1 For each sequence:

i state whether the sequence is increasing, decreasing, or periodic.

ii If the sequence is periodic, write down its order.

a 2, 5, 8, 11, 14 **b** $3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$ **c** 5, 9, 15, 23, 33 **d** 3, −3, 3, −3, 3

2 For each sequence:

i write down the first 5 terms of the sequence

ii state whether the sequence is increasing, decreasing, or periodic.

iii If the sequence is periodic, write down its order.

a $u_n = 20 - 3n$ **b** $u_n = 2^{n-1}$ **c** $u_n = \cos(180n°)$

d $u_n = (-1)^n$ **e** $u_{n+1} = u_n - 5, u_1 = 20$ **f** $u_{n+1} = 5 - u_n, u_1 = 20$

g $u_{n+1} = \frac{2}{3}u_n, u_1 = k$

3 The sequence of numbers $u_1, u_2, u_3, \ldots$ is given by $u_{n+1} = ku_n, u_1 = 5$.
Find the range of values of k for which the sequence is strictly decreasing.

E/P **4** The sequence with recurrence relation $u_{k+1} = pu_k + q, u_1 = 5$, where p is a constant and $q = 13$, is periodic with order 2.
Find the value of p. **(5 marks)**

E/P **5** A sequence has nth term $a_n = \cos(90n°), n \geqslant 1$.

a Find the order of the sequence. **(1 mark)**

b Find $\sum_{r}^{444} a_r$ **(2 marks)**

1 **a** **i** increasing

b **i** decreasing

c **i** increasing

d **i** periodic **ii** 2

2 **a** **i** 17, 14, 11, 8, 5 **ii** decreasing

b **i** 1, 2, 4, 8, 16 **ii** increasing

c **i** $-1, 1, -1, 1, -1$ **ii** periodic

iii 2

d **i** $-1, 1, -1, 1, -1$ **ii** periodic

iii 2

e **i** 20, 15, 10, 5, 0 **ii** decreasing

f **i** $20, -15, 20, -15, 20$ **ii** periodic

iii 2

g **i** $k, \frac{2k}{3}, \frac{4k}{9}, \frac{8k}{27}, \frac{16k}{81}$

ii dependent on value of k

3 $0 < k < 1$ **4** $p = -1$

5 **a** 4 **b** 0

E/P **10** An arithmetic sequence has first term k^2 and common difference k, where $k > 0$. The fifth term of the sequence is 41. Find the value of k, giving your answer in the form $p + q\sqrt{5}$, where p and q are integers to be found. **(4 marks)**

Problem-solving

You will need to make use of the condition $k > 0$ in your answer.

Challenge

The nth term of an arithmetic sequence is $u_n = \ln a + (n - 1) \ln b$ where a and b are integers. $u_3 = \ln 16$ and $u_7 = \ln 256$. Find the values of a and b.

E/P **6** A sequence is given by

$$x_1 = 2$$

$$x_{n+1} = x_n(p - 3x_n)$$

where p is an integer.

a Show that $x_3 = -10p^2 + 132p - 432$. **(2 marks)**

b Given that $x_3 = -288$ find the value of p. **(1 mark)**

c Hence find the value of x_4. **(1 mark)**

E/P **7** A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k$$

$$a_{n+1} = 4a_n + 5$$

a Find a_3 in terms of k. **(2 marks)**

b Show that $\sum_{r=1}^{4} a_r$ is a multiple of 5. **(3 marks)**

10 $-2 + 3\sqrt{5}$

Challenge

$a = 4, b = 2$

6 **a** $x_2 = x_1(p - 3x_1) = 2(p - 3(2)) = 2p - 12$

$x_3 = (2p - 12)(p - 3(2p - 12)) = (2p - 12)(-5p + 36)$

$= -10p^2 + 132p - 432$

b 12 **c** -252288

7 **a** $16k + 25$

b $a_4 = 4(16k + 25) + 5 = 64k + 105$

$\sum_{r=1}^{4} a_r = k + 4k + 5 + 16k + 25 + 64k + 105$

$= 85k + 135 = 5(17k + 27)$