Question 40 (****)

A curve C has implicit equation

$$y = \frac{2x+1}{xy+3}.$$

a) Find an expression for $\frac{dy}{dx}$, in terms of x and y.

b) Show that there is **no** point on C, where the tangent is parallel to the y axis.

$\boxed{}$, $\boxed{\frac{dy}{dx} = \frac{2-y^2}{2xy+3}}$

Question 4 (**+)

The binomial expression $(1+x)^{\frac{1}{3}}$ is to be expanded as an infinite convergent series, in ascending powers of x.

a) Determine the expansion of $(1+x)^{\frac{1}{3}}$, up and including the term in x^3.

b) Use the expansion of part (a) to find the expansion of $(1-3x)^{\frac{1}{3}}$, up and including the term in x^3.

c) Use the expansion of part (a) to find the expansion of $(27-27x)^{\frac{1}{3}}$, up and including the term in x^3.

$\boxed{1+\frac{1}{3}x-\frac{1}{9}x^2+\frac{5}{81}x^3+O(x^4)}$, $\boxed{1-x-x^2-\frac{5}{3}x^3+O(x^4)}$,
$\boxed{3-x-\frac{1}{3}x^2-\frac{5}{27}x^3+O(x^4)}$

Question 8 (***)

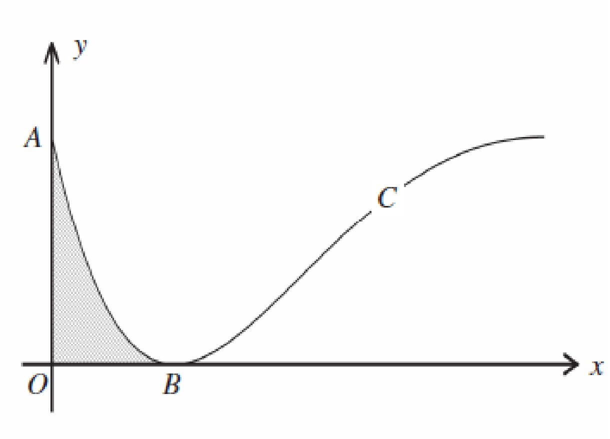
Fine sand is dropping on a horizontal floor at the constant rate of $4 \text{ cm}^3\text{s}^{-1}$ and forms a pile whose volume, $V \text{ cm}^3$, and height, h cm, are connected by the formula

$$V = -8 + \sqrt{h^4 + 64}.$$

Find the rate at which the height of the pile is increasing, when the height of the pile has reached 2 cm.

$\boxed{}$, $\boxed{\sqrt{5} \approx 2.24 \text{ cms}^{-1}}$

Question 12 (***+)



The figure above shows the curve C, with parametric equations

$$x = t^2, \quad y = 1 + \cos t \ , \ 0 \le t \le 2\pi .$$

The curve meets the coordinate axes at the points A and B.

a) Show that the area of the shaded region bounded by C and the coordinate axes is given by the integral

$$\int_{t_1}^{t_2} 2t(1+\cos t) \ dt ,$$

where t_1 and t_2 are constants to be stated.

b) Evaluate the above parametric integral to find an exact value for the area of the shaded region.

$\boxed{t_1 = 0, \ t_2 = \pi}$, $\boxed{\text{area} = \pi^2 - 4}$

Question 49 (****)

The equation of a curve is given implicitly by

$$y^2 - x^2 = 1, \quad |y| \ge 1 \ .$$

Show clearly that

$$\frac{d^2y}{dx^2} = \frac{y^2 - x^2}{y^3}.$$

Question 12 (***)

A curve C is given parametrically by

$$x = 4t - 1, \quad y = \frac{5}{2t} + 10, \quad t \in \mathbb{R}, \; t \neq 0.$$

The curve C crosses the x axis at the point A.

a) Find the coordinates of A.

b) Show that an equation of the tangent to C at A is

$$10x + y + 20 = 0.$$

c) Determine a Cartesian equation for C.

$\boxed{(-2,0)}$, $\boxed{(x+1)(y-10) = 10 \quad \text{or} \quad y = \frac{10(x+2)}{x+1}}$

Question 9

$$f(\theta) \equiv 4\sin\theta + 3\cos\theta, \; \theta \in \mathbb{R}.$$

a) Write the above expression in the form $R\sin(\theta + \alpha)$, $R > 0$, $0 < \alpha < 90°$.

b) Write down the maximum value of $f(\theta)$.

c) Find the smallest positive value of θ for which this maximum value occurs.

$\boxed{f(\theta) \equiv 4\sin\theta + 3\cos\theta \cong 5\sin(\theta + 36.9°)}$, $\boxed{f(\theta)_{\text{max}} = 5}$, $\boxed{\theta \approx 53.1°}$