1. A herbalist claims that a particular remedy is successful in curing a particular disease in 52% of cases.

   A random sample of 25 people who took the remedy is taken.

   a Find the probability that more than 12 people in the sample were cured. **(2 marks)**

   A second random sample of 300 people was taken and 170 were cured.

   b Assuming the herbalist's claim is true, use a suitable approximation to find the probability that at least 170 people were cured. **(4 marks)**

   c Using your answer to part **b**, comment on the herbalist's claim. **(1 mark)**

2. The time a mobile phone battery lasts before needing to be recharged is assumed to be normally distributed with a mean of 48 hours and a standard deviation of 8 hours.

   a Find the probability that a battery will last for more than 60 hours. **(2 marks)**

   b Find the probability that the battery lasts less than 35 hours. **(1 mark)**

   A random sample of 30 phone batteries is taken.

   c Find the probability that 3 or fewer last less than 35 hours. **(2 marks)**

3. Climbing rope produced by a manufacturer is known to be such that one-metre lengths have breaking strengths that are normally distributed with mean 170.2 kg and standard deviation 10.5 kg. Find, to 3 decimal places, the probability that:

   a a one-metre length of rope chosen at random from those produced by the manufacturer will have a breaking strength of 175 kg to the nearest kg **(2 marks)**

   b a random sample of 50 one-metre lengths will have a mean breaking strength of more than 172.4 kg. **(3 marks)**

   A new component material is added to the ropes being produced. The manufacturer believes that this will increase the mean breaking strength without changing the standard deviation. A random sample of 50 one-metre lengths of the new rope is found to have a mean breaking strength of 172.4 kg.

   c Perform a significance test at the 5% level to decide whether this result provides sufficient evidence to confirm the manufacturer's belief that the mean breaking strength is increased. State clearly the null and alternative hypotheses that you are using. **(3 marks)**

4. Historical information finds that nationally 30% of cars fail a brake test.

   a Give a reason to support the use of a binomial distribution as a suitable model for the number of cars failing a brake test. **(1 mark)**

   b Find the probability that, of 5 cars taking the test, all of them pass the brake test. **(2 marks)**

   A garage decides to conduct a survey of their cars. A randomly selected sample of 10 of their cars is tested. Two of them fail the test.

   c Test, at the 5% level of significance, whether or not there is evidence to support the suggestion that cars in this garage fail less than the national average. **(7 marks)**

5. The owner of a local corner shop calculates that the probability of a customer buying a newspaper is 0.40.

   A random sample of 100 customers is recorded.

   a Give two reasons why a normal approximation may be used in this situation. **(2 marks)**

   b Write down the parameters of the normal distribution used. **(2 marks)**

   c Use this approximation to estimate the probability that at least half the customers bought a newspaper. **(2 marks)**

6 The waiting time at a doctor's surgery is assumed to be normally distributed with standard deviation of 3.8 minutes. Given that the probability of waiting more than 15 minutes is 0.0446, find:

a the mean waiting time **(2 marks)**

b the probability of waiting less than 5 minutes. **(2 marks)**

7 A machine fills 1 kg packets of sugar. The actual weight of sugar delivered to each packet can be assumed to be normally distributed. The manufacturer requires that,

i the mean weight of the contents of a packet is 1010 g, and

ii 95% of all packets filled by the machine contain between 1000 g and 1020 g of sugar.

a Show that this is equivalent to demanding that the variance of the sampling distribution, to 2 decimal places, is equal to 26.03 g^2. **(3 marks)**

A sample of 8 packets was selected at random from those filled by the machine. The weights, in grams, of the contents of these packets were

1012.6	1017.7	1015.2	1015.7	1020.9	1005.7	1009.9	1011.4

Assuming that the variance of the actual weights is 26.03 g^2,

b test at the 2% significance level (stating clearly the null and alternative hypotheses that you are using) to decide whether this sample provides sufficient evidence to conclude that the machine is not fulfilling condition **i**. **(4 marks)**

8 The weights of steel sheets produced by a factory are known to be normally distributed with mean 32.5 kg and standard deviation 2.2 kg.

a Find the percentage of sheets that weigh less than 30 kg. **(1 mark)**

Bob requires sheets that weigh between 31.6 kg and 34.8 kg.

b Find the percentage of sheets produced that satisfy Bob's requirements. **(3 marks)**

9 A single observation x is to be taken from a binomial distribution B(30, p). This observation is used to test H_0: $p = 0.35$ against H_1: $p \neq 0.35$.

a Using a 5% level of significance, find the critical region for this test. The probability of rejecting either tail should be as close as possible to 2.5%. **(3 marks)**

b State the actual significance level of this test. **(2 marks)**

The actual value of X obtained is 4.

c State a conclusion that can be drawn based on this value giving a reason for your answer. **(2 marks)**

10 The random variable X has a normal distribution with mean μ and standard deviation 2. A random sample of 25 observations is taken and the sample mean $\overline{X}$ is calculated in order to test the null hypothesis $\mu = 7$ against the alternative hypothesis $\mu > 7$ using a 5% level of significance. Find the critical region for $\overline{X}$. **(4 marks)**

11 A certain brand of mineral water comes in bottles. The amount of water in a bottle, in millilitres, follows a normal distribution of mean μ and standard deviation 2. The manufacturer claims that μ is 125. In order to maintain standards the manufacturer takes a sample of 15 bottles and calculates the mean amount of water per bottle to be 124.2 millilitres. Test, at the 5% level, whether or not there is evidence that the value of μ is lower than the manufacturer's claim. State your hypotheses clearly. **(4 marks)**

(12) A fair coin is spun 60 times. Use a suitable approximation to estimate the probability of obtaining fewer than 25 heads.

(13) The diameters of eggs of the little-gull are approximately normally distributed with mean 4.11 cm and standard deviation 0.19 cm.

a Calculate the probability that an egg chosen at random has a diameter between 3.9 cm and 4.5 cm. **(3 marks)**

A sample of 8 little-gull eggs was collected from a particular island and their diameters, in cm, were

4.4, 4.5, 4.1, 3.9, 4.4, 4.6, 4.5, 4.1

b Assuming that the standard deviation of the diameters of eggs from the island is also 0.19 cm, test, at the 1% level, whether the results indicate that the mean diameter of little-gull eggs on this island is different from elsewhere. **(4 marks)**

(14) The proportion of defective articles in a certain manufacturing process has been found from long experience to be 0.1.

A random sample of 50 articles was taken in order to monitor the production. The number of defective articles was recorded.

a Using a 5% level of significance, find the critical regions for a two-tailed test of the hypothesis that 1 in 10 articles has a defect. The probability in each tail should be as near 2.5% as possible. **(4 marks)**

b State the actual significance level of the above test. **(2 marks)**

Another sample of 20 articles was taken at a later date. Four articles were found to be defective.

c Test, at the 10% significance level, whether or not there is evidence that the proportion of defective articles has increased. State your hypothesis clearly. **(5 marks)**

(15) The heights of a large group of men are normally distributed with a mean of 178 cm and a standard deviation of 4 cm. A man is selected at random from this group.

a Find the probability that he is taller than 185 cm. **(2 marks)**

b Find the probability that three men, selected at random, are all less than 180 cm tall. **(3 marks)**

A manufacturer of door frames wants to ensure that fewer than 0.005 men have to stoop to pass through the frame.

c On the basis of this group, find the minimum height of a door frame to the nearest centimetre. **(2 marks)**

(16) A pharmaceutical company claims that 85% of patients suffering from a chronic rash recover when treated with a new ointment.

A random sample of 20 patients with this rash is taken from hospital records.

a Write down a suitable distribution to model the number of patients in this sample who recover when treated with the new ointment. **(2 marks)**

b Given that the claim is correct, find the probability that the ointment will be successful for exactly 16 patients. **(2 marks)**

The hospital believes that the claim is incorrect and the percentage who will recover is lower. From the records an administrator took a random sample of 30 patients who had been prescribed the ointment. She found that 20 had recovered.

c Stating your hypotheses clearly, test, at the 5% level of significance, the hospital's belief. **(6 marks)**

(17) The thickness of some plastic shelving produced by a factory is normally distributed. As part of the production process the shelving is tested with two gauges. The first gauge is 7 mm thick and 98.61% of the shelving passes through this gauge. The second gauge is 5.2 mm thick and only 1.02% of the shelves pass through this gauge.

Find the mean and standard deviation of the thickness of the shelving. **(4 marks)**