Pearson Edexcel Level 3

GCE Mathematics

Advanced Level
Paper 2: Pure Mathematics

Practice Paper 13b **Time: 2 hours**	**Paper Reference(s)** **9MA0/02**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Given that

$$f(x) = \ln x, \qquad x > 0$$

sketch on separate axes the graphs of

(i) $y = f(x)$,

(ii) $y = |f(x)|$,

(iii) $y = -f(x - 4)$.

Show, on each diagram, the point where the graph meets or crosses the x-axis.
In each case, state the equation of the asymptote.

(7)

---

2. $f(x) = 25x^2e^{2x} - 16, x \in \mathbb{R}$.

(*a*) Using calculus, find the exact coordinates of the turning points on the curve with equation $y = f(x)$.

(5)

(*b*) Show that the equation $f(x) = 0$ can be written as $x = \pm\frac{4}{5}e^{-x}$.

(1)

The equation $f(x) = 0$ has a root α, where $\alpha = 0.5$ to 1 decimal place.

(*c*) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{4}{5}e^{-x_n}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(*d*) Give an accurate estimate for α to 2 decimal places, and justify your answer.

(2)

3. Given that

$$x = \sec^2 3y, \qquad 0 < y < \frac{\pi}{6}$$

(*a*) find $\frac{dx}{dy}$ in terms of *y*.

(2)

(*b*) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

(4)

(*c*) Find an expression for $\frac{d^2y}{dx^2}$ in terms of *x*. Give your answer in its simplest form.

(4)

---

4. Find algebraically the exact solutions to the equations

(*a*) $\ln(4 - 2x) + \ln(9 - 3x) = 2\ln(x + 1), \qquad -1 < x < 2,$

(5)

(*b*) $2^x e^{3x+1} = 10.$

Give your answer to (*b*) in the form $\frac{a + \ln b}{c + \ln d}$ where *a*, *b*, *c* and *d* are integers.

(5)

---

5. The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = \mathrm{f}(x)$ is shown in Figure 1.

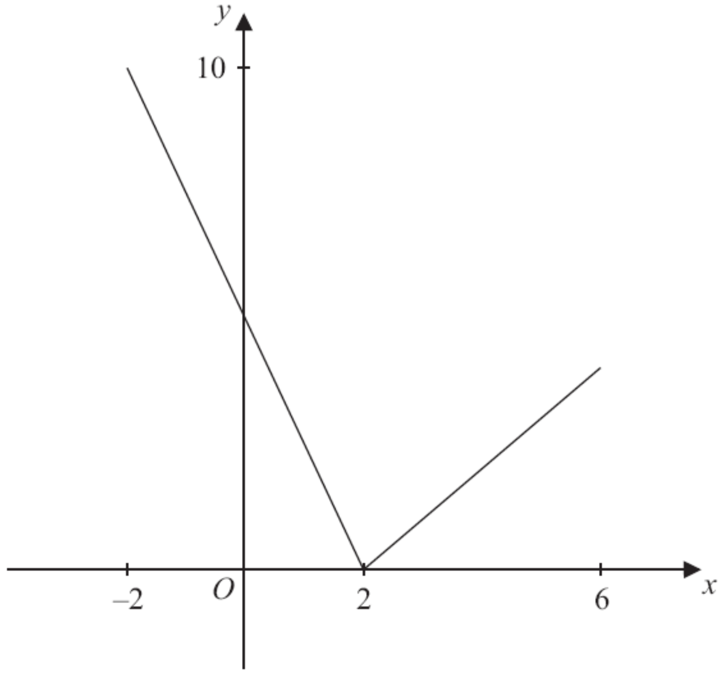

Figure 1

(*a*) Write down the range of f.

(1)

(*b*) Find ff(0).

(2)

The function g is defined by

$$\mathrm{g}: x \rightarrow \frac{4+3x}{5-x}, \quad x \in \mathbb{R}, \qquad x \neq 5.$$

(*c*) Find $\mathrm{g}^{-1}(x)$.

(3)

(*d*) Solve the equation $\mathrm{gf}(x) = 16$.

(5)

6.

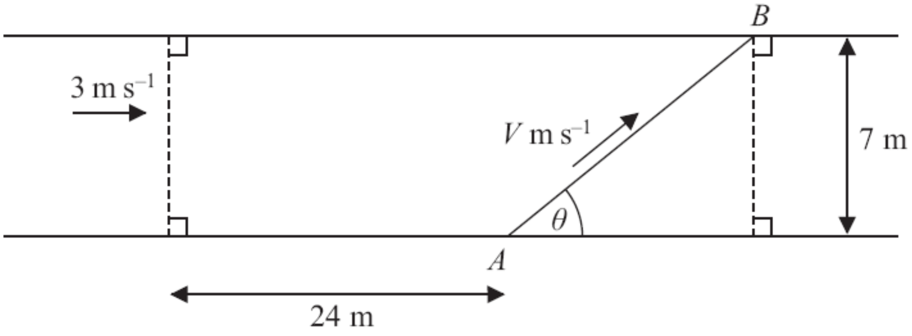

Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1}.

Kate is 24 m ahead of John when she starts to cross the road from the fixed point A.
John passes her as she reaches the other side of the road at a variable point B, as shown in Figure 2.

Kate's speed is V m s^{-1} and she moves in a straight line, which makes an angle θ, $0 < \theta < 150°$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24\sin\theta + 7\cos\theta}, \qquad 0 < \theta < 150°$$

(a) Express $24\sin\theta + 7\cos\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

Given that θ varies,

(b) find the minimum value of V.

(2)

Given that Kate's speed has the value found in part (b),

(c) find the distance AB.

(3)

Given instead that Kate's speed is 1.68 m s^{-1},

(d) find the two possible values of the angle θ, given that $0 < \theta < 150°$.

(6)

---

7. (a) Find $\int x^2 e^x \mathrm{d}x$.

(5)

(b) Hence find the exact value of $\int_0^1 x^2 e^x \mathrm{d}x$.

(2)

8. (*a*) Use the binomial expansion to show that

$$\sqrt{\left(\frac{1+x}{1-x}\right)} \approx 1 + x + \frac{1}{2}x^2, \qquad |x| < 1$$

(6)

(*b*) Substitute $x = \frac{1}{26}$ into

$$\sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{1}{2}x^2$$

to obtain an approximation to $\sqrt{3}$.

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

(3)

---

9. A curve C has parametric equations

$$x = 2\sin t, \qquad y = 1 - \cos 2t, \qquad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

(*a*) Find $\frac{dy}{dx}$ at the point where $t = \frac{\pi}{6}$.

(4)

(*b*) Find a cartesian equation for C in the form

$$y = f(x), \qquad -k \leq x \leq k,$$

stating the value of the constant k.

(3)

(*c*) Write down the range of $f(x)$.

(2)

---

10. A curve is described by the equation

$$x^2 + 4xy + y^2 + 27 = 0$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(5)

A point Q lies on the curve.

The tangent to the curve at Q is parallel to the y-axis.

Given that the x-coordinate of Q is negative,

(*b*) use your answer to part (*a*) to find the coordinates of Q.

(7)