[In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

A particle P moves in such a way that its velocity **v** m s^{-1} at time t seconds is given by **v** = $(3t^2 - 1)$**i** + $(4t - t^2)$**j**.

(a) Find the magnitude of the acceleration of P when t = 1. **(5)**

Given that, when t = 0, the position vector of P is **i** metres,

(b) find the position vector of P when t = 3. **(5)**

Two ships P and Q are travelling at night with constant velocities. At midnight, P is at the point with position vector (20**i** + 10**j**) km relative to a fixed origin O. At the same time, Q is at the point with position vector (14**i** – 6**j**) km. Three hours later, P is at the point with position vector (29**i** + 34**j**) km. The ship Q travels with velocity 12**j** km h^{-1}. At time t hours after midnight, the position vectors of P and Q are **p** km and **q** km respectively. Find

(*a*) the velocity of P, in terms of **i** and **j**,

(2)

(*b*) expressions for **p** and **q**, in terms of t, **i** and **j**.

(4)

At time t hours after midnight, the distance between P and Q is d km.

(*c*) By finding an expression for $\overrightarrow{PQ}$, show that

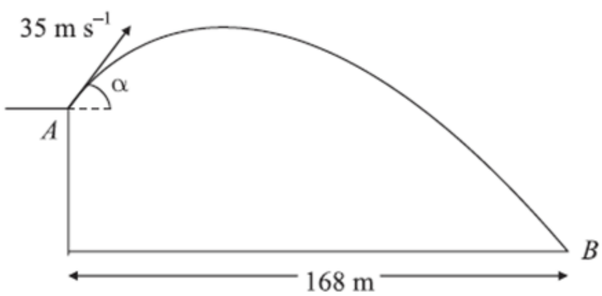
$$d^2 = 25t^2 - 92t + 292.$$

(5)

Weather conditions are such that an observer on P can only see the lights on Q when the distance between P and Q is 15 km or less. Given that when $t = 1$, the lights on Q move into sight of the observer,

(*d*) find the time, to the nearest minute, at which the lights on Q move out of sight of the observer.

Figure 4



A golf ball P is projected with speed 35 m s^{-1} from a point A on a cliff above horizontal ground. The angle of projection is α to the horizontal, where $\tan \alpha = \frac{4}{3}$. The ball moves freely under gravity and hits the ground at the point B, as shown in Figure 4.

(*a*) Find the greatest height of P above the level of A.

(3)

The horizontal distance from A to B is 168 m.

(*b*) Find the height of A above the ground.

(6)

By considering energy, or otherwise,

(*c*) find the speed of P as it hits the ground at B.

(3)

(a)	$\mathbf{v}_P = \{(29\mathbf{i} + 34\mathbf{j}) - (20\mathbf{i} + 10\mathbf{j})\}/3 = \underline{(3\mathbf{i} + 8\mathbf{j})\text{ km h}^{-1}}$	M1 A1 (2)
(b)	$\mathbf{p} = (20\mathbf{i} + 10\mathbf{j}) + (3\mathbf{i} + 8\mathbf{j})t$	M1 A1√
	$\mathbf{q} = (14\mathbf{i} - 6\mathbf{j}) + 12t\mathbf{j}$	M1 A1 (4)
(c)	$\mathbf{q} - \mathbf{p} = (-6 - 3t)\mathbf{i} + (-16 + 4t)\mathbf{j}$	M1 A1 ↓
	$d^2 = (-6 - 3t)^2 + (-16 + 4t)^2$	M1 ↓
	$= 36 + 36t + 9t^2 + 16t^2 - 128t + 256$	M1
	$= 25t^2 - 92t + 292$ (*)	A1 (cso) (5)
(d)	$25t^2 - 92t + 292 = 225$	M1
	$25t^2 - 92t + 67 = 0$	A1 ↓
	$(t - 1)(25t - 67) = 0$	M1
	$t = 67/25$ or 2.68	A1
	time ≈ 161 mins, or 2 hrs 41 mins, or 2.41 am, or 0241	A1 (5)

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		(5)
(a)	$0 = (35 \sin \alpha)^2 - 2gh$ $h = \underline{40\text{ m}}$	M1 A1 A1 (3)
(b)	$x = 168 \Rightarrow 168 = 35 \cos$ □ $. t$ $(\Rightarrow t = 8\text{s})$	M1 A1
	At $t = 8$, $y = 35\sin\alpha \times t - \frac{1}{2}gt^2$ $(= 28.8 - ½.g.8^2 = -89.6\text{ m})$	M1 A1
	Hence height of A = $\underline{89.6\text{ m}}$ or 90 m	DM1 A1 (6)
(c)	$½mv^2 = 1/2.m.35^2 + mg.89.6$	M1 A1
	$\Rightarrow v = \underline{54.6\text{ or }55\text{ m s}^{-1}}$	A1 (3)

Number			
1 **(a)**	$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = 6t\mathbf{i} + (4 - 2t)\mathbf{j}$	M1 A1	Differentiate **v** to obtain **a**. Accept column vector or **i** and **j** components dealt with separately.
	When $t = 1$, $\mathbf{a} = 6\mathbf{i} + 2\mathbf{j}$	DM1	Substitute $t = 1$ into their **a**. Dependent on 1st M1
	$\lvert\mathbf{a}\rvert = \sqrt{6^2 + 2^2} = \sqrt{40} = 6.32\ (\text{m s}^{-2})$	DM1 A1 (5)	Use of Pythagoras to find the magnitude of their **a**. Allow with their t. Dependent on 1st M1 Accept awrt 6.32, 6.3 or exact equivalents.
(b)	$\mathbf{r} = \int (3t^2 - 1)\mathbf{i} + (4t - t^2)\mathbf{j}\ \mathrm{d}t$	M1	Integrate **v** to obtain **r**
	$= (t^3 - t + C)\mathbf{i} + (2t^2 - \frac{1}{3}t^3 + D)\mathbf{j}$	A1	Condone C, D missing
	$t = 0, \mathbf{r} = \mathbf{i} \Rightarrow C = 1, D = 0$	DM1	Use $t = 0$, $\mathbf{r} = \mathbf{i}$ to find C & D
	When $t = 3, \mathbf{r} = 25\mathbf{i} + 9\mathbf{j}$ (m)	DM1 A1 (5) **10**	Substitute $t = 3$ with their C & D to find **r**. Dependent on both previous Ms. cao. Must be a vector.