3 The times taken for a capful of stain remover to remove a standard chocolate stain from a baby's bib are normally distributed with a mean of 185 seconds and a standard deviation of 15 seconds. The manufacturers of the stain remover claim to have developed a new formula which will shorten the time taken for a stain to be removed. A random sample of 25 capfuls of the new formula are tested and the mean time for the sample is 179 seconds.

Test, at the 5% level, whether or not there is evidence that the new formula is an improvement.

Hint You are testing for an improvement, so use a **one-tailed** test.

4 The IQ scores of a population are normally distributed with a mean of 100 and a standard deviation of 15. A psychologist wishes to test the theory that eating chocolate before sitting an IQ test improves your score. A random sample of 80 people are selected and they are each given an identical bar of chocolate to eat before taking an IQ test.

a Find, at the 2.5% level, the critical region for this test, stating your hypotheses clearly.

The mean score on the test for the sample of 80 people was 102.5.

b Comment on this observation in light of the critical region.

5 The diameters of circular cardboard drinks mats produced by a certain machine are normally distributed with a mean of 9 cm and a standard deviation of 0.15 cm. After the machine is serviced a random sample of 30 mats is selected and their diameters are measured to see if the mean diameter has altered.

The mean of the sample was 8.95 cm.
Test, at the 5% level, whether there is significant evidence of a change in the mean diameter of mats produced by the machine.

Hint You are testing for an alteration in either direction, so use a **two-tailed** test.

E/P 6 A machine produces metal bolts of diameter D mm, where D is normally distributed with standard deviation 0.1 mm. Bolts with diameter either less than 5.1 mm or greater than 5.6 mm cannot be sold.

Given that 5% of bolts have a diameter in excess of 5.62 mm,

a find the probability that a randomly chosen bolt can be sold. **(5 marks)**

Twelve bolts are chosen.

b Find the probability that fewer than three cannot be sold. **(2 marks)**

A second machine produces bolts of diameter Y mm, where Y is normally distributed with standard deviation 0.08 mm.

A random sample of 20 bolts produced by this machine is taken and the sample mean of the diameters is found to be 5.52 mm.

c Stating your hypotheses clearly, and using a 2.5% level of significance, test whether the mean diameter of all the bolts produced by the machine is less than 5.7 mm. **(4 marks)**

(E/P) **7** The mass of European water voles, M grams, is normally distributed with standard deviation 12 grams.

Given that 2.5% of water voles have a mass greater than 160 grams,

a find the mean mass of a European water vole. **(3 marks)**

Eight water voles are chosen at random.

b Find the probability that at least 4 have a mass greater than 150 grams. **(3 marks)**

European water rats have mass, N grams, which is normally distributed with standard deviation 85 grams.

A random sample of 15 water rats is taken and the sample mean mass is found to be 875 grams.

c Stating your hypotheses clearly, and using a 10% level of significance, test whether the mean mass of all water rats is different from 860 grams. **(4 marks)**

(E) **8** Daily mean windspeed is modelled as being normally distributed with a standard deviation of 3.1 knots.

A random sample of 25 recorded daily mean windspeeds is taken at Heathrow in 2015.

Given that the mean of the sample is 12.2 knots, test at the 2.5% level of significance whether the mean of the daily mean windspeeds is greater than 9.5 knots.
State your hypotheses clearly. **(4 marks)**

(E) **21** Climbing rope produced by a manufacturer is known to be such that one-metre lengths have breaking strengths that are normally distributed with mean 170.2 kg and standard deviation 10.5 kg. Find, to 3 decimal places, the probability that:

a a one-metre length of rope chosen at random from those produced by the manufacturer will have a breaking strength of 175 kg to the nearest kg **(2 marks)**

b a random sample of 50 one-metre lengths will have a mean breaking strength of more than 172.4 kg. **(3 marks)**

A new component material is added to the ropes being produced. The manufacturer believes that this will increase the mean breaking strength without changing the standard deviation. A random sample of 50 one-metre lengths of the new rope is found to have a mean breaking strength of 172.4 kg.

c Perform a significance test at the 5% level to decide whether this result provides sufficient evidence to confirm the manufacturer's belief that the mean breaking strength is increased. State clearly the null and alternative hypotheses that you are using. **(3 marks)**

(E/P) **22** A machine fills 1 kg packets of sugar. The actual weight of sugar delivered to each packet can be assumed to be normally distributed. The manufacturer requires that,

i the mean weight of the contents of a packet is 1010 g, and

ii 95% of all packets filled by the machine contain between 1000 g and 1020 g of sugar.

a Show that this is equivalent to demanding that the variance of the sampling distribution, to 2 decimal places, is equal to 26.03 g^2. **(3 marks)**

A sample of 8 packets was selected at random from those filled by the machine. The weights, in grams, of the contents of these packets were

1012.6 1017.7 1015.2 1015.7 1020.9 1005.7 1009.9 1011.4

Assuming that the variance of the actual weights is 26.03 g^2,

b test at the 2% significance level (stating clearly the null and alternative hypotheses that you are using) to decide whether this sample provides sufficient evidence to conclude that the machine is not fulfilling condition **i**. **(4 marks)**

3 $\sigma = 15,\ n = 25,\ \bar{x} = 179$

$\mathrm{H_0}: \mu = 185$ (no improvement), $\mathrm{H_1}: \mu < 185$ (shorter time), one-tailed test at the 5% level.

Assume $\mathrm{H_0}$, so that $X \sim \mathrm{N}(185, 15^2)$ and $\bar{X} \sim \left(15, \dfrac{15^2}{25}\right)$ or $\bar{X} \sim (15, 3^2)$

Using the cumulative normal function, $\mathrm{P}(\bar{X} < 179) = 0.0227$ (4 d.p.)

$0.0227 < 0.05$ so significant, so reject $\mathrm{H_0}$.

There is evidence that the new formula is an improvement.

4 a The psychologist wishes to test whether the score has increased (not just changed). Therefore $\mathrm{H_0}: \mu = 100,\ \ \mathrm{H_1}: \mu > 100$, one-tailed test at the 2.5% level.

Assume $\mathrm{H_0}$, so that $X \sim \mathrm{N}(100, 15^2)$ and $\bar{X} \sim \left(100, \dfrac{15^2}{80}\right)$ or $\bar{X} \sim (100, 1.6771^2)$

Using the inverse normal function, $\mathrm{P}(\bar{X} > \bar{x}) = 0.025 \Rightarrow \bar{x} = 103.287\ldots$

So the critical region is $\bar{X} > 103.287\ldots$ or 103.29 (3 s.f.).

b $102.5 < 103.29$ so there is not sufficient evidence to reject $\mathrm{H_0}$, i.e. there is not sufficient evidence to say, at the 2.5% level, that eating chocolate before taking an IQ test improves the result.

5 $\sigma = 0.15,\ n = 30,\ \bar{x} = 8.95$

$\mathrm{H_0}: \mu = 9$ (no change), $\mathrm{H_1}: \mu \neq 9$ (change in mean diameter)

Two-tailed test with 2.5% in each tail

Assume $\mathrm{H_0}$, so that $X \sim \mathrm{N}(9.0, 0.15^2)$ and $\bar{X} \sim \left(9.0, \dfrac{0.15^2}{30}\right)$ or $\bar{X} \sim (9.0, 0.0274^2)$

Using the cumulative normal function, $\mathrm{P}(\bar{X} < 8.95) = 0.0340$ (4 d.p.)

$0.0340 > 0.025$ so not significant, so accept $\mathrm{H_0}$.

There is not enough evidence to conclude that there has been a change in the mean diameter.

6 a First find the mean of the distribution.

$\mathrm{P}(D > 5.62) = 0.05$

Using the inverse normal function (or the percentage points table), $p = 0.05 \Rightarrow z = 1.6449$

Using the formula $z = \dfrac{x-\mu}{\sigma}$, $\dfrac{5.62-\mu}{0.1} = 1.6449 \Rightarrow 5.62 - \mu = 0.16449 \Rightarrow \mu = 5.4555$

The probability that a randomly chosen bolt can be sold is $\mathrm{P}(5.1 \leqslant D \leqslant 5.6)$

Using the cumulative normal function, $\mathrm{P}(5.1 \leqslant D \leqslant 5.6) = 0.92558\ldots$

So the probability that a randomly chosen bolt can be sold is 0.9256 (4 d.p.).

b Use the binomial distribution $N \sim \mathrm{B}(12, 1-0.9256)$ or $N \sim \mathrm{B}(12, 1-0.0744)$.

Using the cumulative binomial function, $\mathrm{P}(N < 3) = \mathrm{P}(N \leqslant 2) = 0.94549\ldots$

So the probability that fewer than three cannot be sold is 0.9455 (4 d.p.).

6 c Test to determine whether the mean diameter is less than 5.7 mm. Therefore
$H_0 : \mu = 5.7$, $H_1 : \mu < 5.7$, one-tailed test at the 2.5% level.

Assume H_0, so that $Y \sim N(5.7, 0.08^2)$ and $\overline{Y} \sim \left(5.7, \dfrac{0.08^2}{20}\right)$ or $\overline{Y} \sim (5.7, 0.01788^2)$

Using the cumulative normal function, $P(\overline{Y} < 5.52) = 4.05 \times 10^{-24}$

$4.05 \times 10^{-24} < 0.025$ so significant, so reject H_0.

There is sufficient evidence to suggest that the mean diameter is less than 5.7 mm.

7 a $P(M > 160) = 0.025$

Using the inverse normal function (or the percentage points table), $p = 0.025 \Rightarrow z = 1.9599$

Using the formula $z = \dfrac{x - \mu}{\sigma}$, $\dfrac{160 - \mu}{12} = 1.9599 \Rightarrow 160 - \mu = 23.52 \Rightarrow \mu = 136.48$

So the mean mass of a European water vole is 136.48 g (2 d.p.).

b Using the cumulative normal function, $P(M > 150) = 0.1299$

Use the binomial distribution $N \sim B(8, 0.1299)$.

Using the cumulative binomial function, $P(N \geqslant 4) = 1 - P(N \leqslant 3) = 1 - 0.98708... = 0.01291...$

The probability that at least 4 voles have a mass greater than 150 g is 0.0129 (4 d.p.).

7 c Test to determine whether the mean mass is different from 860 grams. Therefore
$H_0 : \mu = 860$, $H_1 : \mu \neq 860$, two-tailed test with 5% in each tail.

Assume H_0, so that $N \sim N(860, 85^2)$ and $\overline{N} \sim \left(860, \dfrac{85^2}{15}\right)$ or $\overline{N} \sim (860, 21.946^2)$

Using the cumulative normal function, $P(\overline{N} > 875) = 0.24715...$

$0.24715 > 0.05$ so not significant, accept H_0.

There is insufficient evidence to suggest that the mean mass of all water rats is different from 860 g.

8 Test to determine whether the daily mean windspeed is greater than 9.5 knots. Therefore
$H_0 : \mu = 9.5$, $H_1 : \mu > 9.5$, one-tailed test at the 2.5% level.

Assume H_0, so that $X \sim N(9.5, 3.1^2)$ and $\overline{X} \sim \left(9.5, \dfrac{3.1^2}{25}\right)$ or $\overline{X} \sim (9.5, 0.62^2)$

Using the inverse normal function, $P(\overline{X} > \overline{x}) = 0.025 \Rightarrow \overline{x} = 10.715...$

So the critical region is $\overline{X} \geqslant 10.715...$

$12.2 > 10.715$ so there is sufficient evidence to reject H_0, i.e. there is sufficient evidence to say, at the 2.5% level, that the daily mean windspeed is greater than 9.5 knots.

21 Let B represent the breaking strength, so $B \sim \mathrm{N}(170.2, 10.5^2)$.

a Using the normal CD function, $\mathrm{P}(174.5 < B < 175.5) = 0.03421... = 0.0342$ (3 s.f.)

b $n = 50$ so $\overline{B} \sim \mathrm{N}\left(170.2, \dfrac{10.5^2}{50}\right)$

Using the normal CD function, $\mathrm{P}(\overline{B} > 172.4) = 0.06922... = 0.0692$ (3 s.f.)

c $\mathrm{H}_0 : \mu = 170.2, \mathrm{H}_1 : \mu > 170.2$ one-tailed test at the 5% level.

Assume H_0, so that $B \sim \mathrm{N}(170.2, 10.5^2)$ and $\overline{B} \sim \left(170.2, \dfrac{10.5^2}{50}\right)$ (as before).

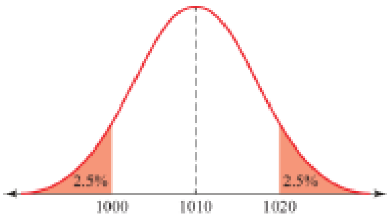
Using the normal CD function, $\mathrm{P}(\overline{B} > 172.4) = 0.06922... = 0.0692$ (3 s.f.)

$0.0692 > 0.05$ so not significant, so accept H_0.

There is insufficient evidence to conclude that the mean breaking strength is increased.

22 Let W represent the weight of sugar in a packet, so $W \sim \mathrm{N}(1010, \sigma^2)$.

a $\mathrm{P}(1000 < W < 1020) = 0.95 \Rightarrow \mathrm{P}(W < 1000) = 0.025 \Rightarrow \mathrm{P}\left(Z < \dfrac{1000 - 1010}{\sigma}\right) = 0.025$



Using the inverse normal function, $z = -1.95996...$

so $-1.95996... = \dfrac{1000 - 1010}{\sigma}$

$$\sigma = \frac{-10}{-1.95996..} = 5.1021...$$

$$\sigma^2 = 26.031... = 26.03 \text{ (2 d.p.)}$$

b $n = 8$ and $\sum x = 8109.1$, so $\overline{x} = 1013.6375$

$\mathrm{H}_0 : \mu = 1010, \mathrm{H}_1 : \mu \neq 1010$, two-tailed test with 1% in each tail.

Assume H_0, so that $W \sim \mathrm{N}(1010, 26.03)$ and $\overline{W} \sim \left(1010, \dfrac{26.03}{8}\right)$

Using the normal CD function, $\mathrm{P}(\overline{W} > 1013.6375) = 0.02187... = 0.0219$ (3 s.f.)

$0.0219 > 0.01$ so not significant, so accept H_0.

There is insufficient evidence of a deviation in the mean from 1010, so we can assume that condition **i** is being met.