(P) 7 Liquid is pouring into a container at a constant rate of $30\,\text{cm}^3\,\text{s}^{-1}$. At time t seconds liquid is leaking from the container at a rate of $\frac{2}{15}V\,\text{cm}^3\,\text{s}^{-1}$, where $V\,\text{cm}^3$ is the volume of the liquid in the container at that time.

Show that $-15\frac{\mathrm{d}V}{\mathrm{d}t} = 2V - 450$.

(P) 8 An electrically-charged body loses its charge, Q coulombs, at a rate, measured in coulombs per second, proportional to the charge Q.
Write down a differential equation in terms of Q and t where t is the time in seconds since the body started to lose its charge.

(P) 9 The ice on a pond has a thickness x mm at a time t hours after the start of freezing. The rate of increase of x is inversely proportional to the square of x.
Write down a differential equation in terms of x and t.

(P) 10 The radius of a circle is increasing at a constant rate of 0.4 cm per second.

a Find $\frac{\mathrm{d}C}{\mathrm{d}t}$, where C is the circumference of the circle, and interpret this value in the context of the model.

b Find the rate at which the area of the circle is increasing when the radius is 10 cm.

c Find the radius of the circle when its area is increasing at the rate of $20\,\text{cm}^2$ per second.

(P) 11 The volume of a cube is decreasing at a constant rate of $4.5\,\text{cm}^3$ per second. Find:

a the rate at which the length of one side of the cube is decreasing when the volume is $100\,\text{cm}^3$

b the volume of the cube when the length of one side is decreasing at the rate of 2 mm per second.

(P) 12 Fluid flows out of a cylindrical tank with constant cross section. At time t minutes, $t > 0$, the volume of fluid remaining in the tank is $V\,\text{m}^3$. The rate at which the fluid flows in $\text{m}^3\,\text{min}^{-1}$ is proportional to the square root of V.
Show that the depth, h metres, of fluid in the tank satisfies the differential equation $\frac{\mathrm{d}h}{\mathrm{d}t} = -k\sqrt{h}$, where k is a positive constant.

(P) 13 At time, t seconds, the surface area of a cube is $A\,\text{cm}^2$ and the volume is $V\,\text{cm}^3$.
The surface area of the cube is expanding at a constant rate of $2\,\text{cm}^2\,\text{s}^{-1}$.

a Write an expression for V in terms of A.

b Find an expression for $\frac{\mathrm{d}V}{\mathrm{d}A}$

c Show that $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1}{2}V^{\frac{1}{3}}$

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