1 Find the sums of the following series.

a $3 + 7 + 11 + 14 + \ldots$ (20 terms)

b $2 + 6 + 10 + 14 + \ldots$ (15 terms)

c $30 + 27 + 24 + 21 + \ldots$ (40 terms)

d $5 + 1 + -3 + -7 + \ldots$ (14 terms)

e $5 + 7 + 9 + \ldots + 75$

f $4 + 7 + 10 + \ldots + 91$

g $34 + 29 + 24 + 19 + \ldots + -111$

h $(x + 1) + (2x + 1) + (3x + 1) + \ldots + (21x + 1)$

> **Hint** For parts **e** to **h**, start by using the last term to work out the number of terms in the series.

2 Find how many terms of the following series are needed to make the given sums.

a $5 + 8 + 11 + 14 + \ldots = 670$

b $3 + 8 + 13 + 18 + \ldots = 1575$

c $64 + 62 + 60 + \ldots = 0$

d $34 + 30 + 26 + 22 + \ldots = 112$

> **Hint** Set the expression for S_n equal to the total and solve the resulting equation to find n.

(P) **3** Find the sum of the first 50 even numbers.

(P) **4** Find the least number of terms for the sum of $7 + 12 + 17 + 22 + 27 + \ldots$ to exceed 1000.

(P) **5** The first term of an arithmetic series is 4. The sum to 20 terms is -15. Find, in any order, the common difference and the 20th term.

(P) **6** The sum of the first three terms of an arithmetic series is 12. If the 20th term is -32, find the first term and the common difference.

(P) **7** Prove that the sum of the first 50 natural numbers is 1275.

> **Problem-solving** Use the same method as Example 4.

(P) **8** Show that the sum of the first $2n$ natural numbers is $n(2n + 1)$.

(P) **9** Prove that the sum of the first n odd numbers is n^2.

(E/P) **10** The fifth term of an arithmetic series is 33. The tenth term is 68. The sum of the first n terms is 2225.

a Show that $7n^2 + 3n - 4450 = 0$. **(4 marks)**

b Hence find the value of n. **(1 mark)**

(E/P) **11** An arithmetic series is given by $(k + 1) + (2k + 3) + (3k + 5) + \ldots + 303$

a Find the number of terms in the series in terms of k. **(1 mark)**

b Show that the sum of the series is given by $\dfrac{152k + 46208}{k + 2}$ **(3 marks)**

c Given that $S_n = 2568$, find the value of k. **(1 mark)**

(E/P) **12** **a** Calculate the sum of all the multiples of 3 from 3 to 99 inclusive,

$$3 + 6 + 9 + \ldots + 99$$ **(3 marks)**

b In the arithmetic series

$$4p + 8p + 12p + \ldots + 400$$

where p is a positive integer and a factor of 100,

i find, in terms of p, an expression for the number of terms in this series.

ii Show that the sum of this series is $200 + \dfrac{20\,000}{p}$ **(4 marks)**

c Find, in terms of p, the 80th term of the arithmetic sequence

$$(3p + 2), (5p + 3), (7p + 4), \ldots,$$

giving your answer in its simplest form. **(2 marks)**