Question 13 (***)

The curve C has equation

$$2\cos 3x \sin y = 1, \quad 0 \le x, y \le \pi .$$

a) Show that

$$\frac{dy}{dx} = 3\tan 3x \tan y .$$

The point $P\left(\frac{\pi}{12}, \frac{\pi}{4}\right)$ lies on C.

b) Show that an equation of the tangent to C at P is

$$y = 3x .$$

Question 25 (***+)

A curve C is given by the parametric equations

$$x = \frac{3t-2}{t-1}, \quad y = \frac{t^2-2t+2}{t-1}, \quad t \in \mathbb{R}, \quad t \ne 1.$$

a) Show clearly that

$$\frac{dy}{dx} = 2t - t^2 .$$

The point $P\left(1, -\frac{5}{2}\right)$ lies on C.

b) Show that the equation of the tangent to C at the point P is

$$3x - 4y - 13 = 0 .$$

Question 5 (**+)

$$f(x)=\frac{5x+3}{(1-x)(1+3x)},\ |x|<\frac{1}{3}.$$

a) Express $f(x)$ into partial fractions.

b) Hence find the series expansion of $f(x)$, up and including the term in x^3.

$\boxed{}$, $\boxed{f(x)=\frac{2}{1-x}+\frac{1}{1+3x}}$, $\boxed{f(x)=3-x+11x^2-25x^3+O(x^4)}$

Question 9 (***)

An oil spillage on the surface of the sea remains circular at all times.

The radius of the spillage, r km, is increasing at the constant rate of $0.5\ \text{km h}^{-1}$.

a) Find the rate at which the area of the spillage, $A\ \text{km}^2$, is increasing, when the circle's radius has reached 10 km.

A different oil spillage on the surface of the sea also remains circular at all times.

The area of this spillage, $A\ \text{km}^2$, is increasing at the rate of $0.5\ \text{km}^2\ \text{h}^{-1}$.

b) Show that when the area of the spillage has reached $10\ \text{km}^2$, the rate at which the radius r of the spillage is increasing is

$$\frac{1}{4\sqrt{10\pi}}\ \text{km h}^{-1}.$$

$\boxed{10\pi \approx 31.4\ \text{km}^2\ \text{h}^{-1}}$

Carry out the following integrations:

1. $\int \frac{1}{2}x e^{4x}\, dx = \frac{1}{8}x e^{4x} - \frac{1}{32}e^{4x} + C$

2. $\int 5x \sin 4x\, dx = -\frac{5}{4}x\cos 4x + \frac{5}{16}\sin 4x + C$

3. $\int (2x+1)\cos 2x\, dx = \frac{1}{2}(2x+1)\sin 2x + \frac{1}{2}\cos 2x + C$

Question 6

$$f(x) \equiv 9\sin x + 12\cos x,\ x \in \mathbb{R}.$$

a) Express $f(x)$ in the form $R\sin(x+\alpha)$, $R>0$, $0<\alpha<\frac{\pi}{2}$.

b) Hence, solve the trigonometric equation

$$9\sin x + 12\cos x = 7.5,\ 0<x<2\pi.$$

$\boxed{f(x) \equiv 9\sin x + 12\cos x \cong 15\sin\left(x+0.927^{c}\right)}$, $\boxed{x \approx 1.69^{c},\ 5.88^{c}}$

Question 39 (****)

The equation of a curve is given implicitly by

$$4y + y^2 e^{3x} = x^3 + C,$$

where C is a non zero constant.

a) Find a simplified expression for $\frac{dy}{dx}$.

The point $P(1,k)$, where $k>0$, is a stationary point of the curve.

b) Find an exact value for C.

$\boxed{}$, $\boxed{\frac{dy}{dx} = \frac{3\left(x^2 - y^2 e^{3x}\right)}{2\left(2+ye^{3x}\right)}}$, $\boxed{C = 4e^{-\frac{3}{2}}}$