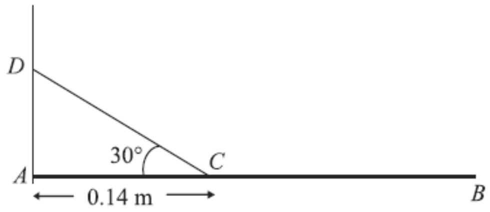
Hinges

1. A uniform beam *AB* of mass 2 kg is freely hinged at one end *A* to a vertical wall. The beam is held in equilibrium in a horizontal position by a rope which is attached to a point *C* on the beam, where *AC* = 0.14 m. The rope is attached to the point *D* on the wall vertically above *A*, where $\angle ACD = 30°$. The beam is modelled as a uniform rod and the rope as a light inextensible string. The tension in the rope is 63 N.

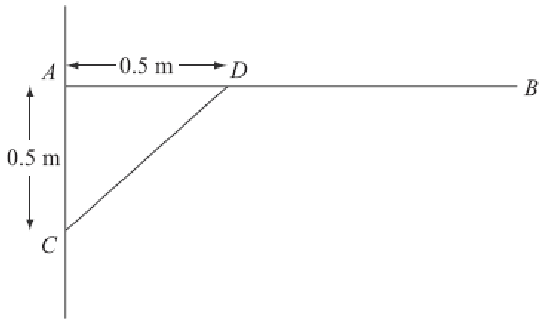


Find

(*a*) the length of *AB*, **(4)**

(*b*) the magnitude of the resultant reaction of the hinge on the beam at *A*. **(5)**

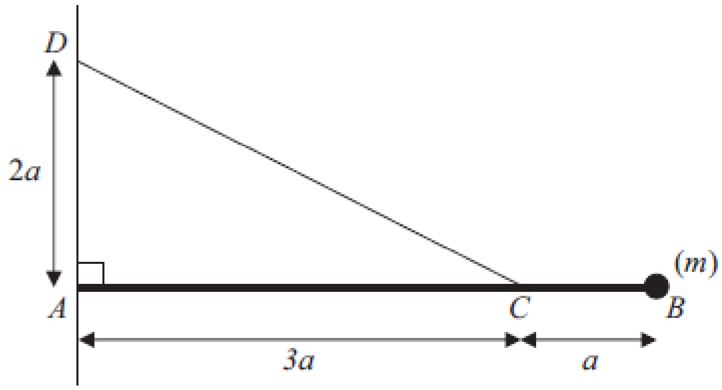
2. A uniform rod *AB*, of length 1.5 m and mass 3 kg, is smoothly hinged to a vertical wall at *A*. The rod is held in equilibrium in a horizontal position by a light strut *CD*. The rod and the strut lie in the same vertical plane, which is perpendicular to the wall. The end *C* of the strut is freely jointed to the wall at a point 0.5 m vertically below *A*. The end *D* is freely joined to the rod so that *AD* is 0.5 m.



(*a*) Find the thrust in *CD*. **(4)**

(*b*) Find the magnitude and direction of the force exerted on the rod *AB* at *A*. **(7)**

3. **The diagram** shows a uniform rod *AB* of mass *m* and length 4*a*. The end *A* of the rod is freely hinged to a point on a vertical wall. A particle of mass *m* is attached to the rod at *B*. One end of a light inextensible string is attached to the rod at *C*, where *AC* = 3*a*. The other end of the string is attached to the wall at *D*, where *AD* = 2*a* and *D* is vertically above *A*. The rod rests horizontally in equilibrium in a vertical plane perpendicular to the wall and the tension in the string is *T*.



(*a*) Show that $T = mg\sqrt{13}$. **(5)**

The particle of mass *m* at *B* is removed from the rod and replaced by a particle of mass *M* which is attached to the rod at *B*. The string breaks if the tension exceeds $2mg\sqrt{13}$. Given that the string does not break,

(*b*) show that $M \leq \frac{5}{2}m$. **(3)**