Inequalities

5 The solution of an inequality is the set of all real numbers x that make the inequality true.

6 To solve a quadratic inequality:

- Rearrange so that the right-hand side of the inequality is 0
- Solve the corresponding quadratic equation to find the critical values
- Sketch the graph of the quadratic function
- Use your sketch to find the required set of values.

7 The values of x for which the curve $y = f(x)$ is **below** the curve $y = g(x)$ satisfy the inequality $f(x) < g(x)$.

The values of x for which the curve $y = f(x)$ is **above** the curve $y = g(x)$ satisfy the inequality $f(x) > g(x)$.

8 $y < f(x)$ represents the points on the coordinate grid below the curve $y = f(x)$.

$y > f(x)$ represents the points on the coordinate grid above the curve $y = f(x)$.

9 If $y > f(x)$ or $y < f(x)$ then the curve $y = f(x)$ is not included in the region and is represented by a dotted line.

If $y \geqslant f(x)$ or $y \leqslant f(x)$ then the curve $y = f(x)$ is included in the region and is represented by a solid line.

Questions

E **9** Give your answers in set notation.

a Solve the inequality $3x - 8 > x + 13$. **(2 marks)**

b Solve the inequality $x^2 - 5x - 14 > 0$. **(4 marks)**

E **10** Find the set of values of x for which $(x - 1)(x - 4) < 2(x - 4)$. **(6 marks)**

E **11** **a** Use algebra to solve $(x - 1)(x + 2) = 18$. **(2 marks)**

b Hence, or otherwise, find the set of values of x for which $(x - 1)(x + 2) > 18$. Give your answer in set notation. **(2 marks)**

12 Find the set of values of x for which:

a $6x - 7 < 2x + 3$ **(2 marks)**

b $2x^2 - 11x + 5 < 0$ **(4 marks)**

c $5 < \frac{20}{x}$ **(4 marks)**

d both $6x - 7 < 2x + 3$ and $2x^2 - 11x + 5 < 0$. **(2 marks)**

E **13** Find the set of values of x that satisfy $\frac{8}{x^2} + 1 \leqslant \frac{9}{x}$, $x \neq 0$ **(5 marks)**

E **14** Find the values of k for which $kx^2 + 8x + 5 = 0$ has real roots. **(3 marks)**

E/P **15** The equation $2x^2 + 4kx - 5k = 0$, where k is a constant, has no real roots. Prove that k satisfies the inequality $-\frac{5}{2} < k < 0$. **(3 marks)**

E **16** **a** Sketch the graphs of $y = f(x) = x^2 + 2x - 15$ and $g(x) = 6 - 2x$ on the same axes. **(4 marks)**

b Find the coordinates of any points of intersection. **(3 marks)**

c Write down the set of values of x for which $f(x) > g(x)$. **(1 mark)**

E **17** Find the set of values of x for which the curve with equation $y = 2x^2 + 3x - 15$ is below the line with equation $y = 8 + 2x$. **(5 marks)**

E **18** On a coordinate grid, shade the region that satisfies the inequalities:

$y > x^2 + 4x - 12$ and $y < 4 - x^2$. **(5 marks)**

E/P **19** **a** On a coordinate grid, shade the region that satisfies the inequalities

$y + x < 6$, $y < 2x + 9$, $y > 3$ and $x > 0$. **(6 marks)**

b Work out the area of the shaded region. **(2 marks)**

Solutions

9 a $3x - x > 13 + 8$

$2x > 21$

$x > 10\frac{1}{2}$

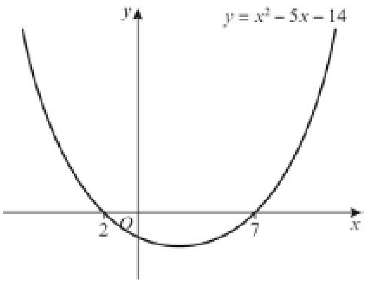
In set notation, the solution is

$\{x : x > \frac{21}{2}\}$

b $x^2 - 5x - 14 = 0$

$(x + 2)(x - 7) = 0$

$x = -2$ or $x = 7$



$x^2 - 5x - 14 > 0$ when $x < -2$ or $x > 7$

In set notation, the solution is

$\{x : x < -2\} \cup \{x : x > 7\}$

10 Multiplying out the brackets:

$x^2 - 5x + 4 < 2x - 8$

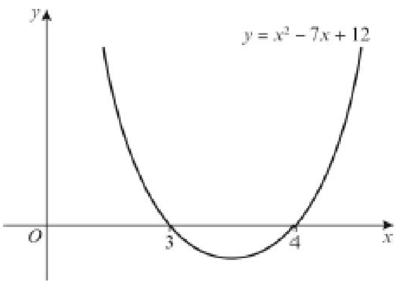
$x^2 - 5x - 2x + 4 + 8 < 0$

$x^2 - 7x + 12 < 0$

$x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

$x = 3$ or $x = 4$



$x^2 - 7x + 12 < 0$ when $3 < x < 4$

11 a $x^2 + x - 2 = 18$

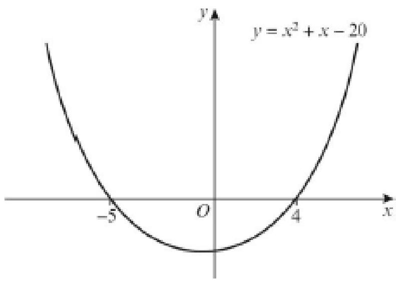
$x^2 + x - 20 = 0$

$(x + 5)(x - 4) = 0$

$x = -5$ or $x = 4$

b $(x - 1)(x + 2) > 18$

$\Rightarrow x^2 + x - 20 > 0$



$x^2 + x - 20 > 0$ when $x < -5$ or $x > 4$

In set notation, the solution is

$\{x : x < -5\} \cup \{x : x > 4\}$

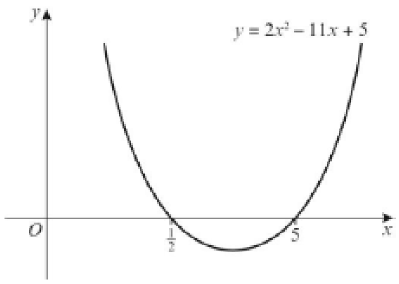
12 a $6x - 2x < 3 + 7$

$4x < 10$

$x < \frac{5}{2}$

b $(2x - 1)(x - 5) = 0$

$x = \frac{1}{2}$ or $x = 5$



$2x^2 - 11x + 5 < 0$ when $\frac{1}{2} < x < 5$

12 c $5 < \frac{20}{x}$

Multiply both sides by x^2

$5x^2 < 20x$

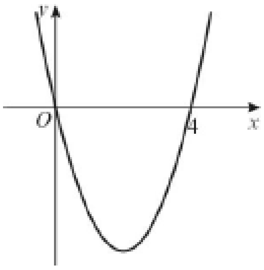
$5x^2 - 20x < 0$

Solve the quadratic to find the critical values:

$5x^2 - 20x = 0$

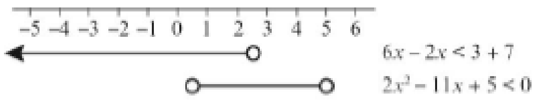
$5x(x - 4) = 0$

$x = 0$ or $x = 4$



The solution is $0 < x < 4$

d



Intersection is $\frac{1}{2} < x < \frac{5}{2}$

13 $\frac{8}{x^2} + 1 \leq \frac{9}{x}$

Multiply both sides by x^2:

$8 + x^2 \leq 9x$

$x^2 - 9x + 8 \leq 0$

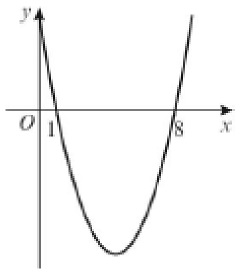
Solve the quadratic to find the critical values:

$x^2 - 9x + 8 = 0$

$(x - 1)(x - 8) = 0$

$x = 1$ or $x = 8$

13



The solution is $1 \leq x \leq 8$

14 $a = k, b = 8, c = 5$

Using the discriminant $b^2 - 4ac \geq 0$:

$8^2 - 4k \times 5 \geq 0$

$64 - 20k \geq 0$

$64 \geq 20k$

$\frac{64}{20} \geq k$

$k \leq \frac{16}{5}$

15 $a = 2, b = 4k, c = -5k$

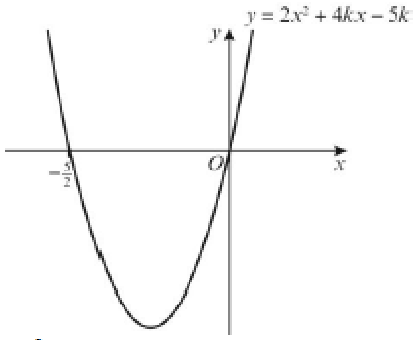
Using the discriminant $b^2 - 4ac < 0$:

$(4k)^2 - 4(2)(-5k) < 0$

$16k^2 + 40k < 0$

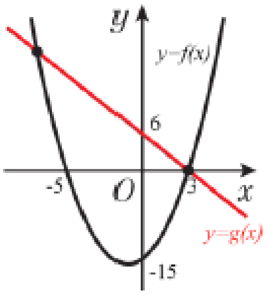
$8k(2k + 5) < 0$

$k = 0$ or $k = -\frac{5}{2}$



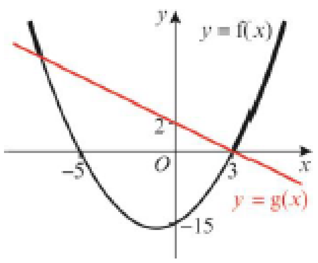
$-\frac{5}{2} < k < 0$

16 a $y = x^2 + 2x - 15$
$y = (x + 5)(x - 3)$
$0 = (x + 5)(x - 3)$
$x = -5$ or $x = 3$
When $x = 0$, $y = -15$



b $x^2 + 2x - 15 = 6 - 2x$
$x^2 + 4x - 21 = 0$
$(x + 7)(x - 3) = 0$
$x = -7$ or $x = 3$
When $x = -7$, $y = 20$
When $x = 3$, $y = 0$
The points of intersection are $(-7, 20)$ and $(3, 0)$.

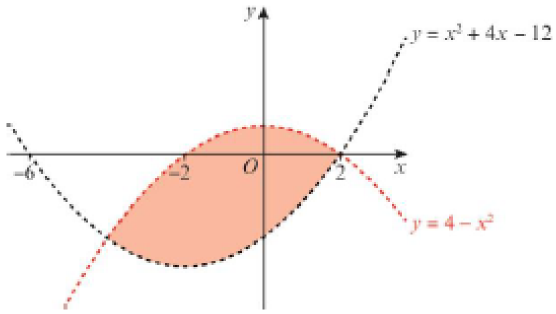
c



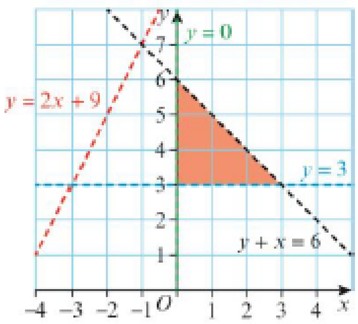
From the graph and the calculated points of intersection, the required values are
$x < -7$ or $x > 3$.

17 $2x^2 + 3x - 15 = 8 + 2x$
$2x^2 + x - 23 = 0$
$$x = \frac{-1 \pm \sqrt{185}}{4} = \tfrac{1}{4}\left(-1 \pm \sqrt{185}\right)$$
$$\tfrac{1}{4}\left(-1 - \sqrt{185}\right) < x < \tfrac{1}{4}\left(-1 + \sqrt{185}\right)$$

18 $y = x^2 + 4x - 12$
$x^2 + 4x - 12 = 0$
$(x + 6)(x - 2) = 0$
$x = -6$ or $x = 2$
$y = 4 - x^2$
$4 - x^2 = 0$
$(2 + x)(2 - x) = 0$
$x = -2$ or $x = 2$



19 a



b Area $= \frac{1}{2} \times 3 \times 3 = 4\frac{1}{2}$ units^2