Question 12 (***)

The binomial $(1+x)^{-\frac{1}{2}}$ is to be expanded as an infinite convergent series, in ascending powers of x.

a) Find the series expansion of $(1+x)^{-\frac{1}{2}}$ up and including the term in x^3.

b) Use the expansion of part (a) to find the expansion of $\frac{1}{\sqrt{1+2x}}$, up and including the term in x^3.

c) State the range of values of x for which the expansion of $\frac{1}{\sqrt{1+2x}}$ is valid.

d) Use the expansion of $\frac{1}{\sqrt{1+2x}}$ with $x=-0.1$ to show that $\sqrt{5}\approx 2.235$.

$\boxed{1-\frac{1}{2}x+\frac{3}{8}x^2-\frac{5}{16}x^3+O\left(x^4\right)}$, $\boxed{1-x+\frac{3}{2}x^2-\frac{5}{2}x^3+O\left(x^4\right)}$, $\boxed{-\frac{1}{2}<x<\frac{1}{2}}$

8. $\int \frac{1}{2}x^3\ln x\,dx=\frac{1}{8}x^4\ln x-\frac{1}{32}x^4+C$

9. $\int x\ln 3x\,dx=\frac{1}{2}x^2\ln 3x-\frac{1}{4}x^2+C$

10. $\int \frac{\ln x}{x^3}\,dx=-\frac{\ln x}{2x^2}-\frac{1}{4x^2}+C$

Question 13 (***)

A curve C is given parametrically by

$$x=3t-1,\quad y=\frac{1}{t},\quad t\in\mathbb{R},\quad t\neq 0.$$

Show that an equation of the normal to C at the point where C crosses the y axis is

$$y=\frac{1}{3}x+3.$$

Question 49 (****)

A curve C is given by the parametric equations

$$x = \sec\theta, \quad y = \ln(1+\cos 2\theta), \quad 0 \le \theta < \frac{\pi}{2}.$$

a) Show clearly that

$$\frac{dy}{dx} = -2\cos\theta.$$

The straight line L is a tangent to C at the point where $\theta = \frac{\pi}{3}$.

b) Find an equation for L, giving the answer in the form $y + x = k$, where k is an exact constant to be found.

c) Show that a Cartesian equation of C is

$$x^2 e^y = 2.$$

Question 14 (***)

A curve has equation

$$3x^2 - xy + y^2 + 2x - 4y = 1.$$

a) Show clearly that

$$\frac{dy}{dx} = \frac{2+6x-y}{4-2y+x}.$$

b) Hence show further that the value of x at the stationary points of the curve satisfies the equation

$$x^2 = \frac{5}{33}.$$

Question 24 (***+)

A curve C is defined by the parametric equations

$$x = \cos t, \quad y = \cos 2t \ , \ 0 \le t \le \pi .$$

a) Find $\frac{dy}{dx}$ in its simplest form.

b) Find a Cartesian equation for C.

c) Sketch the graph of C.

The sketch must include

- the coordinates of the endpoints of the graph.
- the coordinates of any points where the graph meets the coordinates axes.

$\boxed{\frac{dy}{dx} = 4\cos t}$, $\boxed{y = 2x^2 - 1}$, $\boxed{(-1,1)(1,1),(0,-1),\left(-\frac{\sqrt{2}}{2},0\right)\left(\frac{\sqrt{2}}{2},0\right)}$

Question 10

$$f(x) \equiv \sin x - \sqrt{3}\cos x, \ x \in \mathbb{R}.$$

a) Express $f(x)$ in the form $R\sin(x-\alpha)$, $R > 0$, $0 < \alpha < \frac{\pi}{2}$.

b) Write down the maximum value of $f(x)$.

c) Find the smallest positive value of x for which this maximum value occurs.

$\boxed{f(x) \equiv \sin x - \sqrt{3}\cos x \equiv 2\sin\left(x - \frac{\pi}{3}\right)}$, $\boxed{f(x)_{\text{max}} = 2}$, $\boxed{x = \frac{5\pi}{6}}$

Question 12 (***+)

Two variables x and y are related by

$$y = \tfrac{1}{4}\pi x^2 (4 - x).$$

The variable y is changing with time t, at the constant rate of 0.2, in suitable units.

Find the rate at which x is changing with respect to t, when $x = 2$.

$\boxed{\frac{1}{5\pi} \approx 0.0637}$

Question 15

$$f(x) \equiv 3\sin x + \cos x, \ x \in \mathbb{R}$$

a) Express $f(x)$ in the form $R\cos(x-\alpha)$, $R>0$, $0<\alpha<\frac{\pi}{2}$.

b) Solve the equation

$$f(x) = 2 \ \text{for} \ 0 < x < 2\pi.$$

c) Write down the minimum value of $f(x)$.

d) Find the smallest positive value of x for which this minimum value occurs.

$\boxed{f(x) \cong \sqrt{10}\cos\left(x - 1.249^c\right)}$, $\boxed{x = 0.363^c, 2.135^c}$, $\boxed{f(x)_{\text{min}} = -\sqrt{10}}$, $\boxed{x = 4.391^c}$

Question 11 (***)

$$y = \sqrt{4-12x}, \ -\frac{1}{3} < x < \frac{1}{3}.$$

a) Find the binomial expansion of y in ascending powers of x up and including the term in x^3, writing all coefficients in their simplest form.

b) Hence find the coefficient of x^2 in the expansion of

$$(12x-4)(4-12x)^{\frac{1}{2}}.$$

$\boxed{}$, $\boxed{y = 2 - 3x - \frac{9}{4}x^2 - \frac{27}{8}x^3 + O\left(x^4\right)}$, $\boxed{-27}$

Question 13 (***+)

The variables y, x and t are related by the equations

$$y = 10e^{\frac{1}{5}x-1} \ \text{and} \ x = \sqrt{6t+1}, \ t \geq 0.$$

Find the value of $\frac{dy}{dt}$, when $t = 4$.

$\boxed{}$, $\boxed{\left.\frac{dy}{dt}\right|_{t=4} = \frac{6}{5}}$