① Find an exact value for $\int_0^{\pi/4} x\sec^2 x \, dx$

③ The curve C has equation

$$4x^2 - y^3 - 4xy + 2^y = 0$$

The point P with coordinates $(-2, 4)$ lies on C.

(a) Find the exact value of $\frac{dy}{dx}$ at the point P.

(6)

The normal to C at P meets the y-axis at the point A.

(b) Find the y coordinate of A, giving your answer in the form $p + q\ln 2$, where p and q are constants to be determined.

(3)

② Solve the following trigonometric equation

$$\pi + 3\arccos(x+1) = 0.$$

⑤ (i) (a) Show that $2\tan x - \cot x = 5\operatorname{cosec} x$ may be written in the form

$$a\cos^2 x + b\cos x + c = 0$$

stating the values of the constants a, b and c.

(4)

(b) Hence solve, for $0 \leqslant x < 2\pi$, the equation

$$2\tan x - \cot x = 5\operatorname{cosec} x$$

giving your answers to 3 significant figures.

(4)

(ii) Show that

$$\tan\theta + \cot\theta \equiv \lambda \operatorname{cosec} 2\theta, \qquad \theta \neq \frac{n\pi}{2}, \quad n \in \mathbb{Z}$$

stating the value of the constant λ.

(4)

4.

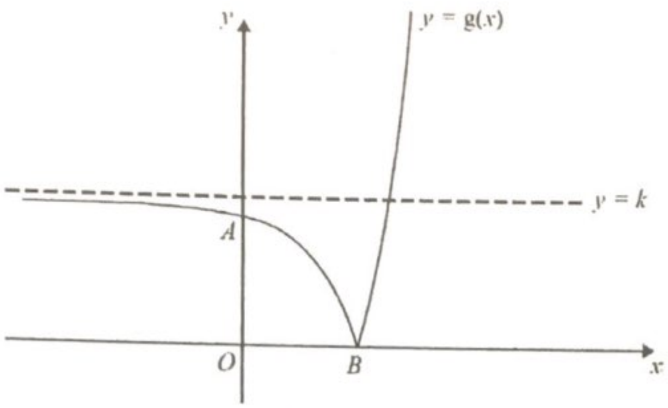


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = g(x)$, where

$$g(x) = |4e^{2x} - 25|, \qquad x \in \mathbb{R}.$$

The curve cuts the y-axis at the point A and meets the x-axis at the point B. The curve has an asymptote $y = k$, where k is a constant, as shown in Figure 1.

(*a*) Find, giving each answer in its simplest form,

(i) the y coordinate of the point A,

(ii) the exact x coordinate of the point B,

(iii) the value of the constant k.

(5)

The equation $g(x) = 2x + 43$ has a positive root at $x = \alpha$.

(*b*) Show that α is a solution of $x = \frac{1}{2}\ln\left(\frac{1}{2}x + 17\right)$.

(2)

The iteration formula

$$x_{n+1} = \frac{1}{2}\ln\left(\frac{1}{2}x_n + 17\right)$$

can be used to find an approximation for α.

(*c*) Taking $x_0 = 1.4$, find the values of x_1 and x_2. Give each answer to 4 decimal places.

(2)

(d) By choosing a suitable interval, show that $\alpha = 1.437$ to 3 decimal places.

(2)

(Total 11 marks)

(6.)

APPLIE

(1)

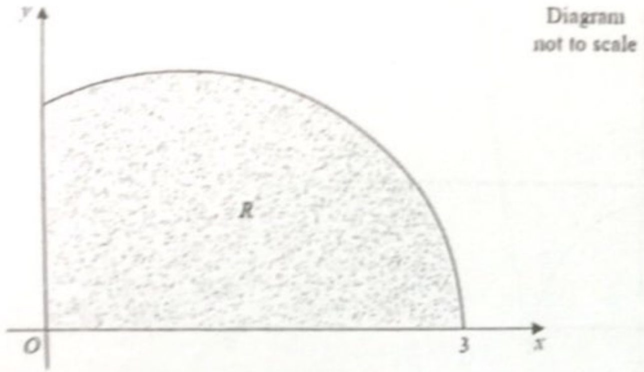


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{(3-x)(x+1)}$, $0 \le x \le 3$.

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis, and the y-axis.

(*a*) Use the substitution $x = 1 + 2\sin\theta$ to show that

$$\int_0^3 \sqrt{(3-x)(x+1)}\ \mathrm{d}x = k\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\ \mathrm{d}\theta,$$ where k is a constant to be determined. **(5)**

(*b*) Hence find, by integration, the exact area of R. **(3)**

(7) (2)

(*a*) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$ Give the exact value of R and give the value of α to 2 decimal places.

(3)

(*b*) Hence solve, for $0 \le \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15.$$

Give your answers to one decimal place.

(5)

(*c*) Use your solutions to parts (*a*) and (*b*) to deduce the smallest positive value of θ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15.$$

Give your answer to one decimal place.

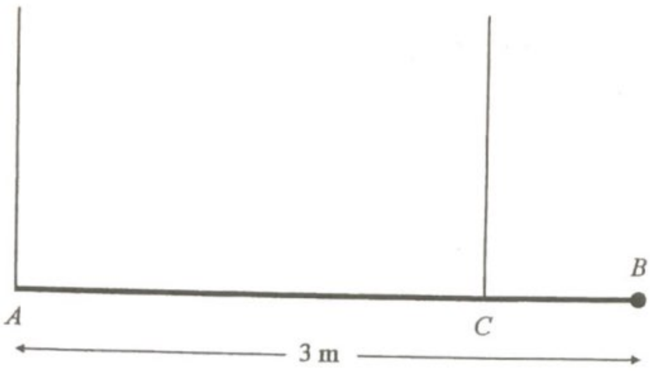
(2)

(Total 10 marks)

APPLIED

(1)

Figure 1



A plank AB has mass 40 kg and length 3 m. A load of mass 20 kg is attached to the plank at B. The loaded plank is held in equilibrium, with AB horizontal, by two vertical ropes attached at A and C, as shown in Figure 1. The plank is modelled as a uniform rod and the load as a particle. Given that the tension in the rope at C is three times the tension in the rope at A, calculate

(*a*) the tension in the rope at C,

(2)

(*b*) the distance CB.

(5)

(2)

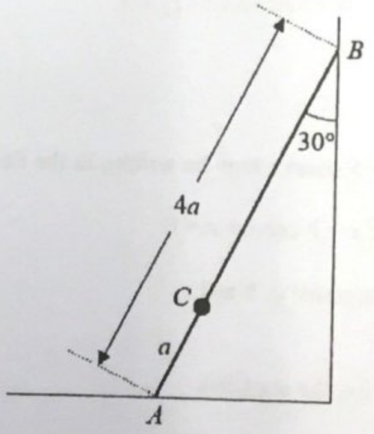


Figure 2

A ladder AB, of mass m and length $4a$, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass $3m$ is fixed on the ladder at the point C, where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium making an angle of 30° with the wall, as shown in Figure 2.

Find the coefficient of friction between the ladder and the ground.

(10)

APPLIED

③ A loom makes table cloths with an average thickness of 2.5 mm. The thickness, T mm, can be modelled using a normal distribution. Given that 65% of table cloths are less than 2.55 mm thick, find:

a the standard deviation of the thickness **(2 marks)**

b the proportion of table cloths with thickness between 2.4 mm and 2.6 mm. **(1 mark)**

A table cloth can be sold if the thickness is between 2.4 mm and 2.6 mm. A sample of 20 table cloths is taken.

c Find the probability that at least 15 table cloths can be sold. **(3 marks)**

④ **a** Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution. **(2 marks)**

A company sells orchids of which 45% produce pink flowers.

A random sample of 20 orchids is taken and X produce pink flowers.

b Find $P(X = 10)$. **(1 mark)**

A second random sample of 240 orchids is taken.

c Using a suitable approximation, find the probability that fewer than 110 orchids produce pink flowers. **(3 marks)**

d The probability that at least q orchids produce pink flowers is 0.2. Find q. **(3 marks)**