1 At time t seconds, a particle P has position vector $\mathbf{r}$ m with respect to a fixed origin O, where

$$\mathbf{r} = (3t - 4)\mathbf{i} + (t^3 - 4t)\mathbf{j},\ t \geqslant 0$$

Find:

a the velocity of P when $t = 3$

b the acceleration of P when $t = 3$.

2 A particle P of mass 3 grams moving in a plane is acted on by a force $\mathbf{F}$ N. Its velocity at time t seconds is given by $\mathbf{v} = (t^2\mathbf{i} + (2t - 3)\mathbf{j})\ \text{m s}^{-1},\ t \geqslant 0$.

Find $\mathbf{F}$ when $t = 4$.

3 In this question $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors east and north respectively.

A particle P is moving in a plane. At time t seconds, the position vector of P, $\mathbf{r}$ m, relative to a fixed origin O is given by $\mathbf{r} = 5e^{-\frac{1}{3}t}\mathbf{i} + 2\mathbf{j},\ t \geqslant 0$.

a Find the time at which the particle is directly north-east of O.

b Find the speed of the particle at this time.

c Explain why the particle is always moving directly west.

4 At time t seconds, a particle P has position vector $\mathbf{r}$ m with respect to a fixed origin O, where

$$\mathbf{r} = 4t^2\mathbf{i} + (24t - 3t^2)\mathbf{j},\ t \geqslant 0$$

a Find the speed of P when $t = 2$. **(3 marks)**

b Show that the acceleration of P is a constant and find the magnitude of this acceleration. **(3 marks)**

5 A particle P of mass 0.5 kg is initially at a fixed origin O. At time $t = 0$, P is projected from O and moves so that, at time t seconds after projection, its position vector $\mathbf{r}$ m relative to O is given by

$$\mathbf{r} = (t^3 - 12t)\mathbf{i} + (4t^2 - 6t)\mathbf{j},\ t \geqslant 0$$

Find:

a the speed of projection of P **(5 marks)**

b the value of t at the instant when P is moving parallel to $\mathbf{j}$ **(3 marks)**

c the position vector of P at the instant when P is moving parallel to $\mathbf{j}$. **(3 marks)**

The motion of the particle is due to it being acted on by a single variable force, $\mathbf{F}$ N.

d Given that the mass of the particle is 0.5 kg, find the magnitude of $\mathbf{F}$ when $t = 5$ s. **(4 marks)**

1 A particle P starts from rest at a fixed origin O. The acceleration of P at time t seconds (where $t \geqslant 0$) is $(6t^2\mathbf{i} + (8 - 4t^3)\mathbf{j})\,\text{m s}^{-2}$. Find:

a the velocity of P when $t = 2$ (3 marks)

b the position vector of P when $t = 4$. (3 marks)

2 A particle P is moving in a plane with velocity $\mathbf{v}\,\text{m s}^{-1}$ at time t seconds where

$$\mathbf{v} = (3t^2 + 2)\mathbf{i} + (6t - 4)\mathbf{j},\ t \geqslant 0$$

When $t = 2$, P has position vector $9\mathbf{j}$ m with respect to a fixed origin O. Find:

a the distance of P from O when $t = 0$ (4 marks)

b the acceleration of P at the instant when it is moving parallel to the vector $\mathbf{i}$. (4 marks)

3 At time t seconds, where $t \geqslant 0$, the particle P is moving in a plane with velocity $\mathbf{v}\,\text{m s}^{-1}$ and acceleration $\mathbf{a}\,\text{m s}^{-2}$, where $\mathbf{a} = (2t - 4)\mathbf{i} + 6\sin t\mathbf{j}$.

Given that P is instantaneously at rest when $t = \frac{\pi}{2}$ seconds, find:

a $\mathbf{v}$ in terms of π and t (5 marks)

b the exact speed of P when $t = \frac{3\pi}{2}$ (3 marks)

4 At time t seconds (where $t \geqslant 0$), the particle P is moving in a plane with acceleration $\mathbf{a}\,\text{m s}^{-2}$, where

$$\mathbf{a} = (5t - 3)\mathbf{i} + (8 - t)\mathbf{j}$$

When $t = 0$, the velocity of P is $(2\mathbf{i} - 5\mathbf{j})\,\text{m s}^{-1}$. Find:

a the velocity of P after t seconds (3 marks)

b the value of t for which P is moving parallel to $\mathbf{i} - \mathbf{j}$ (4 marks)

c the speed of P when it is moving parallel to $\mathbf{i} - \mathbf{j}$. (3 marks)

5 At time t seconds (where $t \geqslant 0$), a particle P is moving in a plane with acceleration $(2\mathbf{i} - 2t\mathbf{j})\,\text{m s}^{-2}$. When $t = 0$, the velocity of P is $2\mathbf{j}\,\text{m s}^{-1}$ and the position vector of P is $6\mathbf{i}$ m with respect to a fixed origin O.

a Find the position vector of P at time t seconds. (5 marks)

At time t seconds (where $t \geqslant 0$), a second particle Q is moving in the plane with velocity $((3t^2 - 4)\mathbf{i} - 2t\mathbf{j})\,\text{m s}^{-1}$. The particles collide when $t = 3$.

b Find the position vector of Q at time $t = 0$. (4 marks)

1 A particle P is moving in a plane. At time t seconds, the position vector of P, $\mathbf{r}$ m, is given by $\mathbf{r} = (3t^2 - 6t + 4)\mathbf{i} + (t^3 + kt^2)\mathbf{j}$, where k is a constant.

When $t = 3$, the speed of P is 12.5 ms^{-1}.

a Find the two possible values of k. (6 marks)

b For each of these values of k, find the magnitude of the acceleration of P when $t = 1.5$. (4 marks)

2 Relative to a fixed origin O, the position vector of a particle P at time t seconds is $\mathbf{r}$ metres, where

$$\mathbf{r} = 6t^2\mathbf{i} + t\mathbf{j}, \quad t \geqslant 0$$

At the instant when $t = 4$, find:

a the speed of P (5 marks)

b the acceleration of P, giving your answer as a vector. (2 marks)

3 A particle P moves in a horizontal plane. At time t seconds, the position vector of P is $\mathbf{r}$ metres relative to a fixed origin O where $\mathbf{r}$ is given by

$$\mathbf{r} = (18t - 4t^3)\mathbf{i} + ct^2\mathbf{j}, \quad t \geqslant 0,$$

where c is a positive constant. When $t = 1.5$, the speed of P is 15 ms^{-1}. Find:

a the value of c (6 marks)

b the acceleration of P when $t = 1.5$. (3 marks)

4 At time $t = 0$ a particle P is at rest at a point with position vector $(4\mathbf{i} - 6\mathbf{j})$ m with respect to a fixed origin O. The acceleration of P at time t seconds (where $t \geqslant 0$) is $((4t - 3)\mathbf{i} - 6t^2\mathbf{j})$ ms^{-2}. Find:

a the velocity of P when $t = \frac{1}{2}$ (5 marks)

b the position vector of P when $t = 6$. (5 marks)

5 At time t seconds (where $t \geqslant 0$) the particle P is moving in a plane with acceleration $\mathbf{a}$ ms^{-2}, where $\mathbf{a} = (8t^3 - 6t)\mathbf{i} + (8t - 3)\mathbf{j}$.

When $t = 2$, the velocity of P is $(16\mathbf{i} + 3\mathbf{j})$ ms^{-1}. Find:

a the velocity of P after t seconds (4 marks)

b the value of t when P is moving parallel to $\mathbf{i}$. (3 marks)

6 At time t seconds the velocity of a particle P is $((4t - 3)\mathbf{i} + 4\mathbf{j})$ ms^{-1}. When $t = 0$, the position vector of P is $(\mathbf{i} + 2\mathbf{j})$ m, relative to a fixed origin O.

a Find an expression for the position vector of P at time t seconds. (4 marks)

A second particle Q moves with constant velocity $(5\mathbf{i} + 4\mathbf{j})$ m s^{-1}.

When $t = 0$, the position vector of Q is $(10\mathbf{i} + 3\mathbf{j})$ m.

b Given that the particles P and Q collide, find:

i the value of k

ii the position vector of the point of collision. (6 marks)