TRACKING TEST 2 ½

Name……………………………………………..

Question	Max	Score
1	13	
2	13	
3	11	
4	12	
5	9	
6	9	
7	8	
TOTAL	75	
PERCENTAGE	100	

1. (*a*) Find $\int \tan^2 x \ \mathrm{d}x$. **(2)**

(*b*) Use integration by parts to find $\int \frac{1}{x^3} \ln x \ \mathrm{d}x$. **(4)**

(*c*) Use the substitution $u = 1 + \mathrm{e}^x$ to show that

$$\int \frac{\mathrm{e}^{3x}}{1+\mathrm{e}^x} \ \mathrm{d}x = \frac{1}{2}\mathrm{e}^{2x} - \mathrm{e}^x + \ln(1 + \mathrm{e}^x) + k,$$

where k is a constant. **(7)**

2. (*a*) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8 \sin^3 \theta - 6 \sin \theta + 1 = 0.$$

Give your answers in terms of π.

(5)

(*b*) Using $\sin (\theta - \alpha) = \sin \theta \cos \alpha - \cos \theta \sin \alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)

3. $\mathrm{f}(x) = 3x\mathrm{e}^x - 1.$

The curve with equation $y = \mathrm{f}(x)$ has a turning point P.

(*a*) Find the exact coordinates of P. **(5)**

The equation $\mathrm{f}(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$.

(*b*) Use the iterative formula $x_{n+1} = \frac{1}{3}\mathrm{e}^{-x_n}$.

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3. **(3)**

(*c*) By choosing a suitable interval, show that a root of $\mathrm{f}(x) = 0$ is $x = 0.2576$ correct to 4 decimal places. **(3)**

4. (*a*) Express $3 \cos \theta + 4 \sin \theta$ in the form $R \cos (\theta - \alpha)$,
where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. **(4)**

(*b*) Hence find the maximum value of $3 \cos \theta + 4 \sin \theta$ and the smallest positive value of θ for which this maximum occurs. **(3)**

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3 \cos (15t)° + 4 \sin (15t)°,$$

where t is the time in hours from midday and $0 \leq t < 24$.

(*c*) Calculate the minimum temperature of the warehouse as given by this model. **(2)**

(*d*) Find the value of t when this minimum temperature occurs. **(3)**

5. A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(*a*) Find the coordinates of the two points on the curve where $x = -8$. **(3)**

(*b*) Find the gradient of the curve at each of these points. **(6)**

6.

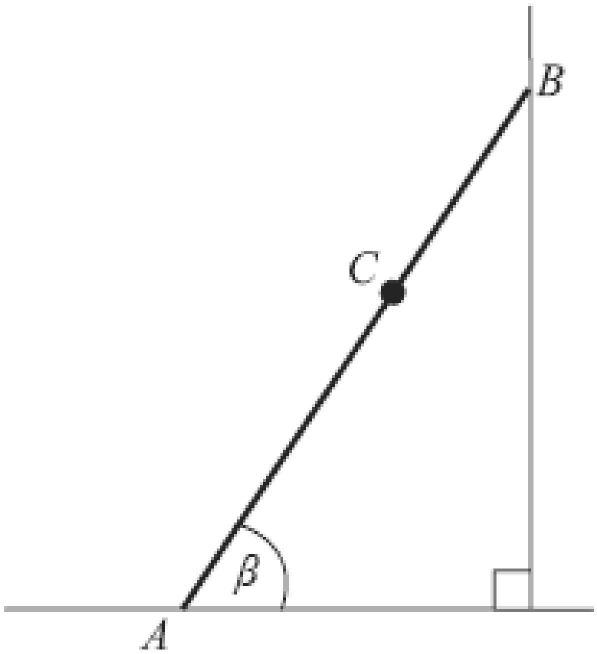

Figure 1

Figure 1 shows a ladder *AB*, of mass 25 kg and length 4 m, resting in equilibrium with one end *A* on rough horizontal ground and the other end *B* against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the ladder and the ground is $\frac{11}{25}$. The ladder makes an angle β with the ground. When Reece, who has mass 75 kg, stands at the point *C* on the ladder, where $AC = 2.8$ m, the ladder is on the point of slipping. The ladder is modelled as a uniform rod and Reece is modelled as a particle.

(*a*) Find the magnitude of the frictional force of the ground on the ladder. **(3)**

(*b*) Find, to the nearest degree, the value of β. **(6)**

7. The heights of a group of athletes are modelled by a normal distribution with mean 180 cm and a standard deviation 5.2 cm. The weights of this group of athletes are modelled by a normal distribution with mean 85 kg and standard deviation 7.1 kg.
Find the probability that a randomly chosen athlete

(*a*) is taller than 188 cm, **(3)**

(*b*) weighs less than 97 kg. **(2)**

(*c*) Assuming that for these athletes height and weight are independent, find the probability that a randomly chosen athlete is taller than 188 cm and weighs more than 97 kg. **(3)**