GCE A level Mathematics (9MA0)
Pure Mathematics 14b mark scheme

Guidance on the use of codes within this document

M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.

A1 – accuracy mark. This mark is generally given for a correct answer following correct working.

B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.

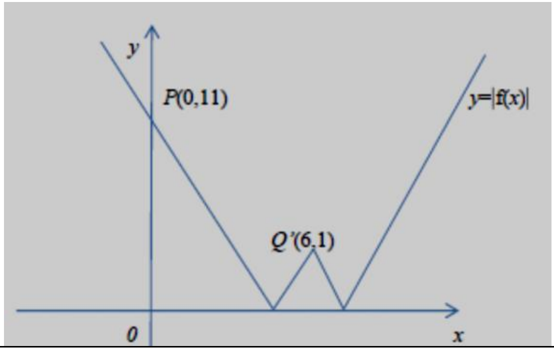
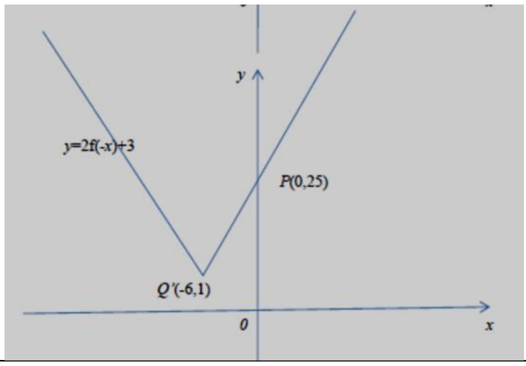
Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

1. (a)	$\frac{1}{2}(9x+6x)4x$ or $2x\times15x$ or $\left(\frac{1}{2}4x\times(9x-6x)+6x\times4x\right)$ or $6x^2+24x^2$ or $\left(9x\times4x-\frac{1}{2}4x\times(9x-6x)\right)$ or $36x^2-6x^2$	**M1A1cso**
	$\Rightarrow 30x^2y=9600 \Rightarrow y=\frac{9600}{30x^2} \Rightarrow y=\frac{320}{x^2}$ *	
		[2]
(b)	$(S=)\ \frac{1}{2}(9x+6x)4x+\frac{1}{2}(9x+6x)4x+6xy+9xy+5xy+4xy$	**M1A1**
	$y=\frac{320}{x^2} \Rightarrow (S=)\ 30x^2+30x^2+24x\left(\frac{320}{x^2}\right)$	**M1**
	So, $(S=)\ 60x^2+\frac{7680}{x}$ *	**A1* cso**
		[4]
10(c)	$\frac{dS}{dx}=120x-7680x^{-2}\ \left\{=120x-\frac{7680}{x^2}\right\}$	**M1**
		A1 aef
	$120x-\frac{7680}{x^2}=0$ $\Rightarrow x^3=\frac{7680}{120};=64 \Rightarrow x=4$	**M1A1cso**
	$\{x=4,\}\quad S=60(4)^2+\frac{7680}{4}=2880\ (\text{cm}^2)$	**ddM1**
		A1 cao and cso
		[6]
10(d)	$\frac{d^2S}{dx^2}=120+\frac{15360}{x^3}>0$ $\Rightarrow$ Minimum	**M1A1ft**
	Note parts (c) and (d) can be marked together.	**[2]**
		Total 14

2.(a)	$\mathrm{f}(x)=\frac{4x+1}{x-2}, \quad x>2$	
	Applies $\frac{vu'-uv'}{v^2}$ to get $\frac{(x-2)\times 4-(4x+1)\times 1}{(x-2)^2}$	**M1A1**
	$=\frac{-9}{(x-2)^2}$	**A1***
		(3)
(b)	$\frac{-9}{(x-2)^2}=-1 \Rightarrow x=..$	**M1**
	(5,7)	**A1,A1**
		(3)
		6 marks

3.(a)	$2\ln(2x+1)-10=0 \Rightarrow \ln(2x+1)=5 \quad \Rightarrow 2x+1=e^5 \Rightarrow x=..$	**M1**
	$\Rightarrow x=\frac{e^5-1}{2}$	**A1**
		(2)
(b)	$3^x e^{4x}=e^7 \Rightarrow \ln(3^x e^{4x})=\ln e^7$	
	$\ln 3^x+\ln e^{4x}=\ln e^7 \Rightarrow x\ln 3+4x\ln e=7\ln e$	**M1,M1**
	$x(\ln 3+4)=7 \Rightarrow x=...$	**dM1**
	$x=\frac{7}{(\ln 3+4)}$ oe	**A1**
		(4)
		6 marks

4.(a)	$x = 8\frac{\pi}{8}\tan\left(2\times\frac{\pi}{8}\right) = \pi$	**B1***
		(1)
(b)	$\frac{dx}{dy} = 8\tan 2y + 16y\sec^2(2y)$	**M1A1A1**
	At P $\frac{dx}{dy} = 8\tan 2\frac{\pi}{8} + 16\frac{\pi}{8}\sec^2\left(2\times\frac{\pi}{8}\right) = \{8+4\pi\}$	**M1**
	$\frac{y-\frac{\pi}{8}}{x-\pi} = \frac{1}{8+4\pi}$, accept $y - \frac{\pi}{8} = 0.049(x-\pi)$	**M1A1**
	$\Rightarrow (8+4\pi)y = x + \frac{\pi^2}{2}$	**A1**
		(7)
		(8 marks)

5.(a)	 'W' Shape (0, 11) and (6, 1)	**B1** **B1**
		(2)
(b)	 'V' shape (-6,1) (0,25)	**B1** **B1** **B1**
		(3)
(c)	One of a = 2 or b = 6	**B1**
	a = 2 and b = 6	**B1**
		(2)
		(7 marks)

6.(a)	$y_{2.1} = -0.224$, $y_{2.2} = (+)0.546$	**M1**
	Change of sign$\Rightarrow$Q lies between	**A1**
		(2)
(b)	$\frac{dy}{dx} = -2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$ At R	**M1A1**
	$-2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3 = 0 \Rightarrow$ $x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$ cso	**M1A1***
		(4)
(c)	$x_1 = \sqrt{1 + \frac{2}{3} \times 1.3\sin\left(\frac{1}{2} \times 1.3^2\right)}$	**M1**
	$x_1 = \text{awrt } 1.284$ $x_2 = \text{awrt } 1.276$	**A1**
		(2)
		(8 marks)

7.(a)	$\text{cosec}2x + \cot 2x = \dfrac{1}{\sin 2x} + \dfrac{\cos 2x}{\sin 2x}$	**M1**
	$= \dfrac{1+\cos 2x}{\sin 2x}$	**M1**
	$= \dfrac{1+2\cos^2 x - 1}{2\sin x \cos x}$	
	$= \dfrac{2\cos^2 x}{2\sin x \cos x}$	**M1 A1**
	$= \dfrac{\cos x}{\sin x} = \cot x$	**A1***
		(5)
(b)	$\text{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3}$	
	$\cot(2\theta \pm ..°) = \sqrt{3}$	**M1**
	$2\theta \pm ... = 30° \Rightarrow \theta = 12.5°$	**dM1, A1**
	$2\theta \pm ... = 180 + PV° \Rightarrow \theta = ..°$	**dM1**
	$\theta = 102.5°$	**A1**
		(5)
		(10 marks)

8.(a)	$P=\frac{800e^0}{1+3e^0},=\frac{800}{1+3}=200$	**M1,A1**
		(2)
(b)	$250=\frac{800e^{0.1t}}{1+3e^{0.1t}}$	
	$250(1+3e^{0.1t})=800e^{0.1t}\Rightarrow 50e^{0.1t}=250,\ \Rightarrow e^{0.1t}=5$	**M1,A1**
	$t=\frac{1}{0.1}\ln(5)$	**M1**
	$t=10\ln(5)$	**A1**
		(4)
(c)	$P=\frac{800e^{0.1t}}{1+3e^{0.1t}}\Rightarrow\frac{dP}{dt}=\frac{(1+3e^{0.1t})\times 800\times 0.1e^{0.1t}-800e^{0.1t}\times 3\times 0.1e^{0.1t}}{(1+3e^{0.1t})^2}$	**M1,A1**
	At t=10	
	$\frac{dP}{dt}=\frac{(1+3e)\times 80e-240e^2}{(1+3e)^2}=\frac{80e}{(1+3e)^2}$	**M1,A1**
		(4)
(d)	$P=\frac{800e^{0.1t}}{1+3e^{0.1t}}=\frac{800}{e^{-0.1t}+3}\Rightarrow P_{max}=\frac{800}{3}=266$. Hence P cannot be 270	**B1**
		(1)
		(11 marks)

9.(a)	$R=\sqrt{20}$	**B1**
	$\tan\alpha=\frac{4}{2}\Rightarrow\alpha=\text{awrt}\,1.107$	**M1A1**
		(3)
(b)(i)	$'4+5R^2'=104$	**B1ft**
(ii)	$3\theta-'1.107'=\frac{\pi}{2}\Rightarrow\theta=\text{awrt }0.89$	**M1A1**
		(3)
(c)(i)	4	**B1**
(ii)	$3\theta-'1.107'=2\pi\Rightarrow\theta=\text{awrt }2.46$	**M1A1**
		(3)
		(9 marks)

10.	$x^3 + 2xy - x - y^3 - 20 = 0$	
(a)	$\left\{\frac{\cancel{dy}}{\cancel{dx}} \cancel{\times}\right\}$ $\underline{3x^2} + \left(\underline{\underline{2y + 2x\frac{dy}{dx}}}\right) \underline{- 1 - 3y^2 \frac{dy}{dx} = 0}$	M1 A1 B1
	$3x^2 + 2y - 1 + (2x - 3y^2)\frac{dy}{dx} = 0$	dM1
	$\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$	A1 **cso**
		[5]
(b)	At $P(3, -2)$, $m(\mathbf{T}) = \frac{dy}{dx} = \frac{3(3)^2 + 2(-2) - 1}{3(-2)^2 - 2(3)}; = \frac{22}{6}$ or $\frac{11}{3}$	
	and either T: $y - -2 = "\frac{11}{3}"(x - 3)$ **see notes**	M1
	or $(-2) = \left(\frac{11}{3}\right)(3) + c \Rightarrow c = \dots,$	
	T: $11x - 3y - 39 = 0$ or $K(11x - 3y - 39) = 0$	A1 **cso**
		[2] **7**

11.	$\left\{(1+kx)^{-4} = 1 + (-4)(kx) + \frac{(-4)(-4-1)}{2!}(kx)^2 + ...\right\}$		
(a)	**Either** $(-4)k = -6$ **or** $(1+kx)^{-4} = 1 + (-4)(kx)$ **see notes**	M1	
	leading to $k = \frac{3}{2}$ $\qquad k = \frac{3}{2}$ or 1.5 or $\frac{6}{4}$	A1	
			[2]
(b)	$\frac{(-4)(-5)}{2}(k)^2$ **Either** $\frac{(-4)(-5)}{2!}$ **or** $(k)^2$ **or** $(kx)^2$	M1	
	Either $\frac{(-4)(-5)}{2!}(k)^2$ **or** $\frac{(-4)(-5)}{2!}(kx)^2$	M1	
	$\left\{A = \frac{(-4)(-5)}{2!}\left(\frac{3}{2}\right)^2\right\} \Rightarrow A = \frac{45}{2}$ $\qquad \frac{45}{2}$ or 22.5	A1	
			[3]
			5

12.	x: 1, 2, 3, 4; y: 1.42857, 0.90326, ***0.682116...***, 0.55556	$y = \dfrac{10}{2x+5\sqrt{x}}$	
(a)	$\{\text{At } x = 3,\}\ y =$ 0.68212 (5 dp)	0.68212	**B1 cao**
			[1]
(b)	$\frac{1}{2}\times 1\times\left[1.42857 + 0.55556 + 2(0.90326 + \text{their } 0.68212)\right]$	Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$	B1 aef
		For structure of $[\ldots\ldots\ldots\ldots]$	M1
	$\{= \frac{1}{2}(5.15489)\} = 2.577445 = 2.5774$ (4 dp)	anything that rounds to 2.5774	A1
			[3]
(c)	• Overestimate **and** a reason such as • {top of} trapezia lie above the curve • a diagram which gives reference to the extra area • concave or convex • $\frac{d^2y}{dx^2} > 0$ (can be implied) • bends inwards • curves downwards		B1
			[1]
(d)	$\{u = \sqrt{x} \Rightarrow\}\ \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{dx}{du} = 2u$		B1
	$\int \frac{10}{2u^2+5u}.\ 2u\ du$	Either $\left\{\int\right\}\frac{\pm ku}{\alpha u^2 \pm \beta u}\{du\}$ or $\left\{\int\right\}\frac{\pm k}{u(\alpha u^2 \pm \beta u)}\{du\}$	M1
	$\left\{= \int \frac{20}{2u+5}\ du\right\} = \frac{20}{2}\ln(2u+5)$	$\pm\lambda\ln(2u+5)$ or $\pm\lambda\ln\left(u+\frac{5}{2}\right)$, $\lambda \neq 0$ **with no other terms.**	M1
		$\frac{20}{2u+5} \to \frac{20}{2}\ln(2u+5)$ or $10\ln\left(u+\frac{5}{2}\right)$	A1 **cso**
	$\left\{\left[\frac{20}{2}\ln(2u+5)\right]_1^2\right\} = 10\ln(2(2)+5) - 10\ln(2(1)+5)$	Substitutes limits of 2 and 1 in u (or 4 and 1 in x) and subtracts the correct way round.	M1
	$10\ln 9 - 10\ln 7$ or $10\ln\left(\frac{9}{7}\right)$ or $20\ln 3 - 10\ln 7$		A1 oe **cso**
			[6]
			11