Section A: TT3 Corrections

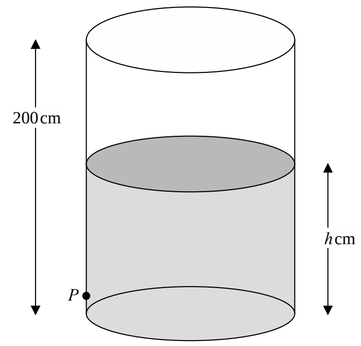
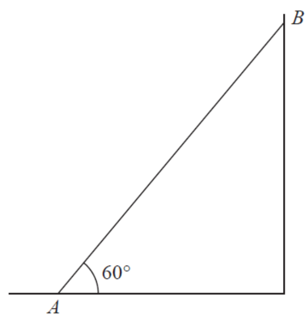


Diagram not drawn to scale

Figure 3

1. The diagram shows a vertical cylindrical tank of height 200 cm containing water.
Water is leaking from a hole *P* on the side of the tank.
At time *t* minutes after the leaking starts, the height of water in the tank is *h* cm.
The height *h* cm of the water in the tank satisfies the differential equation $\frac{dh}{dt} = k(h-9)^{\frac{1}{2}}$ $9 < h \leqslant 200$
where *k* is a constant.
Given that, when *h* = 130, the height of the water is falling at a rate of 1.1 cm per minute,
(*a*) show that the value of *k* is -0.1
Given that the tank was full of water when the leaking started,
(*b*) solve the differential equation with your value of *k*, to find the value of *t* when *h* = 50



2. A non-uniform rod, *AB*, of mass *m* and length 2*l*, rests in equilibrium with one end *A* on a rough horizontal floor and the other end *B* against a rough vertical wall. The rod is in a vertical plane perpendicular to the wall and makes an angle of 60° with the floor as shown in the diagram. The coefficient of friction between the rod and the floor is $\frac{1}{4}$ and the coefficient of friction between the rod and the wall is $\frac{2}{3}$. The rod is on the point of slipping at both ends.
(*a*) Find the magnitude of the vertical component of the force exerted on the rod by the floor. Write your answer in the format kmg where k is a rational number which should be stated.
The centre of mass of the rod is at *G*.
(*b*) Find the distance *AG*. Write your answer in the format pl where p is a real number which should be stated correct to 3 significant figures.

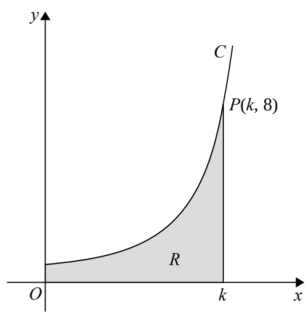


Diagram not drawn to scale

Figure 4

3. The diagram shows a sketch of part of the curve *C* with parametric equations

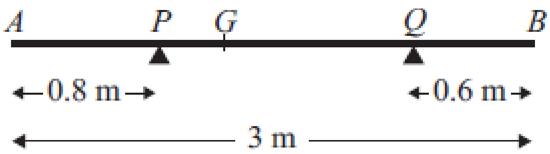
$x = 3\theta\sin\theta$, $y = \sec^3\theta$, $0 \leqslant \theta < \frac{\pi}{2}$

The point *P*(*k*, 8) lies on *C*, where *k* is a constant.
(*a*) Find the exact value of *k*.
The finite region *R*, shown shaded in Figure 4, is bounded by the curve *C*, the *y*-axis,
the *x*-axis and the line with equation *x* = *k*.
(*b*) Show that the area of *R* can be expressed in the form

$\lambda \int_{\alpha}^{\beta} (\theta\sec^2\theta + \tan\theta\sec^2\theta)\,d\theta$ where *λ*, *α* and *β* are constants to be determined.
(*c*) Hence use integration to find the exact value of the area of *R*.

Answers
1b) 148 minutes
2a) $\frac{6}{7}mg$ b) 1.03*l*
3a) $\frac{\pi\sqrt{3}}{2}$ b) $\frac{9}{2} + \pi\sqrt{3} - 3\ln 2$

Section B: Moments

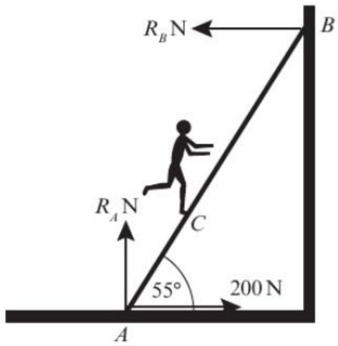


4. A non-uniform rod *AB* has length 3 m and mass 4.5 kg. The rod rests in equilibrium, in a horizontal position, on two smooth supports at *P* and at *Q*, where *AP* = 0.8 m and *QB* = 0.6 m, as shown in the diagram. The centre of mass of the rod is at G. Given that the magnitude of the reaction of the support at *P* on the rod is twice the magnitude of the reaction of the support at *Q* on the rod, find

(*a*) the magnitude of the reaction of the support at *Q* on the rod,

(*b*) the distance *AG*.

5. A ladder, AB, is leaning against a smooth vertical wall and on rough horizontal ground at an angle of 55° to the horizontal. The ladder has length 10 m and mass 20 kg. A man of mass 80 kg is standing at the point C on the ladder. Given that the magnitude of the frictional force at A is 200 N, find the distance AC. Model the ladder as a uniform rod and the man as a particle.



Answers

4. a) 15 N b) $\frac{4}{3}$

5. 2.39 m

Section C: Modulus graphs

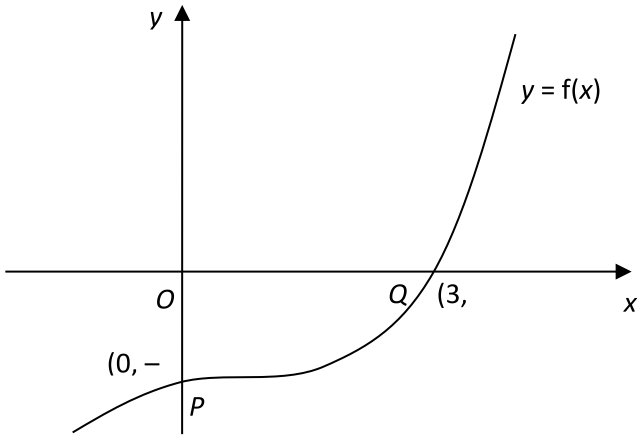
6. The diagram shows part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$, where f is an increasing function of x. The curve passes through the points $P(0, -2)$ and $Q(3, 0)$ as shown.

In separate diagrams, sketch the curve with equation

(*a*) $y = |\mathrm{f}(x)|$,

(*b*) $y = \mathrm{f}^{-1}(x)$,

(*c*) $y = \frac{1}{2}\mathrm{f}(3x)$.



7. The diagram shows the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$,
The graph consists of two line segments that meet at the point P.
The graph cuts the y-axis at the point Q and the x-axis at the points $(-3, 0)$ and R.
Sketch, on separate diagrams, the graphs of

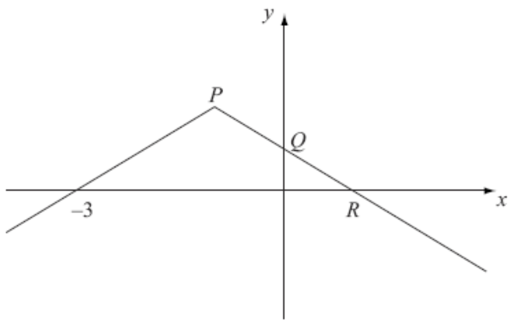
(*a*) $y = |\mathrm{f}(x)|$,

(*b*) $y = \mathrm{f}(-x)$.

Given that $\mathrm{f}(x) = 2 - |x + 1|$,

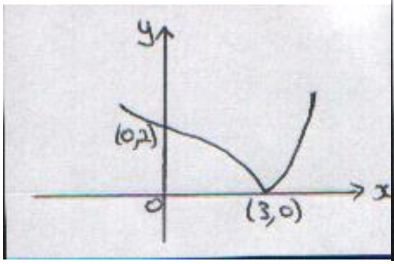
(*c*) find the coordinates of the points P, Q and R,

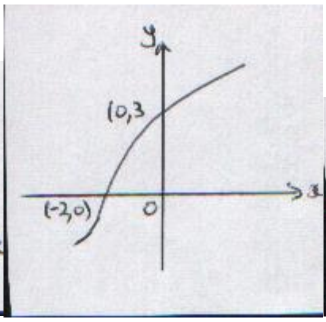
(*d*) solve $\mathrm{f}(x) = \frac{1}{2}x$.

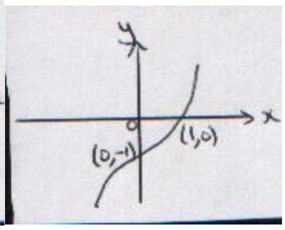


Answers

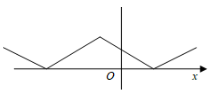
6.

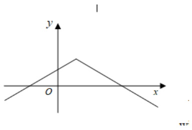






7.



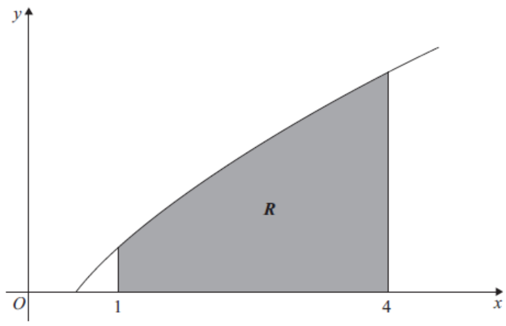


c) P(-1, 2) Q(0,1) R(1,0) d) x = -6

Section D: Integration

8. Given that $y = 2$ at $x = \frac{\pi}{4}$, solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{y\cos^2 x}$.

9. The diagram shows a sketch of part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$.



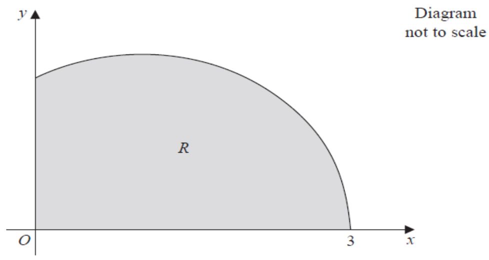
The finite region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$.

(*a*) Use the trapezium rule, with 3 strips of equal width, to find an estimate for the area of R, giving your answer to 2 decimal places.

(*b*) Find $\int x^{\frac{1}{2}} \ln 2x \ \mathrm{d}x$.

(*c*) Hence find the exact area of R, giving your answer in the form $a \ln 2 + b$, where a and b are exact constants.

10. The diagram shows a sketch of the curve with equation $y = \sqrt{(3-x)(x+1)}$, $0 \le x \le 3$.



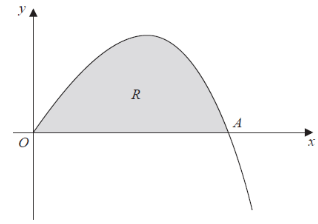
The finite region R, shown shaded in the diagram, is bounded by the curve, the x-axis, and the y-axis.

(*a*) Use the substitution $x = 1 + 2\sin\theta$ to show that

$$\int_0^3 \sqrt{(3-x)(x+1)}\ \mathrm{d}x = k\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta \ \mathrm{d}\theta,$$ where k is a constant to be determined.

(*b*) Hence find, by integration, the exact area of R.

11. Figure 1 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \ge 0$.



The curve meets the x-axis at the origin O and cuts the x-axis at the point A.

(*a*) Find, in terms of ln 2, the x coordinate of the point A.

(*b*) Find $\int xe^{\frac{1}{2}x} \ \mathrm{d}x$.

The finite region R, shown shaded in Figure 1, is bounded by the x-axis and the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \ge 0$.

(*c*) Find, by integration, the exact value for the area of R. Give your answer in terms of ln 2.

Answers

8. $\frac{1}{2}y^2 = 3\tan x - 1$

9. a) 7.49 b) $\frac{2}{3}x^{\frac{3}{2}}\ln 2x - \frac{4}{9}x^{\frac{3}{2}} + c$ c) $\frac{46}{3}\ln 2 - \frac{28}{9}$

10. b) $\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$

11. a) 4 ln 2 b) $2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x} + c$ c) $32(\ln 2)^2 - 32(\ln 2) + 12$