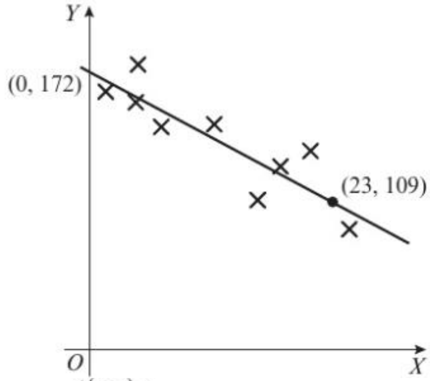
Exponential data

1 Data are coded using $Y = \log y$ and $X = \log$
to give a linear relationship. The equation of
the regression line for the coded data is $Y = 1.2 + 0.4X$.

a State whether the relationship between y and x is of the form $y = ax^n$ or $y = kb^x$.

b Write down the relationship between y and x and find the values of the constants.

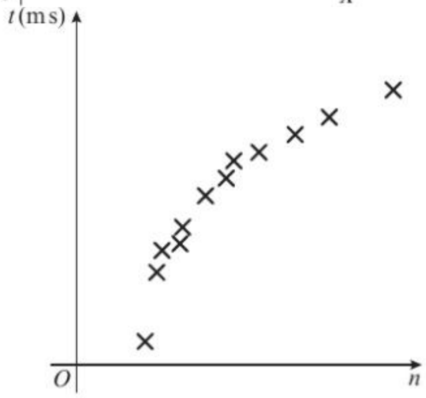
2 The scatter diagram shows the relationship between two sets of coded data, X and Y, where $X = \log x$ and $Y = \log y$. The regression line of Y on X is shown, and passes through the points (0, 172) and (23, 109).



The relationship between the original data sets is modelled by an equation of the form $y = ax^n$. Find, correct to 3 decimal places, the values of a and n.

3

The time, t m s, needed for a computer algorithm to determine whether a number, n, is prime is recorded for different values of n. A scatter graph of t against n is drawn.



a Explain why a model of the form $t = a + bn$ is unlikely to fit these data.

The data are coded using the changes of variable $y = \log t$ and $x = \log n$. The regression line of y on x is found to be $y = -0.301 + 0.6x$.

b Find an equation for t in terms of n, giving your answer in the form $t = an^k$, where a and k are constants to be found.

4

The heights, h cm, and masses, m kg, of a sample of Galapagos penguins are recorded. The data are coded using $y = \log m$ and $x = \log h$ and it is found that a linear relationship exists between x and y. The equation of the regression line of y on x is $y = 0.0023 + 1.8x$.

Find an equation to describe the relationship between m and h, giving your answer in the form $m = ah^n$, where a and n are constants to be found.

5

The table shows some data collected on the temperature, t °C, of a colony of insect larvae and the growth rate, g, of the population.

Temp, t (°C)	13	17	21	25	26	28
Growth rate, g	5.37	8.44	13.29	20.91	23.42	29.38

The data are coded using the changes of variable $x = t$ and $y = \log g$. The regression line of y on x is found to be $y = 0.09 + 0.05x$.

a Given that the data can be modelled by an equation of the form $g = ab^t$ where a and b are constants, find the values of a and b. **(3 marks)**

b Give an interpretation of the constant b in this equation. **(1 mark)**

c Explain why this model is not reliable for estimating the growth rate of the population when the temperature is 35 °C. **(1 mark)**

Answers to exponential data

1 **a** As noted at the beginning of Section 1.1, the equation $Y = 1.2 + 0.4X$ can be rewritten as $\log y = 1.2 + 0.4\log x$, which is of the form $\log y = \log a + n\log x$ and so $y = ax^n$.

b $Y = 1.2 + 0.4X$

$\Rightarrow \log y = 1.2 + 0.4\log x$

$\Rightarrow y = 10^{1.2+0.4\log x} = 10^{1.2} \times 10^{0.4\log x}$

$\Rightarrow y = 10^{1.2} \times 10^{\log x^{0.4}} = 10^{1.2} \times x^{0.4}$

Therefore $a = 10^{1.2} \approx 15.8$ (3 s.f.) and $n = 0.4$

2

In the linear model $Y = mX + c$, where m and c are constants,
$Y = \log y$ and $X = \log x$, so $\log y = m\log x + c$

Therefore $c = \log a$

The point (0, 172) lies on the line, so $c = 172$ and $\log a = 172 \Rightarrow a = 10^{172}$

(23, 109) lies on $Y = mX + 172$:

$109 = 23m + 172$

$\Rightarrow 23m = 109 - 172$

$\Rightarrow m = \frac{-63}{23} \approx -2.739$ (3 d.p.).

3

a The equation $t = a + bn$ is the equation of a straight line, but the data on the scatter diagram are not close to a straight line.

b $y = -0.301 + 0.6x$

$\Rightarrow \log t = -0.301 + 0.6\log n$

$\Rightarrow t = 10^{-0.301+0.6\log n} = 10^{-0.301} \times 10^{0.6\log n}$

$\Rightarrow t = 10^{-0.301} \times 10^{\log n^{0.6}}$

$\Rightarrow t = 10^{-0.301} \times n^{0.6}$

Therefore $a = 10^{-0.301} \approx 0.5$ (3 s.f.) and $k = 0.6$.

4 $y = 0.0023 + 1.8x$

$\Rightarrow \log m = 0.0023 + 1.8\log h$

$\Rightarrow m = 10^{0.0023+1.8\log h} = 10^{0.0023} \times 10^{1.8\log h}$

$\Rightarrow m = 10^{0.0023} \times 10^{\log h^{1.8}} = 10^{0.0023} \times h^{1.8}$

Therefore $a = 10^{0.0023} \approx 1.0$ (3 s.f.) and $n = 1.8$.

5 **a** $y = 0.09 + 0.05x$

$\Rightarrow \log g = 0.09 + 0.05t$

$\Rightarrow g = 10^{0.09+0.05t} = 10^{0.09} \times 10^{0.05t}$

$\Rightarrow g = 10^{0.09} \times \left(10^{0.05}\right)^t$

Therefore $a = 10^{0.09} \approx 1.23$ and $b \approx 1.12$ (3 s.f.)

b If you increase the temperature by 1 °C, b is the increase in the growth rate g, i.e. b is the rate of change of g per degree.

c 35 °C is outside of the range of data (extrapolation).