Pearson Edexcel Level 3

GCE Mathematics

Advanced Level

Paper 1: Pure Mathematics

Practice Paper 15a	**Paper Reference(s)**
Time: 2 hours	**9MA0/01**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Solve the simultaneous equations

$$y - 2x - 4 = 0$$

$$4x^2 + y^2 + 20x = 0$$

(7)

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2. (i) A sequence $U_1, U_2, U_3, \ldots$ is defined by

$$U_{n+2} = 2U_{n+1} - U_n, \quad n \geq 1,$$

$$U_1 = 4 \text{ and } U_2 = 4.$$

Find the value of

(*a*) U_3,

(1)

(*b*) $\sum_{n=1}^{20} U_n$.

(2)

(ii) Another sequence $V_1, V_2, V_3, \ldots$ is defined by

$$V_{n+2} = 2V_{n+1} - V_n, \quad n \geq 1,$$

$$V_1 = k \text{ and } V_2 = 2k, \text{ where } k \text{ is a constant.}$$

(*a*) Find V_3 and V_4 in terms of k.

(2)

Given that $\sum_{n=1}^{5} V_n = 165$,

(*b*) find the value of k.

(3)

3. The equation

$$(p-1)x^2 + 4x + (p-5) = 0, \text{ where } p \text{ is a constant,}$$

has no real roots.

(*a*) Show that p satisfies $p^2 - 6p + 1 > 0$.

(3)

(*b*) Hence find the set of possible values of p.

(4)

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4. The curve C has equation

$$y = \frac{(x^2+4)(x-3)}{2x}, \quad x \neq 0.$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form.

(5)

(*b*) Find an equation of the tangent to C at the point where $x = -1$.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

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5. Given that $y = 2^x$,

(*a*) express 4^x in terms of y.

(1)

(*b*) Hence, or otherwise, solve

$$8(4^x) - 9(2^x) + 1 = 0.$$

(4)

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6. Jess started work 20 years ago. In year 1 her annual salary was £17 000. Her annual salary increased by £1500 each year, so that her annual salary in year 2 was £18 500, in year 3 it was £20 000 and so on, forming an arithmetic sequence. This continued until she reached her maximum annual salary of £32000 in year k. Her annual salary then remained at £32 000.

(*a*) Find the value of the constant k.

(2)

(*b*) Calculate the total amount that Jess has earned in the 20 years.

(5)

7. A circle C with centre at the point $(2, -1)$ passes through the point A at $(4, -5)$.

(a) Find an equation for the circle C.

(3)

(b) Find an equation of the tangent to the circle C at the point A, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

8.

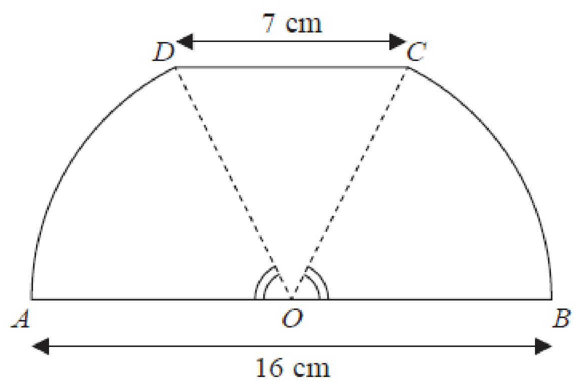

Figure 1

Figure 1 shows a sketch of a design for a scraper blade. The blade $AOBCDA$ consists of an isosceles triangle COD joined along its equal sides to sectors OBC and ODA of a circle with centre O and radius 8 cm. Angles AOD and BOC are equal. AOB is a straight line and is parallel to the line DC. DC has length 7 cm.

(a) Show that the angle COD is 0.906 radians, correct to 3 significant figures.

(2)

(b) Find the perimeter of $AOBCDA$, giving your answer to 3 significant figures.

(3)

(c) Find the area of $AOBCDA$, giving your answer to 3 significant figures.

(3)

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9. Given that

$$\tan \theta° = p, \text{ where } p \text{ is a constant, } p \neq \pm 1,$$

use standard trigonometric identities, to find in terms of p,

(*a*) $\tan 2\theta°$,

(2)

(*b*) $\cos \theta°$,

(2)

(*c*) $\cot (\theta - 45)°$.

(2)

Write each answer in its simplest form.

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10. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2.$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

(*a*) find the exact value of p.

(1)

The tangent to the curve at P cuts the y-axis at the point A.

(*b*) Use calculus to find the coordinates of A.

(6)

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11. (i) Use logarithms to solve the equation $8^{2x+1} = 24$, giving your answer to 3 decimal places.

(3)

(ii) Find the values of y such that

$$\log_2 (11y - 3) - \log_2 3 - 2 \log_2 y = 1, \qquad y > \frac{3}{11}.$$

(6)

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12. (i) Solve, for $0 \leq \theta < \pi$, the equation

$$\sin 3\theta - \sqrt{3} \cos 3\theta = 0,$$

giving your answers in terms of π.

(3)

(ii) Given that

$$4 \sin^2 x + \cos x = 4 - k, \qquad 0 \leq k \leq 3,$$

(*a*) find $\cos x$ in terms of k.

(3)

(*b*) When $k = 3$, find the values of x in the range $0 \leq x < 360°$.

(3)

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13. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of $75\pi \text{ cm}^3$.

The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

(*a*) show that the cost of the polishing, £C, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}.$$

(4)

(*b*) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound.

(5)

(*c*) Justify that the answer that you have obtained in part (*b*) is a minimum.

(1