Pearson Edexcel Level 3

GCE Mathematics

Advanced Level

Paper 2: Pure Mathematics

Practice Paper 14b Time: 2 hours	Paper Reference(s) 9MA0/02

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1.

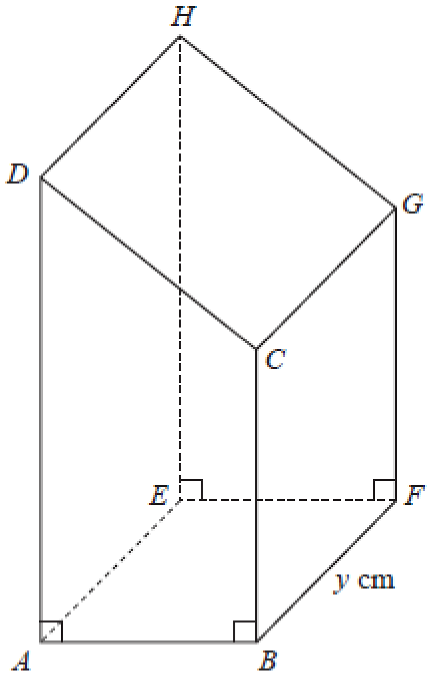

Figure 4

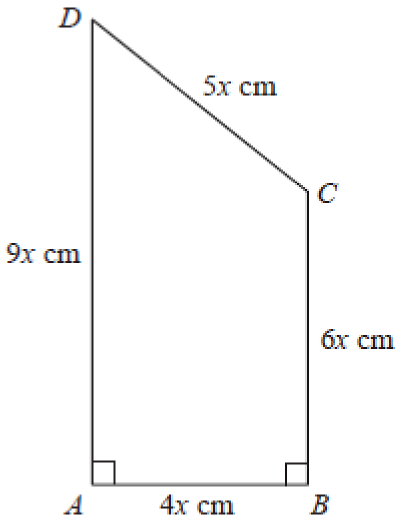

Figure 5

Figure 4 shows a closed letter box *ABFEHGCD*, which is made to be attached to a wall of a house.

The letter box is a right prism of length *y* cm as shown in Figure 4. The base *ABFE* of the prism is a rectangle. The total surface area of the six faces of the prism is *S* cm^2.

The cross section *ABCD* of the letter box is a trapezium with edges of lengths *DA* = 9*x* cm, *AB* = 4*x* cm, *BC* = 6*x* cm and *CD* = 5*x* cm as shown in Figure 5.

The angle *DAB* = 90° and the angle *ABC* = 90°. The volume of the letter box is 9600 cm^3.

(*a*) Show that $y = \dfrac{320}{x^2}$.

(2)

(*b*) Hence show that the surface area of the letter box, *S* cm^2, is given by $S = 60x^2 + \dfrac{7680}{x}$.

(4)

(*c*) Use calculus to find the minimum value of *S*.

(6)

(*d*) Justify, by further differentiation, that the value of *S* you have found is a minimum.

(2)

2. The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{4x+1}{x-2}, \qquad x > 2$$

(*a*) Show that

$$\mathrm{f}'(x) = \frac{-9}{(x-2)^2}$$

(3)

Given that P is a point on C such that $\mathrm{f}'(x) = -1$,

(*b*) find the coordinates of P.

(3)

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3. Find the exact solutions, in their simplest form, to the equations

(*a*) $2 \ln (2x + 1) - 10 = 0$

(2)

(*b*) $3^x e^{4x} = e^7$

(4)

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4. The curve C has equation $x = 8y \tan 2y$.

The point P has coordinates $\left(\pi, \frac{\pi}{8}\right)$.

(*a*) Verify that P lies on C.

(1)

(*b*) Find the equation of the tangent to C at P in the form $ay = x + b$, where the constants a and b are to be found in terms of π.

(7)

5.

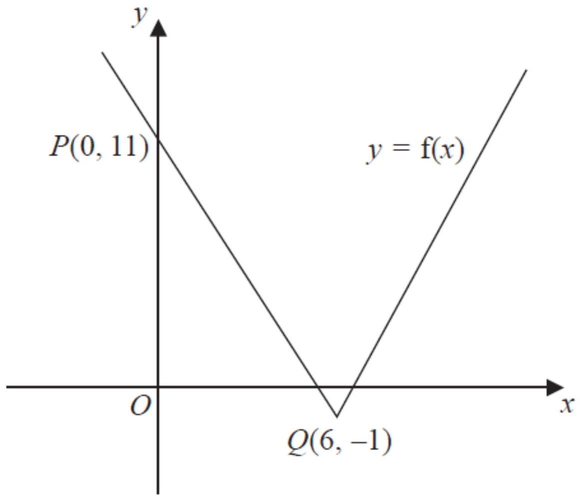

Figure 1

Figure 1 shows part of the graph with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $Q(6, -1)$.

The graph crosses the y-axis at the point $P(0, 11)$.

Sketch, on separate diagrams, the graphs of

(*a*) $y = |\mathrm{f}(x)|$

(2)

(*b*) $y = 2\mathrm{f}(-x) + 3$

(3)

On each diagram, show the coordinates of the points corresponding to P and Q.

Given that $\mathrm{f}(x) = a|x - b| - 1$, where a and b are constants,

(*c*) state the value of a and the value of b.

(2)

6.

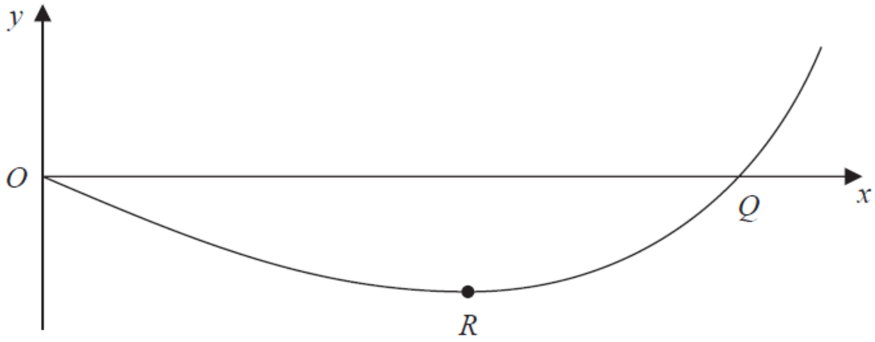

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 2\cos\left(\frac{1}{2}x^2\right) + x^3 - 3x - 2$$

The curve crosses the x-axis at the point Q and has a minimum turning point at R.

(a) Show that the x coordinate of Q lies between 2.1 and 2.2.

(2)

(b) Show that the x coordinate of R is a solution of the equation

$$x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$$

(4)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{2}{3}x_n\sin\left(\frac{1}{2}x_n{}^2\right)}, \qquad x_0 = 1.3$$

(c) find the values of x_1 and x_2 to 3 decimal places.

(2)

7. (*a*) Show that

$$\text{cosec } 2x + \cot 2x = \cot x, \qquad x \neq 90n°, \qquad n \in \mathbb{Z}$$

(5)

(*b*) Hence, or otherwise, solve, for $0 \leq \theta < 180°$,

$$\text{cosec } (4\theta + 10°) + \cot (4\theta + 10°) = \sqrt{3}$$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

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8. A rare species of primrose is being studied. The population, P, of primroses at time t years after the study started is modelled by the equation

$$P = \frac{800e^{0.1t}}{1 + 3e^{0.1t}}, \qquad t \geq 0, \quad t \in \mathbb{R}$$

(*a*) Calculate the number of primroses at the start of the study.

(2)

(*b*) Find the exact value of t when $P = 250$, giving your answer in the form $a \ln(b)$ where a and b are integers.

(4)

(*c*) Find the exact value of $\frac{dP}{dt}$ when $t = 10$. Give your answer in its simplest form.

(4)

(*d*) Explain why the population of primroses can never be 270.

(1)

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9. (*a*) Express $2\sin\theta - 4\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(3)

$$H(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

Find

(*b*) (i) the maximum value of $H(\theta)$,

(ii) the smallest value of θ, for $0 \leq \theta \leq \pi$, at which this maximum value occurs.

(3)

Find

(*c*) (i) the minimum value of $H(\theta)$,

(ii) the largest value of θ, for $0 \leq \theta \leq \pi$, at which this minimum value occurs.

(3)

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10. A curve C has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(*a*) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(*b*) Find an equation of the tangent to C at the point $(3, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

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11. Given that the binomial expansion of $(1 + kx)^{-4}$, $|kx| < 1$, is

$$1 - 6x + Ax^2 + \ldots$$

(*a*) find the value of the constant k,

(2)

(*b*) find the value of the constant A, giving your answer in its simplest form.

(3)

12.

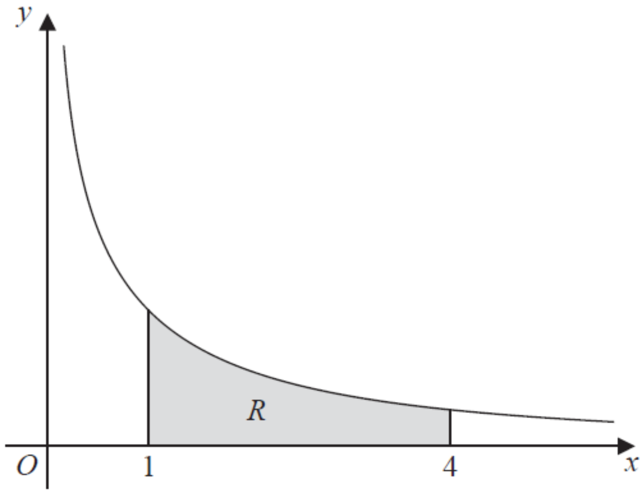

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{10}{2x + 5\sqrt{x}}$, $x > 0$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, and the lines with equations $x = 1$ and $x = 4$.

The table below shows corresponding values of x and y for $y = \dfrac{10}{2x + 5\sqrt{x}}$.

x	1	2	3	4
y	1.42857	0.90326		0.55556

(*a*) Complete the table above by giving the missing value of y to 5 decimal places.

(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for the area of R, giving your answer to 4 decimal places.

(3)

(*c*) By reference to the curve in Figure 1, state, giving a reason, whether your estimate in part (*b*) is an overestimate or an underestimate for the area of R.

(1)

(*d*) Use the substitution $u = \sqrt{x}$, or otherwise, to find the exact value of

$$\int_1^4 \frac{10}{2x + 5\sqrt{x}}\,dx$$

(6)