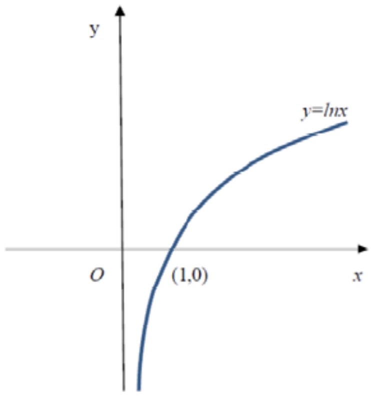
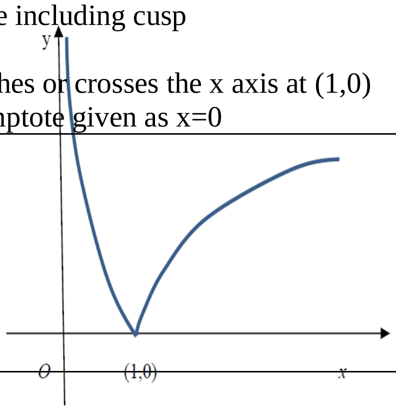
GCE A level Mathematics (9MA0)
Pure Mathematics 13b mark scheme

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

<table>
<tr><td>1(i)</td><td>
</td><td>B1</td></tr>
<tr><td>2(ii)</td><td>Shape including cusp

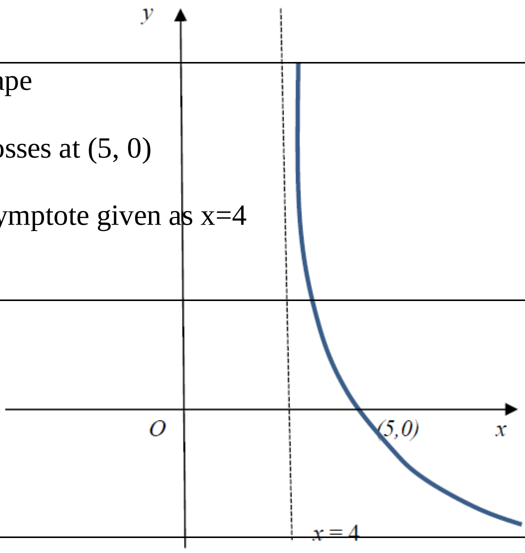
Touches or crosses the x axis at (1,0)
Asymptote given as x=0</td><td>B1ft

B1ft
B1</td></tr>
<tr><td></td><td>
</td><td></td></tr>
<tr><td>2(iii)</td><td>Shape

Crosses at (5, 0)

Asymptote given as x=4</td><td>B1

B1ft

B1</td></tr>
<tr><td></td><td>
</td><td>[7]</td></tr>
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2(a)	$f'(x) = 50x^2e^{2x} + 50xe^{2x}$ oe.	**M1A1**
	Puts $f'(x) = 0$ to give x = -1 and x = 0 or one coordinate	**dM1A1**
	Obtains $(0,-16)$ and $(-1, 25e^{-2}-16)$ CSO	**A1**
		(5)
(b)	Puts $25x^2e^{2x} - 16 = 0 \Rightarrow x^2 = \frac{16}{25}e^{-2x} \Rightarrow x = \pm\frac{4}{5}e^{-x}$	**B1***
		(1)
(c)	Subs $x_0 = 0.5$ into $x = \frac{4}{5}e^{-x} \Rightarrow x_1 = \text{awrt } 0.485$	**M1A1**
	$\Rightarrow x_2 = \text{awrt } 0.492,\ x_3 = \text{awrt } 0.489$	**A1** (3)
(d)	$\alpha = 0.49$ $f(0.485) = -\ 0.487,\ f(0.495) = (+)0.485$, sign change and deduction	**B1** **B1**
		(2) **[11]**

3(a)	$\frac{dx}{dy} = 2\times 3\sec 3y \sec 3y \tan 3y = \left(6\sec^2 3y \tan 3y\right)$ $\left(\text{oe } \frac{6\sin 3y}{\cos^3 3y}\right)$	**M1A1** **(2)**
(b)	Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to obtain $\frac{dy}{dx} = \frac{1}{6\sec^2 3y \tan 3y}$	**M1**
	$\tan^2 3y = \sec^2 3y - 1 = x - 1$	**B1**
	Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in just x.	**M1**
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$ CSO	**A1*** **(4)**
(c)	$\frac{d^2y}{dx^2} = \frac{0 - [6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$	**M1A1**
	$\frac{d^2y}{dx^2} = \frac{6-9x}{36x^2(x-1)^{\frac{3}{2}}} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$	**dM1A1**
		(4)
		[10]

4(a)		
	$\ln(4-2x)(9-3x), = \ln(x+1)^2$	**M1, M1**
	So $36-30x+6x^2 = x^2+2x+1$ and $5x^2-32x+35=0$	**A1**
	Solve $5x^2-32x+35=0$ to give $x=\frac{7}{5}$ oe (Ignore the solution $x=5$)	**M1A1**
		(5)
(b)	Take loge's to give $\ln 2^x + \ln e^{3x+1} = \ln 10$	**M1**
	$x\ln 2 + (3x+1)\ln e = \ln 10$	**M1**
	$x(\ln 2 + 3\ln e) = \ln 10 - \ln e \Rightarrow x = ..$	**dM1**
	and uses lne = 1	**M1**
	$x = \frac{-1+\ln 10}{3+\ln 2}$	**A1**
		(5)
		[10]

5(a)	$0 \le f(x) \le 10$	**B1**
		(1)
(b)	ff(0) = f(5), = 3	**B1,B1**
		(2)
(c)	$y = \dfrac{4+3x}{5-x} \Rightarrow y(5-x) = 4+3x$	
	$\Rightarrow 5y-4 = xy+3x$	**M1**
	$\Rightarrow 5y-4 = x(y+3) \Rightarrow x = \dfrac{5y-4}{y+3}$	**dM1**
	$g^{-1}(x) = \dfrac{5x-4}{3+x}$	**A1**
		(3)
(d)	$gf(x) = 16 \Rightarrow f(x) = g^{-1}(16) = 4$ oe	**M1A1**
	$f(x) = 4 \Rightarrow x = 6$	**B1**
	$f(x) = 4 \Rightarrow 5-2.5x = 4 \Rightarrow x = 0.4$ oe	**M1A1**
		(5)
		[11]

6(a)	$R = \sqrt{(7^2 + 24^2)} = 25$	**B1**
	$\tan\alpha = \frac{24}{7}, \quad \Rightarrow \quad \alpha = \text{awrt } 73.74°$	**M1A1**
		(3)
(b)	maximum value of 24sinx + 7cosx = 25 so $V_{\min} = \frac{21}{25} = (0.84)$	**M1A1**
		(2)
(c)	Distance AB = $\frac{7}{\sin\theta}$, with $\theta = \alpha$	**M1, B1**
	So distance = 7.29m $= \frac{175}{24}$ m	**A1**
		(3)
(d)	$R\cos(\theta - \alpha) = \frac{21}{1.68} \Rightarrow \cos(\theta - \alpha) = 0.5$	**M1, A1**
	$\theta - \alpha = 60 \Rightarrow \theta = ..,\theta - \alpha = -60 \Rightarrow \theta = ..$	**dM1, dM1**
	$\theta = \text{awrt } 133.7, 13.7$	**A1, A1**
		(6)
		[14]

7. (a)	$\int x^2 e^x \,dx$, 1st Application: $\left\{ \begin{array}{l} u = x^2 \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array} \right\}$, 2nd Application: $\left\{ \begin{array}{l} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array} \right\}$		
	$= x^2 e^x - \int 2xe^x dx$	$x^2 e^x - \int \lambda x e^x \{dx\}$, $\lambda > 0$	M1
		$x^2 e^x - \int 2xe^x \{dx\}$	A1 oe
	$= x^2 e^x - 2\left(xe^x - \int e^x dx\right)$	**Either** $\pm Ax^2 e^x \pm Bxe^x \pm C\int e^x \{dx\}$ **or** for $\pm K\int xe^x \{dx\} \to \pm K\left(xe^x - \int e^x \{dx\}\right)$	M1
	$= x^2 e^x - 2(xe^x - e^x)\ \{+c\}$	$\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$	M1
		Correct answer, with/without $+c$	A1 **(5)**
(b)	$\left\{\left[x^2 e^x - 2(xe^x - e^x)\right]_0^1\right\}$ $= \left(1^2 e^1 - 2(1e^1 - e^1)\right) - \left(0^2 e^0 - 2(0e^0 - e^0)\right)$	Applies limits of 1 and 0 to an expression of the form $\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$, $A \neq 0$, $B \neq 0$ and $C \neq 0$ and subtracts the correct way round.	M1
	$= e - 2$	$e - 2$ **cso**	A1 oe **(2)** **[7]**

8. (a)	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)}\right\} = (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$ $\qquad (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	**B1**
	$= \left(1+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2+\ldots\right)\times\left(1+\left(-\frac{1}{2}\right)(-x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2+\ldots\right)$	**M1** **A1** **A1**
	$= \left(1+\frac{1}{2}x-\frac{1}{8}x^2+\ldots\right)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2+\ldots\right)$	
	$= 1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{1}{2}x+\frac{1}{4}x^2-\frac{1}{8}x^2+\ldots$	**M1**
	$= 1+x+\frac{1}{2}x^2$ Answer is given in the question.	**A1** *
		(6)
(b)	$\sqrt{\left(\frac{1+\left(\frac{1}{26}\right)}{1-\left(\frac{1}{26}\right)}\right)} = 1+\left(\frac{1}{26}\right)+\frac{1}{2}\left(\frac{1}{26}\right)^2$	**M1**
	ie: $\frac{3\sqrt{3}}{5} = \frac{1405}{1352}$	**B1**
	so, $\sqrt{3} = \frac{7025}{4056}$ $\qquad \frac{7025}{4056}$	**A1** **cao**
		(3)
		[9]

Question Number	Scheme	Notes	Marks
9.	$x = 2\sin t, \quad y = 1 - \cos 2t \quad \left\{= 2\sin^2 t\right\}, \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$		
(a)	$\frac{dx}{dt} = 2\cos t, \quad \frac{dy}{dt} = 2\sin 2t \quad \text{or} \quad \frac{dy}{dt} = 4\sin t\cos t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	So, $\frac{dy}{dx} = \frac{2\sin 2t}{2\cos t} \left\{= \frac{4\cos t\sin t}{2\cos t} = 2\sin t\right\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes $t = \frac{\pi}{6}$ into their $\frac{dy}{dx}$.	M1;
	At $t = \frac{\pi}{6}, \quad \frac{dy}{dx} = \frac{2\sin\left(\frac{2\pi}{6}\right)}{2\cos\left(\frac{\pi}{6}\right)}; \; = 1$	Correct value for $\frac{dy}{dx}$ of 1	A1 **cao cso**
			(4)
(b)	$y = 1 - \cos 2t = 1 - (1 - 2\sin^2 t)$ $= 2\sin^2 t$		M1
	So, $y = 2\left(\frac{x}{2}\right)^2 \quad \text{or} \quad y = \frac{x^2}{2} \quad \text{or} \quad y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$	$y = \frac{x^2}{2}$ or equivalent.	A1 **cso isw**
	Either $k = 2$ or $-2 \le x \le 2$		B1
			(3)
(c)	Range: $0 \le f(x) \le 2$ or $0 \le y \le 2$ or $0 \le f \le 2$		B1 B1
			(2) **[9]**

10.	$x^2 + 4xy + y^2 + 27 = 0$	
(a)	$\left\{\frac{dy}{dx} \times\right\}$ $\quad 2x + \left(4y + 4x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$	M1 A1 B1
	$2x + 4y + (4x + 2y)\frac{dy}{dx} = 0$	**dM1**
	$\frac{dy}{dx} = \frac{-2x - 4y}{4x + 2y}$ $\left\{= \frac{-x - 2y}{2x + y}\right\}$	**A1 cso oe**
		(5)
(b)	$4x + 2y = 0$	**M1**
	$y = -2x$	$x = -\frac{1}{2}y$	A1
	$x^2 + 4x(-2x) + (-2x)^2 + 27 = 0$	$\left(-\frac{1}{2}y\right)^2 + 4\left(-\frac{1}{2}y\right)y + y^2 + 27 = 0$	M1*
	$-3x^2 + 27 = 0$	$-\frac{3}{4}y^2 + 27 = 0$	
	$x^2 = 9$	$y^2 = 36$	dM1*
	$x = -3$	$y = 6$	A1
	When $x = -3$, $y = -2(-3)$	When $y = 6$, $x = -\frac{1}{2}(6)$	ddM1*
	$y = 6$	$x = -3$	A1 **cso**
		(7) **[12]**