GCE A level Mathematics (9MA0)
Pure Mathematics 13a mark scheme

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

1(a)	$(a_2 =)\ k(4+2)\ \ (= 6k)$	B1
		(1)
(b)	$a_3 = k(\text{their } a_2 + 2)\ (= 6k^2 + 2k)$	M1
	$a_1 + a_2 + a_3 = 4 + (6k) + (6k^2 + 2k)$	M1
	$4 + (6k) + (6k^2 + 2k) = 2$	A1
	Solves $6k^2 + 8k + 2 = 0$ to obtain $k =$ $(6k^2 + 8k + 2 = 2(3k+1)(k+1))$	M1
	$k = -1/3$	A1
	$k = -1$	B1
		(6)
		[7]

2 (a)	$6x + x > 1 - 8$	M1
	$x > -1$	A1
		(2)
(b)	$(x + 3)(3x - 1)$ [= 0] $\Rightarrow x = -3$ and $\frac{1}{3}$	M1A1
	$-3 < x < \frac{1}{3}$	M1A1ft
		(4)
		[6]

3	(-1, 3) , (11, 12)		
(a)	$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{12-3}{11-(-1)}, = \dfrac{3}{4}$	M1, A1	
	$y - 3 = ¾ (x+1)$ or $y - 12 = ¾ (x - 11)$ or $y = ¾ x + c$ with attempt at substitution to find c	M1	
	$4y - 3x - 15 = 0$	A1	
			(4)
(b)	Solve equation from part (a) and L_2 simultaneously to eliminate one variable	M1	
	$x = 3$ or $y = 6$	A1	
	Both $x = 3$ **and** $y = 6$	A1	
			(3)
			[7]

4(a)	600 = 200 + (N – 1)20 $\Rightarrow N = \dots$	**M1**
	N = 21	**A1 cso**
		(2)
(b)	$S = \frac{21}{2}(2\times 200 + 20\times 20)$ or $\frac{21}{2}(200+600)$ or $S = \frac{20}{2}(2\times 200 + 19\times 20)$ or $\frac{20}{2}(200+580)$ (= 8400 or 7800)	**M1A1**
	$600\times(52 - "N")$ (= 18600)	**M1A1ft**
	So total is 27000	**A1 cao**
		(5)
		[7]

5 (a)	$\left(-\frac{3}{4}, 0\right)$. Accept $x = -\frac{3}{4}$	**B1**
		(1)
(b)	y = 4 x = 0 or 'y-axis'	**B1B1**
		(2)
(c)	$\frac{dy}{dx} = -3x^{-2}$	**M1**
	At $x = -3$, gradient of curve = $-\frac{1}{3}$	**A1**
	Gradient of normal = –1/m	**dM1**
	Normal at P is $(y-3) = 3(x+3)$	**dM1A1**
		(5)
(d)	(–4, 0) and (0, 12).	**B1**
	So AB has length $\sqrt{160}$ or AB2 has length 160	**M1 A1cso**
		(3)
		[11]

6. (a)	$\{r =\}\ \frac{2}{3}$	**B1**
		(1)
6b	8	**B1 cao**
6c	$\{S_{15} =\}\ \frac{18\left(1-\left(\frac{2}{3}\right)^{15}\right)}{1-\frac{2}{3}}$	**M1**
	$\{S_{15} = 53.87668...\} \Rightarrow S_{15} = \text{awrt } 53.877$	**A1**

7. (a)	**Usually answered in radians: Uses** either $\frac{1}{2}ab\sin(\text{angle})$ **or** $\frac{1}{2}(12)^2(\text{angle})$ or both	M1
	Area $= \frac{1}{2}(23)(12)\sin 0.64$ **or** $\frac{1}{2}(12)^2(\pi - 0.64)$ $\{= 82.41297091...$ **or** $180.1146711...\}$	A1
	Area $= \frac{1}{2}(23)(12)\sin 0.64 + \frac{1}{2}(12)^2(\pi - 0.64)$ $\{= 82.41297091... + 180.1146711...\}$	A1
	$\{\text{Area} = 262.527642...\}$ = awrt 262.5 (m^2) or 262.4(m^2) or 262.6 (m^2)	A1
		(4)
(b)	$CDE = 12\times(angle),\ = 12(\pi - 0.64)\{\Rightarrow CDE = 30.01911...\}$	M1, A1
	$AE^2 = 23^2 + 12^2 - 2(23)(12)\cos(0.64) \Rightarrow AE^2 =$ or $AE =$ $\{AE = 15.17376...\}$	M1
	Perimeter $= 23 + 12 + 15.17376... + 30.01911...$	M1
	$= 80.19287... =$ awrt 80.2 (m)	A1
		(5)
		[9]

8. (a)	Seeing -4 and 2.	**B1**
		(1)
(b)	$x(x+4)(x-2) = \underline{x^3 + 2x^2 - 8x}$ or $\underline{x^3 - 2x^2 + 4x^2 - 8x}$ (without simplifying)	**B1**
	$\int (x^3 + 2x^2 - 8x)\,dx = \frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2} \{+c\}$ or $\frac{x^4}{4} - \frac{2x^3}{3} + \frac{4x^3}{3} - \frac{8x^2}{2} \{+c\}$	**M1A1ft**
	$\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}\right]_{-4}^{0} = (0) - \left(64 - \frac{128}{3} - 64\right)$ or $\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}\right]_{0}^{2} = \left(4 + \frac{16}{3} - 16\right) - (0)$ One integral $= \pm 42\frac{2}{3}$ (42.6 or awrt 42.7) or other integral $= \pm 6\frac{2}{3}$ (6.6 or awrt 6.7)	**dM1**
		A1
	Hence Area $=$ "$their\,42\frac{2}{3}$" $+$ "$their\,6\frac{2}{3}$" or Area $=$ "$their\,42\frac{2}{3}$" $-$ "$-their\,6\frac{2}{3}$"	**dM1**
	$= 49\frac{1}{3}$ or 49.3 or $\frac{148}{3}$ (NOT $-\frac{148}{3}$)	**A1**
	(An answer of $= 49\frac{1}{3}$ may not get the final two marks – check solution carefully)	**(7)**
		[8]

9. (i)	$\log_2\left(\frac{2x}{5x+4}\right) = -3$ or $\log_2\left(\frac{5x+4}{2x}\right) = 3$, or $\log_2\left(\frac{5x+4}{x}\right) = 4$ (see special case 2)	**M1**
	$\left(\frac{2x}{5x+4}\right) = 2^{-3}$ or $\left(\frac{5x+4}{2x}\right) = 2^3$ or $\left(\frac{5x+4}{x}\right) = 2^4$ or $\left(\log_2\left(\frac{2x}{5x+4}\right)\right) = \log_2\left(\frac{1}{8}\right)$	**M1**
	$16x = 5x + 4 \Rightarrow x =$ (depends on previous Ms and must be this equation or equivalent)	**dM1**
	x = $\frac{4}{11}$ or exact recurring decimal $0.\dot{3}\dot{6}$ after correct work	**A1 cso (4)**
(ii)	$\log_a y + \log_a 2^3 = 5$	M1
	$\log_a 8y = 5$ — Applies product law of logarithms.	dM1
	$y = \frac{1}{8}a^5$ — $y = \frac{1}{8}a^5$	A1cao
		(3) **[7]**

10. (i)	$(\lvert\alpha\rvert = 56.3099...)$	
	$x = \{\alpha + 40 = 96.309993...\} =$ **awrt 96.3**	**B1**
	$x - 40^\circ = -180 + "56.3099"...$ or $x - 40^\circ = -\pi + "0.983"...$	**M1**
	$x = \{-180 + 56.3099... + 40 = -83.6901...\} =$ **awrt -83.7**	**A1**
		(3)
(ii)(a)	$\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right) = 3\cos\theta + 2$	**M1**
	$\left(\frac{1-\cos^2\theta}{\cos\theta}\right) = 3\cos\theta + 2$	**dM1**
	$1-\cos^2\theta = 3\cos^2\theta + 2\cos\theta \Rightarrow 0 = 4\cos^2\theta + 2\cos\theta - 1$ *	**A1 cso ***
		(3)
(b)	$\cos\theta = \frac{-2 \pm \sqrt{4 - 4(4)(-1)}}{8}$ or $4(\cos\theta \pm \frac{1}{4})^2 \pm q \pm 1 = 0$, or $(2\cos\theta \pm \frac{1}{2})^2 \pm q \pm 1 = 0$, $q \neq 0$ so $\cos\theta = ...$	**M1**
	One solution is 72° or 144°, Two solutions are 72° and 144°	**A1, A1**
	$\theta = \{72, 144, 216, 288\}$	**M1 A1**
		(5)
		[11]

11. (a)	$\left\{\frac{dy}{dx}=\right\}\ 2x-16x^{-\frac{1}{2}}$	**M1 A1**
	$2x-16x^{-\frac{1}{2}}=0 \Rightarrow\ x^{\frac{3}{2}}=,\ x^{-\frac{3}{2}}=$,or $2x-=16x^{-\frac{1}{2}}$ then squared then obtain $x^3=$ [or $2x-16x^{-\frac{1}{2}}=0 \Rightarrow x=4$(no wrong work seen)]	**M1**
	$(x^{\frac{3}{2}}=8 \Rightarrow)x=\ 4$	**A1**
	$x=4,\ y=\ 4^2-32\sqrt{4}+20=-28$ (ignore y = 100 as second answer)	**M1 A1**
		(6)
(b)	$\left\{\frac{d^2y}{dx^2}=\right\}\ 2+8x^{-\frac{3}{2}}$	**M1 A1**
	$(\frac{d^2y}{dx^2}>0 \Rightarrow)y$ is a minimum (there should be no wrong reasoning)	**A1**
		(3)
		[9]

12. (a)		
	Equation of form $(x \pm 5)^2 + (y \pm 9)^2 = k$, k >0	**M1**
	Equation of form $(x - a)^2 + (y - b)^2 = 5^2$, with values for a and b	**M1**
	$(x+5)^2 + (y-9)^2 = 25 = 5^2$	**A1**
		(3)
(b)	$P(8, -7)$. Let centre of circle $= X(-5, 9)$	
	$PX^2 = \left(8 - "-5"\right)^2 + \left(-7 - "9"\right)^2$ or $PX = \sqrt{(8 - -5)^2 + (-7-9)^2}$	M1
	($PX = \sqrt{425}$ or $5\sqrt{17}$) $PT^2 = \left(PX\right)^2 - 5^2$ with numerical PX	dM1
	$PT \left\{= \sqrt{400}\right\} = 20$ (allow 20.0)	A1 **cso**
		(3) **[6]**

13(a)	$2\cos x\cos 50 - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$	**M1**
	$\sin x(\cos 40 + 2\sin 50) = \cos x(2\cos 50 - \sin 40)$	
	$\div\cos x \Rightarrow \tan x(\cos 40 + 2\sin 50) = 2\cos 50 - \sin 40$	**M1**
	$\tan x = \dfrac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50}$, (or numerical answer awrt 0.28)	**A1**
	States or uses cos50 = sin40 and cos40 =sin50 and so $\tan x^{o} = \frac{1}{3}\tan 40^{o}$ * cao	**A1 *** **(4)**
(b)	Deduces $\tan 2\theta = \frac{1}{3}\tan 40$	**M1**
	$2\theta =$ 15.6 so $\theta =$ awrt 7.8(1) One answer	**A1**
	Also $2\theta =$ 195.6, 375.6 , 555.6 Þ $\theta =$ .. $\theta =$ awrt 7.8 , 97.8, 187.8, 277.8 All 4 answers	**M1** **A1**
		(4)
		[8]