

1.

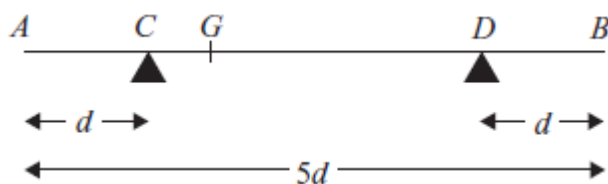


Figure 1

A non-uniform rod  $AB$ , of mass  $m$  and length  $5d$ , rests horizontally in equilibrium on two supports at  $C$  and  $D$ , where  $AC = DB = d$ , as shown in Figure 1. The centre of mass of the rod is at the point  $G$ . A particle of mass  $\frac{5}{2}m$  is placed on the rod at  $B$  and the rod is on the point of tipping about  $D$ .

(a) Show that  $GD = \frac{5}{2}d$ . (4)

The particle is moved from  $B$  to the mid-point of the rod and the rod remains in equilibrium.

(b) Find the magnitude of the normal reaction between the support at  $D$  and the rod. (5)

Answer (b)  $\frac{17}{12}mg$

2.

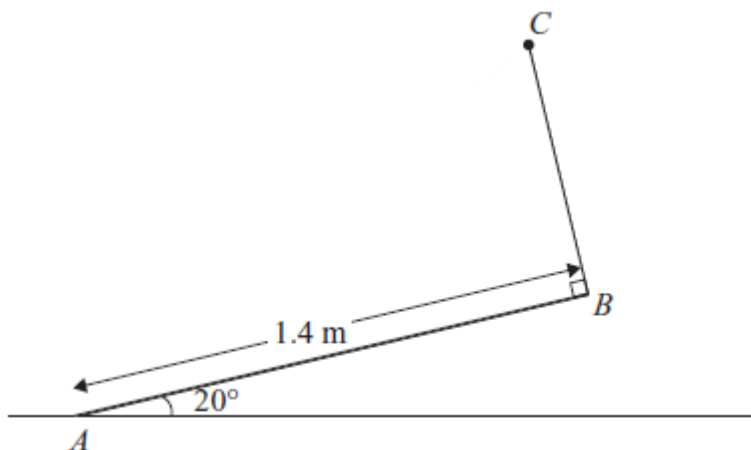


Figure 2

A uniform rod  $AB$  has mass  $4\text{ kg}$  and length  $1.4\text{ m}$ . The end  $A$  is resting on rough horizontal ground. A light string  $BC$  has one end attached to  $B$  and the other end attached to a fixed point  $C$ . The string is perpendicular to the rod and lies in the same vertical plane as the rod. The rod is in equilibrium, inclined at  $20^\circ$  to the ground, as shown in Figure 2.

(a) Find the tension in the string. (4)

Given that the rod is about to slip,

(b) find the coefficient of friction between the rod and the ground. (7)

Answer (a)  $18.4\text{ N}$  (b)  $0.29$

3. The length of time,  $L$  hours, that a phone will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours.
- (a) Find  $P(L > 127)$ . (3)
- (b) Find the value of  $d$  such that  $P(L < d) = 0.10$ . (3)
- Alice is about to go on a 6 hour journey. Given that it is 127 hours since Alice last charged her phone,
- (c) find the probability that her phone will not need charging before her journey is completed. (4)

**Answer** a) 0.0359 b)  $d = 80.776$  c) = 0.3871

4.

The probability that a certain type of rose bush will exceed 2 metres in height is 0.25.

Sixty such rose bushes are planted.

Using a distributional approximation, find the probability that 14 to 18 of these bushes, both ends inclusive, will exceed a height of 2 metres.

0.524

Handwritten solution for question 4:

Approximate by  $Y \sim N(15, 11.25)$

$E(X) = 60 \times 0.25 = 15$

$Var(X) = 60 \times 0.25 \times 0.75 = 11.25$

$\rightarrow MP(1-p)$

Normal distribution curve with mean 15 and standard deviation 11.25. The area between 13.5 and 18.5 is shaded.

Binomial distribution approximation:

$X = \text{"tall" rose bush}$

$X \sim B(60, 0.25)$

Calculations:

$$P(14 \leq X \leq 18)$$

$$= P(13.5 < Y < 18.5)$$

$$= P(Y < 18.5) - P(Y < 13.5)$$

$$= [P(Y < 18.5) - P(Y > 13.5)] - 1$$

$$= P(Y < 18.5) + P(Y > 13.5) - 1$$

$$= P(Z < \frac{18.5 - 15}{\sqrt{11.25}}) + P(Z > \frac{13.5 - 15}{\sqrt{11.25}}) - 1$$

$$= \Phi(1.0435) + \Phi(-0.4472) - 1$$

$$= 0.8508 + 0.6736 - 1$$

$$= 0.524$$