Parametric Integration

1. The curve C has parametric equations $x = t^3, y = t^2, t \geq 0$. Show that the exact area of the region bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$ is $k\sqrt[3]{2}$, where k is a rational constant to be found.

2. The curve C has parametric equations $x = \sin t, y = \sin 2t, 0 \leq t \leq \frac{\pi}{2}$
The finite region R is bounded by the curve and the x-axis. Find the exact area of R.

3. The curve C has parametric equations $x = \ln(t+2), y = \frac{1}{t+1}, t > -1$
The finite region R between the curve C and the x-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown the diagram.

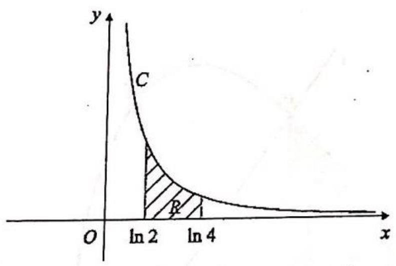
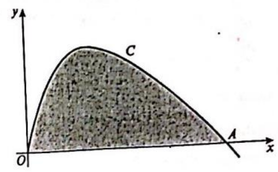


Figure 3

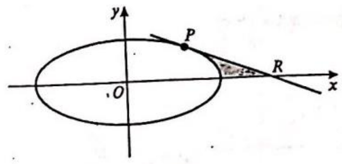
a) Show the area R is given by the integral $\int_0^2 \frac{1}{(t+1)(t+2)} dt$
b) hence find an exact value for this area.
c) Find a Cartesian equation for the curve C, in the form $y = f(x)$
d) State the domain of values of xfor this curve.

4. The diagram shows the curve C with parametric equations
$x = 3t^2, y = \sin 2t, t \geq 0$



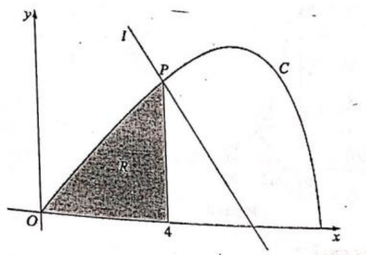
a) Write down the value of t at the point A where the curve crosses the x-axis.
b) Find in terms of π, the exact area of the shaded region bounded by C and the x-axis.

5. The curve shown has parametric equations
$x = 5\cos\theta, y = 4\sin\theta, 0 \leq \theta \leq 2\pi$



a) Find the gradient of the curve at the point P where $\theta = \frac{\pi}{4}$.
b) Find the equation of the tangent to the curve at the point P.
c) Find the exact area of the shaded region bounded by the tangent PR, the curve and the x-axis.

6. The diagram shows the curve C with parametric equations
$x = 8\cos t,\ y = 4\sin 2t, 0 \leq t \leq \frac{\pi}{2}$



The point P lies on C and has coordinates $(4, 2\sqrt{3})$
a) Find the value of t at this point.
The line l is a normal to C at P
b) Show that the equation of l is $y = -x\sqrt{3} + 6\sqrt{3}$
The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in the diagram.
c) Show that the area of R is given by $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64\sin^2 t\cos t\, dt$
d) use this integral to find the area of R , giving your answer in the form $a + b\sqrt{3}$ where a and b are constants to be determined.