REVISION FOR TTH

1

(a) Express $2 \sin x - 3 \cos x$ in the form $R \sin (x - a)$, where $R > 0$ and $0 \leq a \leq \frac{\pi}{2}$

(b) Hence find the greatest value of $(2 \sin x - 3 \cos x)^2$ and find, the smallest positive value of x for which this maximum occurs

(c) Solve, for $0 \leq \theta \leq 2\pi$,

$$2 \sin x - 3 \cos x = 1$$

Give your answers to 3 decimal places.

2

A particle P moves in a straight line such that at t seconds, $t \geq 0$, its velocity, v ms^{-1} is given by:

$$v = 12 - 2t^2$$

Find:

(a) the distance travelled by P in the first second,

(b) the value of t when P changes direction of motion,

(c) the value of t at the instant P returns to its starting point.

3

A teacher selects a random sample of 56 students and records, to the nearest hour, the time spent watching television in a particular week.

Hours	1–10	11–20	21–25	26–30	31–40	41–59
Frequency	6	15	11	13	8	3
Mid-point	5.5	15.5		28		50

(*a*) Find the mid-points of the 21–25 hour and 31–40 hour groups.

A histogram was drawn to represent these data. The 11–20 group was represented by a bar of width 4 cm and height 6 cm.

(*b*) Find the width and height of the 26–30 group.

(*c*) Use your calculator to estimate the mean and standard deviation of the time spent watching television by these students.

(*d*) Use linear interpolation to estimate the median length of time spent watching television by these students.

e) Suggest how the teacher may have carried out a random sample?

f) State one advantage of random sampling

g) State and explain two other sampling methods they could have used?

(4) In an experiment testing solid rocket fuel, some fuel is burned and the waste products are collected. Throughout the experiment the sum of the masses of the unburned fuel and waste products remains constant.

Let x be the mass of waste products, in kg, at time t minutes after the start of the experiment. It is known that at time t minutes, the rate of increase of the mass of waste products, in kg per minute, is k times the mass of unburned fuel remaining, where k is a positive constant.

The differential equation connecting x and t may be written in the form

$$\frac{dx}{dt} = k(M - x), \text{ where } M \text{ is a constant.}$$

(a) Explain, in the context of the problem, what $\frac{dx}{dt}$ and M represent.

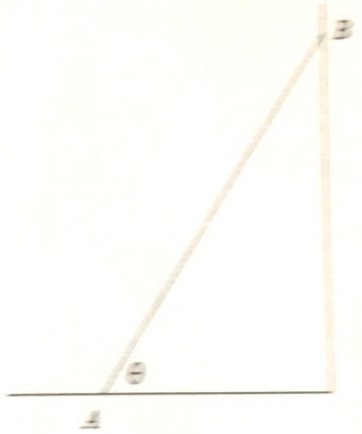
Given that initially the mass of waste products is zero,

(b) solve the differential equation, expressing x in terms of k, M and t.

Given also that $x = \frac{1}{2}M$ when $t = \ln 4$,

(c) find the value of x when $t = \ln 9$, expressing x in terms of M, in its simplest form.

(5) A uniform ladder AB of mass 20 kg and length 5 m is at rest with one end A on rough horizontal ground and the other end B against a smooth vertical wall.
The coefficient of friction between the ladder and the ground is 0.4.
The ladder makes an angle θ with the ground.



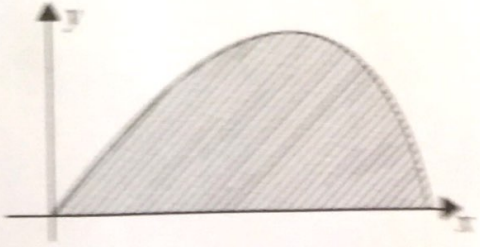
Given that the ladder is on the point of slipping.

(a) Find the magnitude of the frictional force of the ground on the ladder (2)

(b) Find the value of θ, to 1 decimal place. (5)

(6) The curve C has the parametric equations

$$x = 5\cos t \qquad y = 3\sin 2t \qquad 0 \le t \le \frac{\pi}{2}$$



(a) Find the points where the curve meets the x axis

(b) Find the area between the curve and the x axis

1 a) $\sqrt{13}\sin(x-0.983)$

b) 13, 2.55

c) 1.264, 3.843

2 a) $\frac{34}{3}$

b) $\sqrt{6}$

c) $3\sqrt{2}$

3 a) 23, 35.5

b) 10.4

c) 23.5, 107

d) 23.7 or 23.9

e) allocate number then use random number function on calculator

f) avoids bias

g) opportunity or quota or stratified + explanation

4 a) $\frac{dx}{dt}$ is the rate of increase of the mass of waste products

M is the total mass of unburned fuel and waste fuel

b) $x = M - Me^{-kt}$

c) $\frac{2}{3}M$

5 a) 8g

b) 51.3°

6 a) (5, 0) (0, 0)

b) 10