1. *(a)* Use integration by parts to find:-

$$\int x e^{2x-2}\, dx$$

(3)

(*b*) Use integration by parts to find $\int \frac{1}{x^3} \ln x \, dx$.

(4)

2. (*a*) For $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, sketch the graph of $y = g(x)$ where

$$g(x) = \arcsin x, \qquad -1 \leq x \leq 1.$$

(2)

(*b*) Find the exact value of x for which

$$3g(x+1) + \pi = 0.$$

(3)

3.

A curve has equation $3x^2 - y^2 + xy = 4$. The points P and Q lie on the curve. The gradient of the tangent to the curve is $\frac{8}{3}$ at P and at Q.

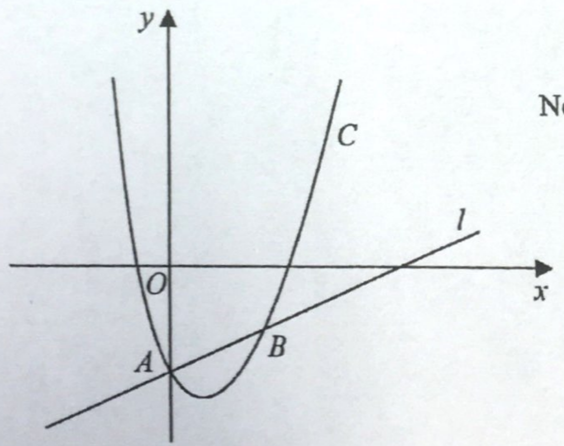
(a) Use implicit differentiation to show that $y - 2x = 0$ at P and at Q.

(6)

(b) Find the coordinates of P and Q.

(3)

4



Not to scale

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = e^{-2x} + x^2 - 3$$

The curve C crosses the y-axis at the point A.

The line l is the normal to C at the point A.

(*a*) Find the equation of l, writing your answer in the form $y = mx + c$, where m and c are constants. **(5)**

The line l meets C again at the point B, as shown in Figure 1.

(*b*) Show that the x coordinate of B is a solution of

$$x = \sqrt{1 + \frac{1}{2}x - e^{-2x}}$$

(2)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{1}{2}x_n - e^{-2x_n}}$$

with $x_1 = 1$

(*c*) find x_2 and x_3 to 3 decimal places. **(2)**

5 (i) Using the identity for tan $(A \pm B)$, solve, for $-90° < x < 90°$,

$$\frac{\tan 2x + \tan 32°}{1 - \tan 2x \tan 32°} = 5$$

Give your answers, in degrees, to 2 decimal places. **(4)**

(ii) (*a*) Using the identity for tan $(A \pm B)$, show that

$$\tan(3\theta - 45°) \equiv \frac{\tan 3\theta - 1}{1 + \tan 3\theta}, \qquad \theta \neq (60n + 45)°, n \in \mathbb{Z}$$

(2)

(*b*) Hence solve, for $0 < \theta < 180°$,

$$(1 + \tan 3\theta)\tan(\theta + 28°) = \tan 3\theta - 1$$

(4)

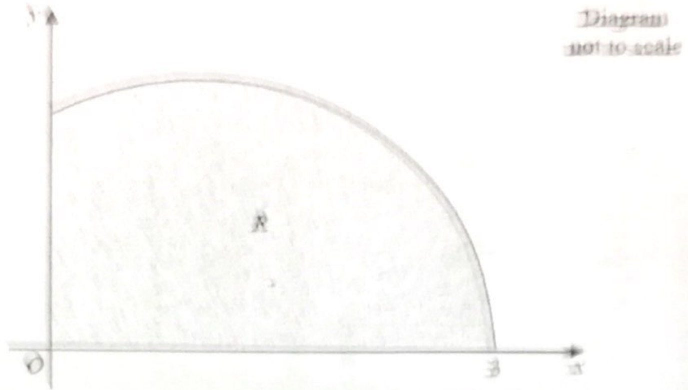


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{(3-x)(x+1)}$, $0 \le x \le 3$.

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis, and the y-axis.

(*a*) Use the substitution $x = 1 + 2\sin\theta$ to show that

$$\int_0^3 \sqrt{(3-x)(x+1)}\ \mathrm{d}x = k\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\ \mathrm{d}\theta,$$

where k is a constant to be determined.

(5)

(*b*) Hence find, by integration, the exact area of R. (3)

7a) Express $2\sin\theta - 4\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, R > 0 and $0 < \frac{\pi}{2} < \frac{\pi}{2}$. Give the exact value of R and the value of α, in radians to 3 decimal places. (3)

$$H(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

Find:

b) i) the maximum value of $H(\theta)$

ii) the smallest value of θ, for $0 \le \theta \le \pi$, at which this maximum occurs. (3)

Find:

c) i) the minimum value of $H(\theta)$

ii) the largest value of θ, for $0 \le \theta \le \pi$, at which this minimum occurs (3)