# GCE A level Mathematics (9MA0)
# Pure Mathematics 14a mark scheme

| **Guidance on the use of codes within this document** |
|---|
| M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.<br><br>A1 – accuracy mark. This mark is generally given for a correct answer following correct working.<br><br>B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.<br><br>Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer). |

| | | |
|---|---|---|
| **1.** | (a) $3x-7>3-x$ | |
| | $4x>10$ | M1 |
| | $x>2.5,\quad x>\frac{5}{2},\quad \frac{5}{2}<x$ o.e. | A1 |
| | | (2) |
| | (b) Obtain $x^2-9x-36$ and attempt to solve $x^2-9x-36=0$ | |
| | e.g. $(x-12)(x+3)$ = 0 so $x$ = , or $x=\frac{9\pm\sqrt{81+144}}{2}$ | M1 |
| | 12, –3 | A1 |
| | $-3\leq x\leq 12$ | M1A1 |
| | | (4) |
| | (c ) $2.5<x\leq 12$ | A1cso |
| | | (1) |
| | | **(7 marks)** |

| | | |
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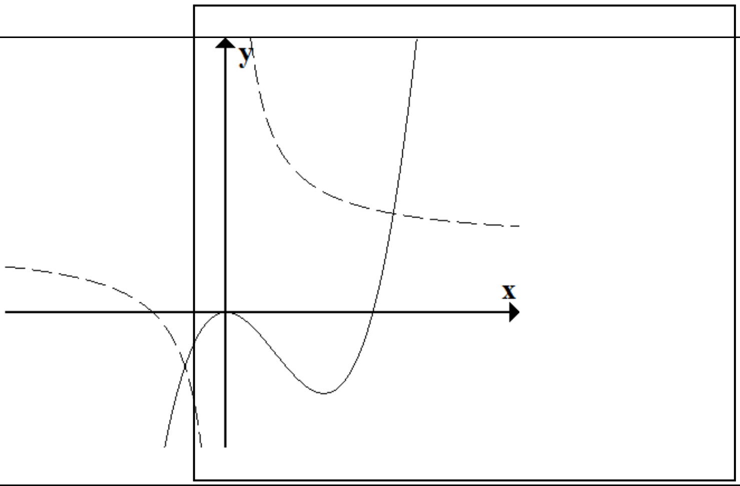
| **2.** | (a) - 1 accept (–1, 0) | **B1** |
| | | **(1)** |
| | (b) | |
| | | |
| | <br>Shape<br>Touches at (0,0)<br>Crosses at (2,0) **only** | **B1**<br>**B1**<br>**B1** |
| | | **(3)** |
| | (c ) 2 solutions as curves cross twice | **B1 ft** |
| | | **(1)** |
| | | **(5 marks)** |
| | | |

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| **3.** | (a) $7 = 5a_1 - 3 \quad \Rightarrow a_1 = ..$ | **M1** |
| | $a_1 = 2$ | **A1** |
| | | **(2)** |
| | (b) $a_3 = "32" \quad \text{and} \quad a_4 = "157"$ | **M1** |
| | | |
| | $\sum_{r=1}^{r=4} a_r = a_1 + a_2 + a_3 + a_4$ | |
| | | |
| | = "2"+ "7"+ "32"+ "157" | **dM1** |
| | | |
| | = 198 | **A1** |
| | | |
| | | **(3)** |
| | | |
| | | **(5 marks)** |
| | | |
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| **4.** | (a) $80 = 5\times16$ | | |
| | $\sqrt{80} = 4\sqrt{5}$ | | **B1** |
| | | | **(1)** |
| | Method 1 | Method 2 | |
| | (b) $\dfrac{\sqrt{80}}{\sqrt{5}+1}$ or $\dfrac{c\sqrt{5}}{\sqrt{5}+1}$ | $(p+q\sqrt{5})(\sqrt{5}+1)=\sqrt{80}$ | B1ft |
| | $=\dfrac{\sqrt{80}}{\sqrt{5}+1}\times\dfrac{\sqrt{5}-1}{\sqrt{5}-1}$ or $\dfrac{\sqrt{80}}{1+\sqrt{5}}\times\dfrac{1-\sqrt{5}}{1-\sqrt{5}}$ | $p\sqrt{5}+q\sqrt{5}+p+5q = 4\sqrt{5}$ | M1 |
| | $=\dfrac{20-4\sqrt{5}}{4}$ or $\dfrac{4\sqrt{5}-20}{-4}$ | $p + 5q = 0$<br>$p + q = 4$ | A1 |
| | $=5-\sqrt{5}$ | $p = 5,\ q = -1$ | A1cao |
| | | | (4)<br>**(5 marks)** |

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|---|---|---|
| **5.** | (a) Use $n^{th}\text{term} = a + (n-1)d$ with d = 10; a = 150 and n = 8, or a = 160 and n = 7, or a = 170 and n = 6 : = 150+7×10 or 160 +6 ×10 or 170 + 5 ×10 | **M1** |
| | = 220* (Or gives clear list – see note) | **A1*** |
| | | **(2)** |
| **Or** | If answer 220 is assumed and 150 + (n – 1) 10 =220 or variation is solved for n= | **M1** |
| | Then n = 8, so 2007 is the year (must conclude the year) | **A1*** **(2)** |
| | (b) Use $S_n = \frac{n}{2}\{2a + (n-1)10\}$ Or $S_n = \frac{n}{2}\{a + l\}$ **and** $l = a + (n-1)10$<br>= 7(300+13×10) or 7(150 + 280)<br>= 7×430<br>= 3010<br><br>(c) Cost in year $n$ = 900+($n$–1)×-20<br>Sales in year $n$ = 150+($n$–1)×10<br><br>Cost =3×Sales ⇒ 900+($n$–1)×-20 = 3×(150+($n$–1)×10)<br>900–20$n$+20 = 450+30$n$–30<br>500 = 50$n$<br>$n$ =10<br>Year is 2009<br>As $n$ is not defined they may work correctly from another base year to get the answer 2009 and their $n$ may not equal 10. If doubtful – send to review. | M1<br>A1<br><br>A1<br>(3)<br>M1<br><br><br>M1<br><br><br>M1<br>A1<br><br>(4)<br>(9 marks) |

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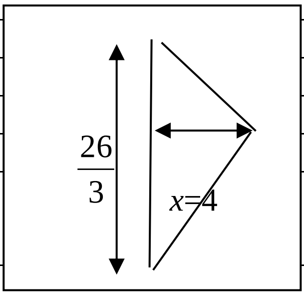
| **6.** | (a) $2x+3y=26 \Rightarrow 3y=26\pm 2x$ and attempt to find m from y = mx + c | M1 |
| | | |
| | ( $\Rightarrow y=\frac{26}{3}-\frac{2}{3}x$ ) so gradient = $-\frac{2}{3}$ | A1 |
| | Gradient of perpendicular = $\frac{-1}{\text{their gradient}}$ (= $\frac{3}{2}$ ) | M1 |
| | Line goes through (0,0) so $y=\frac{3}{2}x$ | A1 |
| | | (4 ) |
| | (b) Solves their $y=\frac{3}{2}x$ with their $2x+3y=26$ to form equation in x or in y | M1 |
| | Solves their equation in x or in y to obtain x = or y = | dM1 |
| | | |
| | x=4 or any equivalent e.g. 156/39 or y = 6 o.a.e | A1 |
| | B= (0, $\frac{26}{3}$ ) used or stated in (b) | B1 |
| | | |
| |  Method 1 ( see other methods in notes below) | |
| | | |
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| | | |
| | Area = $\frac{1}{2}\times"4"\times\frac{"26"}{3}$ | dM1 |
| | | |
| | $=\frac{52}{3}$ (oe with integer numerator and denominator) | A1 |
| | | (6) |
| | | (10 marks) |

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| **7.(a)** | $f(x) = 2x^3 - 7x^2 + 4x + 4$ | |
| | $f(2) = 2(2)^3 - 7(2)^2 + 4(2) + 4$ | M1 |
| | $= 0$, and so $(x - 2)$ is a factor. | A1 |
| | | [2] |
| **(b)** | $f(x) = \{(x-2)\}(2x^2 - 3x - 2)$ | M1 A1 |
| | $= (x-2)(x-2)(2x+1)$ or $(x-2)^2(2x+1)$<br>or equivalent e.g.<br>$= 2(x-2)(x-2)(x+\frac{1}{2})$ or $2(x-2)^2(x+\frac{1}{2})$ | dM1 A1 |
| | | [4] |
| | | Total 6 |

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| **8.** | $\left\{\int\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} \quad = \frac{x^4}{6(4)}+\frac{x^{-1}}{(3)(-1)}$ | M1A1A1 |
| | $\left\{\int_1^{\sqrt{3}}\left(\frac{x^3}{6}+\frac{1}{3x^2}\right)\mathrm{d}x\right\} = \left(\frac{\left(\sqrt{3}\right)^4}{24}+\frac{\left(\sqrt{3}\right)^{-1}}{-1(3)}\right)-\left(\frac{(1)^4}{24}+\frac{(1)^{-1}}{-1(3)}\right)$ | dM1 |
| | $=\left(\frac{9}{24}-\frac{1}{3\sqrt{3}}\right)-\left(\frac{1}{24}-\frac{1}{3}\right)=\frac{2}{3}-\frac{1}{9}\sqrt{3}$ | A1cso |
| | | Total 5 |

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| **9.(a)** | Area BDE $=\frac{1}{2}(5)^2(1.4)$ | | M1A1 |
| | $=17.5\ (\text{cm}^2)$ | | |
| | | | [2] |
| **(b)** | Parts (b) and (c) can be marked together | | |
| | $6.1^2=5^2+7.5^2-\left(2\times5\times7.5\cos DBC\right)$ or $\cos DBC=\frac{5^2+7.5^2-6.1^2}{2\times5\times7.5}$ (or equivalent) | | M1 |
| | | | |
| | Angle $DBC=0.943201...$ | awrt $0.943$ | A1 |
| | | | **[2]** |
| **(c)** | **Note that candidates may work in degrees in (c) (**Angle $DBC=54.04....\text{degrees}$) | | |
| | Area $CBD=\frac{1}{2}5(7.5)\sin(0.943)$ | | |
| | Angle $EBA=\pi-1.4-"0.943"$ **(Maybe seen on the diagram)** | Area $CBD=\frac{1}{2}5(7.5)\sin(\text{their }0.943)$ or awrt 15.2. (Note area of $CBD$ $=15.177...$) A correct method for the area of triangle $CBD$ which can be implied by awrt 15.2 | M1 |
| | $\pi-1.4-"\text{their }0.943"$ | | M1 |
| | $AB=5\cos(\pi-1.4-"0.943")$ or $AE=5\sin(\pi-1.4-"0.943")$ | | |
| | $AB=5\cos(\pi-1.4-\text{their }0.943)$ <br> $AB=5\cos(0.79859...)=3.488577938...$ <br> Or <br> $AE=5\sin(\pi-1.4-\text{their }0.943)$ <br> $AE=5\sin(0.79859...)=3.581874365688...$ | | M1 |
| | Area $EAB=\frac{1}{2}5\cos(\pi-1.4-"0.943")\times5\sin(\pi-1.4-"0.943")$ | | **d**M1 |
| | Area $ABCDE=15.17...+17.5+6.24...=38.92...$ | | |
| | | awrt 38.9 | A1**cso** |
| | | | **[5]** |
| | | | **Total 9** |

| | | |
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| **10**(a) | $S_\infty = \dfrac{20}{1-\frac{7}{8}}$ ; $= 160$ | **M1A1** |
| | | **[2]** |
| (b) | $S_{12} = \dfrac{20\left(1-(\frac{7}{8})^{12}\right)}{1-\frac{7}{8}}$ ; $= 127.77324...$ | **M1A1** |
| | | **[2]** |
| (c) | $160 - \dfrac{20(1-(\frac{7}{8})^N)}{1-\frac{7}{8}} < 0.5$ | **M1** |
| | $160\left(\dfrac{7}{8}\right)^N < (0.5)$ or $\left(\dfrac{7}{8}\right)^N < \left(\dfrac{0.5}{160}\right)$ | **dM1** |
| | $N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{160}\right)$ | **M1** |
| | $N > \dfrac{\log\left(\frac{0.5}{160}\right)}{\log\left(\frac{7}{8}\right)} = 43.19823... \Rightarrow N = 44$ | **A1 cso** |
| | | **[4]** |
| | | **Total 8** |

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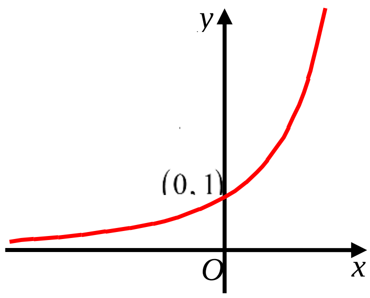
| **11.** | (i) $9\sin(\theta+60^\circ)=4$; $0 \le \theta < 360^\circ$<br>(ii) $2\tan x - 3\sin x = 0$; $-\pi \le x < \pi$ | |
| **(i)** | $\sin(\theta+60^\circ)=\frac{4}{9}$, so $(\theta+60^\circ)=26.3877...$<br>$(\alpha = 26.3877...)$ | M1 |
| | So, $\theta + 60^\circ = \{153.6122...,\ 386.3877...\}$ | M1 |
| | and $\theta = \{93.6122...,\ 326.3877...\}$ | A1 A1 |
| | **Both answers are cso and must come from correct work** | |
| | | **[4]** |
| **(ii)** | $2\left(\frac{\sin x}{\cos x}\right) - 3\sin x = 0$ | M1 |
| | $2\sin x - 3\sin x\cos x = 0$<br>$\sin x(2-3\cos x)=0$ | |
| | $\cos x = \frac{2}{3}$ | A1 |
| | $x = \text{awrt}\{0.84, -0.84\}$ | A1A1ft |
| | $\{\sin x = 0 \Rightarrow\}$ $x=0$ **and** $-\pi$ | B1 |
| | | **[5]** |
| | | **Total 9** |

| | | |
|---|---|---|
| **12.** | Graph of $y = 3^x$ and solving $3^{2x} - 9(3^x) + 18 = 0$ | |
| (a) |  | **B1**<br>**B1** |
| | | **[2]** |
| (b) | $\left(3^x\right)^2 - 9\left(3^x\right) + 18 = 0$<br>or<br>$y = 3^x \Rightarrow y^2 - 9y + 18 = 0$ | **M1** |
| | $\{(y-6)(y-3) = 0$ or $\left(3^x - 6\right)\left(3^x - 3\right) = 0\}$ | |
| | $y = 6,\ \ y = 3$ or $3^x = 6,\ \ 3^x = 3$ | **A1** |
| | $\left\{3^x = 6 \Rightarrow\right\}\ x\log 3 = \log 6$<br>or $x = \dfrac{\log 6}{\log 3}$ or $x = \log_3 6$ | **dM1** |
| | $x = 1.63092...$ | **A1cso** |
| | | |
| | $x = 1$ | **B1** |
| | | **[5]** |
| | | **Total 7** |
| | Mark (a) and (b) together | |

| | | |
|---|---|---|
| **13**. (a) | $OQ^2 = \left(6\sqrt{5}\right)^2 + 4^2$ or $OQ = \sqrt{\left(6\sqrt{5}\right)^2 + 4^2}$ $\{= 14\}$ | **M1** |
| | $y_Q = \sqrt{14^2 - 11^2}$ | **dM1** |
| | $= \sqrt{75}$ or $5\sqrt{3}$ | **A1cso** |
| | | **[3]** |
| (b) | $\left(x - 11\right)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$ | **M1A1** |
| | | **[2]** |
| | | **Total 5** |

| | | |
|---|---|---|
| **14.**(a) | $x^2+x-6=(x+3)(x-2)$ | **B1** |
| | $\dfrac{x}{x+3}+\dfrac{3(2x+1)}{(x+3)(x-2)}=\dfrac{x(x-2)+3(2x+1)}{(x+3)(x-2)}$ | **M1** |
| | $=\dfrac{x^2+4x+3}{(x+3)(x-2)}$ | **A1** |
| | $=\dfrac{\cancel{(x+3)}(x+1)}{\cancel{(x+3)}(x-2)}$ | |
| | $=\dfrac{(x+1)}{(x-2)}$ cso | **A1*** |
| | | **(4)** |
| (b) | One end either $(y)>1, (y)\geqslant 1$ or $(y)<4, (y)\leqslant 4$ | **B1** |
| | $1<y<4$ | **B1** |
| | | **(2)** |
| (c ) | Attempt to set | |
| | Either $g(x)=x$ or $g(x)=g^{-1}(x)$ or $g^{-1}(x)=x$ or $g^2(x)=x$ | |
| | $\dfrac{(x+1)}{(x-2)}=x$ $\quad \dfrac{x+1}{x-2}=\dfrac{2x+1}{x-1}$ $\quad \dfrac{2x+1}{x-1}=x$ $\quad \dfrac{\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-2}=x$ | **M1** |
| | | |
| | $x^2-3x-1=0 \Rightarrow x=\ldots$ | **A1, dM1** |
| | $a=\dfrac{3+\sqrt{13}}{2}$ oe $\left(1.5+\sqrt{3.25}\right)$ cso | **A1** |
| | | **(4)** |
| | | **(10 marks)** |
| | | |