= awrt\ 2.24)$ and Perimeter = 23+(16 $\times\theta$)	**M1**
	accept awrt 40.9 (cm)	**A1** **(3)**
(c)	Either Way 1: (Use of Area of two sectors + area of triangle)	
	Area of triangle = $\frac{1}{2}\times8\times8\times\sin0.906$ (or 25.1781155 accept awrt 25.2)or $\frac{1}{2}\times8\times7\times\sin1.118$ or $\frac{1}{2}\times7\times h$ after h calculated from correct Pythagoras or trig.	**M1**
	Area of sector = $\frac{1}{2}8^2\times"1.117979732"$ (or 35.77535142 accept awrt 35.8)	**M1**
	Total Area = Area of two sectors + area of triangle =awrt 96.7 or 96.8 or 96.9 (cm^2)	**A1** **(3)**

9.(a)	$\tan 2\theta° = \dfrac{2\tan\theta°}{1-\tan^2\theta°} = \dfrac{2p}{1-p^2}$	M1A1 **(2)**
(b)	$\cos\theta° = \dfrac{1}{\sec\theta°} = \dfrac{1}{\sqrt{1+\tan^2\theta°}} = \dfrac{1}{\sqrt{1+p^2}}$	M1A1 **(2)**
(c)	$\cot(\theta-45)° = \dfrac{1}{\tan(\theta-45)°} = \dfrac{1+\tan\theta°\tan 45°}{\tan\theta°-\tan 45°} = \dfrac{1+p}{p-1}$	M1A1 **(2)** **(6 marks)**

10.(a)	$p = 4\pi^2$ $p = 4\pi^2$ or $(2\pi)^2$	B1
		(1)
(b)	$x = (4y - \sin 2y)^2 \Rightarrow \frac{dx}{dy} = 2(4y - \sin 2y)(4 - 2\cos 2y)$	M1A1
	Sub $y = \frac{\pi}{2}$ into $\frac{dx}{dy} = 2(4y - \sin 2y)(4 - 2\cos 2y)$	
	Þ $\frac{dx}{dy} = 24\pi$ $(= 75.4)$ / $\frac{dy}{dx} = \frac{1}{24\pi}(= 0.013)$	M1
	Equation of tangent $y - \frac{\pi}{2} = \frac{1}{24\pi}(x - 4\pi^2)$	M1
	Using $y - \frac{\pi}{2} = \frac{1}{24\pi}(x - 4\pi^2)$ with $x = 0$ Þ $y = \frac{\pi}{3}$ cso	M1, A1
		(6)
		(7 marks)

11. (i)	$(2x+1)\log 8 = \log 24$ or $(2x+1) = \log_8 24$ | or $8^{2x} = 3$ and so $(2x)\log 8 = \log 3$ or $(2x) = \log_8 3$	M1	
	$x = \frac{1}{2}\left(\frac{\log 24}{\log 8} - 1\right)$ or $x = \frac{1}{2}(\log_8 24 - 1)$ | $x = \frac{1}{2}\left(\frac{\log 3}{\log 8}\right)$ or $x = \frac{1}{2}(\log_8 3)$ o.e.	dM1	
	=0.264	A1	**(3)**
(ii)	$\log_2(11y-3) - \log_2 3 - 2\log_2 y = 1$ $\log_2(11y-3) - \log_2 3 - \log_2 y^2 = 1$.	M1	
	$\log_2 \frac{(11y-3)}{3y^2} = 1$ or $\log_2 \frac{(11y-3)}{y^2} = 1 + \log_2 3 = 2.58496501$	dM1	
	$\log_2 \frac{(11y-3)}{3y^2} = \log_2 2$ or $\log_2 \frac{(11y-3)}{y^2} = \log_2 6$ (allow awrt 6 if replaced by 6 later)	B1	
	Obtains $6y^2 - 11y + 3 = 0$ o.e. i.e. $6y^2 = 11y - 3$ for example	A1	
	Solves quadratic to give $y =$	ddM1	
	$y = \frac{1}{3}$ and $\frac{3}{2}$ (need both- one should not be rejected)	A1	**(6)** **[9]**

12. (i)	Divides by $\cos 3\theta$ to give $\tan 3\theta = \sqrt{3}$ so $(3\theta) = \frac{\pi}{3}$	M1
	Adds π or 2π to previous value of angle (to give $\frac{4\pi}{3}$ or $\frac{7\pi}{3}$)	M1
	So $\theta = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}$ (all three, no extra in range)	A1 **(3)**
(ii)(a)	$4(1-\cos^2 x) + \cos x = 4 - k$ Applies $\sin^2 x = 1 - \cos^2 x$	M1
	Attempts to solve $4\cos^2 x - \cos x - k = 0$, to give $\cos x =$	dM1
	$\cos x = \frac{1 \pm \sqrt{1+16k}}{8}$ or $\cos x = \frac{1}{8} \pm \sqrt{\frac{1}{64} + \frac{k}{4}}$ or other correct equivalent	A1 **(3)**
(b)	$\cos x = \frac{1 \pm \sqrt{49}}{8} = 1$ and $-\frac{3}{4}$ (see the note below if errors are made)	M1
	Obtains two solutions from 0 , 139 , 221 (0 or 2.42 or 3.86 in radians)	dM1
	x = 0 and 139 and 221 (allow awrt 139 and 221) must be in degrees	A1 **(3)** **[9]**

13. (a)	Either: (Cost of polishing top and bottom (two circles) is) $3\times 2\pi r^2$ **or (**Cost of polishing curved surface area is) $2\times 2\pi rh$ or both - just need to see at least one of these products	B1
	Uses volume to give $(h=)\dfrac{75\pi}{\pi r^2}$ or $(h=)\dfrac{75}{r^2}$ (simplified) (if V is misread – see below)	B1ft
	$(C)=6\pi r^2+4\pi r\left(\dfrac{75}{r^2}\right)$ — Substitutes expression for h into area or cost expression of form Ar^2+Brh	M1
	$C=6\pi r^2+\dfrac{300\pi}{r}$ *	A1* (4)
(b)	$\left\{\dfrac{dC}{dr}=\right\}12\pi r-\dfrac{300\pi}{r^2}$ or $12\pi r-300\pi r^{-2}$ (then isw)	M1 A1 ft
	$12\pi r-\dfrac{300\pi}{r^2}=0$ **so** r^k = value where $k=\pm 2,\pm 3,\pm 4$	dM1
	Use **cube** root to obtain r = $\left(\mathit{their}\ \frac{300}{12}\right)^{\frac{1}{3}}$ $(=2.92)$- allow r = 3, and thus C =	ddM1
	Then C = awrt 483 or 484	A1cao (5)
(c)	$\left\{\dfrac{d^2C}{dr^2}=\right\}12\pi+\dfrac{600\pi}{r^3}>0$ **so minimum**	B1ft (1)
		[10]