Connected Rates of Change

1) The radius, $r\ cm$, of a circle is increasing at the rate of $3\ ms^{-1}$. Find the rate at which the area is increasing when its radius is $13.5\ cm$.

2) The side of a cube of length $x\ cm$ is increasing at the constant rate of $1.5\ cm\ s^{-1}$. Find the rate at which the volume is increasing when its side is $6\ cm$.

3) The surface area, $S\ cm^2$ of a sphere is increasing at the constant rate of $512\ cm^2\ s^{-1}$. The surface area of a sphere is given by $S = 4\pi r^2$. Find the rate at which the radius, $r\ cm$, is increasing when it's the sphere's radius has reached $8\ cm$.

4) $x = 4\sin\theta + 7\cos\theta$. The value of θ is increasing at the constant rate of $0.5\ units\ s^{-1}$. Find the rate at which x is changing when $\theta = \frac{\pi}{2}$.

5) Fine sand is dropping on a horizontal floor at the constant rate of $4\ cm^3 s^{-1}$ and forms a pile whose volume, $V\ cm^3$, and height $h\ cm$ are connected by the formula $V = -8 + \sqrt{h^4 + 64}$. Find the rate at which the height of the pile is increasing when the height of the pile has reached $2\ cm$.

6) Two variables, x and y are related by $y = \frac{1}{4}\pi x^2(4 - x)$. The variable y is changing with time t, at a constant rate of $0.2\ units\ s^{-1}$. Find the rate at which x is changing with respect to t when $x = 2$.

7) Liquid is pouring into a container at the constant rate of $30\ cm^3 s^{-1}$. The container is initially empty and when the height of the liquid in the container is $h\ cm$, the colume of the liquid $V\ cm^3$ is given by $V = 36h^2$.
a) Find the rate at which the height of the liquid in the container is rising when the height of the liquid reaches $3\ cm$.
b) Determine the rate at which the height of the liquid in the container is rising 12.5 minutes after the liquid started pouring in.

8) The radius of a circle $R\ cm$ at time t seconds is given by $R = 10(1 - e^{-kt})$ where k is a positive constant and $t > 0$. Show that if A is the area of the circle in cm^2, then $\frac{dA}{dt} = Pk\left(e^{-kt} - e^{-2kt}\right)$, stating the exact value of P.

$C = 7\pi$
$B = 18\pi$
$J = 81\pi$
$L = 127$
$M = 219$
$I = 162$
$P = 2.55$
$T = 126$
$A = 200\pi$
$I = -7/2$
$P = \frac{5}{36}, \frac{1}{60}$
$W = \frac{7}{36}, \frac{1}{30}$

$A = \frac{1}{5\pi}$
$C = 7/2$
$V = \sqrt{7}$
$J = \sqrt{5}$
$N = 400\pi$
$A = \frac{1}{4\pi}$
$K = 20\pi$
$P = \frac{7}{36}, \frac{1}{60}$
$A =$
$T = \frac{5}{36}, \frac{1}{30}$
$Y = \sqrt{11}$

The correct letters spell, in order, a South American city.