

1 Find the sums of the following series.

- a  $3 + 7 + 11 + 14 + \dots$  (20 terms)      b  $2 + 6 + 10 + 14 + \dots$  (15 terms)  
 c  $30 + 27 + 24 + 21 + \dots$  (40 terms)      d  $5 + 1 + -3 + -7 + \dots$  (14 terms)  
 e  $5 + 7 + 9 + \dots + 75$   
 f  $4 + 7 + 10 + \dots + 91$   
 g  $34 + 29 + 24 + 19 + \dots + -111$   
 h  $(x + 1) + (2x + 1) + (3x + 1) + \dots + (21x + 1)$

**Hint** For parts e to h, start by using the last term to work out the number of terms in the series.

2 Find how many terms of the following series are needed to make the given sums.

- a  $5 + 8 + 11 + 14 + \dots = 670$   
 b  $3 + 8 + 13 + 18 + \dots = 1575$   
 c  $64 + 62 + 60 + \dots = 0$   
 d  $34 + 30 + 26 + 22 + \dots = 112$

**Hint** Set the expression for  $S_n$  equal to the total and solve the resulting equation to find  $n$ .

P 3 Find the sum of the first 50 even numbers.

P 4 Find the least number of terms for the sum of  $7 + 12 + 17 + 22 + 27 + \dots$  to exceed 1000.

P 5 The first term of an arithmetic series is 4. The sum to 20 terms is  $-15$ . Find, in any order, the common difference and the 20th term.

P 6 The sum of the first three terms of an arithmetic series is 12. If the 20th term is  $-32$ , find the first term and the common difference.

P 7 Prove that the sum of the first 50 natural numbers is 1275.

**Problem-solving**  
Use the same method as Example 4.

P 8 Show that the sum of the first  $2n$  natural numbers is  $n(2n + 1)$ .

P 9 Prove that the sum of the first  $n$  odd numbers is  $n^2$ .

E/P 10 The fifth term of an arithmetic series is 33. The tenth term is 68. The sum of the first  $n$  terms is 2225.

- a Show that  $7n^2 + 3n - 4450 = 0$ . (4 marks)  
 b Hence find the value of  $n$ . (1 mark)

E/P 11 An arithmetic series is given by  $(k + 1) + (2k + 3) + (3k + 5) + \dots + 303$

- a Find the number of terms in the series in terms of  $k$ . (1 mark)  
 b Show that the sum of the series is given by  $\frac{152k + 46208}{k + 2}$  (3 marks)  
 c Given that  $S_n = 2568$ , find the value of  $k$ . (1 mark)

E/P 12 a Calculate the sum of all the multiples of 3 from 3 to 99 inclusive,

$$3 + 6 + 9 + \dots + 99$$

b In the arithmetic series

$$4p + 8p + 12p + \dots + 400$$

where  $p$  is a positive integer and a factor of 100,

- i find, in terms of  $p$ , an expression for the number of terms in this series. (3 marks)  
 ii Show that the sum of this series is  $200 + \frac{20\,000}{p}$  (4 marks)  
 c Find, in terms of  $p$ , the 80th term of the arithmetic sequence  $(3p + 2), (5p + 3), (7p + 4), \dots$ , giving your answer in its simplest form. (2 marks)

Exercise 3B

1 a 820      b 450      c -1140  
 d -294      e 1440      f 1425  
 g -1155      h 231x + 21

2 a 20      b 25      c 65      d 40014  
 e 2550

3 a -1      b 6      c 6      d -2

4 a  $-\frac{1}{2}$       b 20th term = -5      c  $a = 6, d = -2$   
 d  $-\frac{1}{2}$       e  $-\frac{1}{2}$       f  $-\frac{1}{2}$

5  $S_5 = 1 + 2 + 3 + \dots + 50$   
 $S_5 = 50 + 49 + 48 + \dots + 1$   
 $2 \times S_5 = 50 + 49 + 48 + \dots + 1$   
 $S_5 = 1 + 2 + 3 + \dots + 2n$   
 $S_5 = 2n + (2n - 1) + (2n - 2) + \dots + 1$   
 $2 \times S_5 = 2n(2n + 1) = S_5 = n(2n + 1)$   
 $S_5 = 1 + 3 + 5 + \dots + (2n - 3) + (2n - 1)$   
 $2 \times S_5 = n(2n - 1) + (2n - 3) + \dots + 3 + 1$   
 $S_5 = n(2n - 1) = S_5 = n^2$   
 $2 \times S_5 = n(2n) = S_5 = n^2$   
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 $S_5 = 1 + 3 + 5 + \dots + (2n - 3) + (2n - 1)$   
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 $S_5 = 1 + 2 + 3 + \dots + 2n$   
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11 a  $\frac{304}{k + 2}$   
 b  $S_n = \frac{k + 2}{152}(k - 1 + 303) = \frac{k + 2}{152}(405 - k)$   
 c 17

12 a  $3 + 6 + 9 + \dots + 99$   
 b  $4p + 8p + 12p + \dots + 400$   
 where  $p$  is a positive integer and a factor of 100,  
 i find, in terms of  $p$ , an expression for the number of terms in this series. (3 marks)  
 ii Show that the sum of this series is  $200 + \frac{20\,000}{p}$  (4 marks)  
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