Further Trigonometry (R Forms)

1. $f(x) = 12 \cos x - 4 \sin x$.

Given that $f(x) = R \cos (x + \alpha)$, where $R \geq 0$ and $0 \leq \alpha \leq 90°$,

(*a*) find the value of R and the value of α. **(4)**

(*b*) Hence solve the equation $12 \cos x - 4 \sin x = 7$ for $0 \leq x < 360°$, giving your answers to one decimal place. **(5)**

(*c*) (i) Write down the minimum value of $12 \cos x - 4 \sin x$. **(1)**

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs. **(2)**

2. **A** curve satisfies the equation $y = \sqrt{3} \cos x + \sin x$.

(*a*) Express the equation of the curve in the form $y = R \sin (x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(4)**

(*b*) Find the values of x, $0 \leq x < 2\pi$, for which $y = 1$. **(4)**

3. (*a*) Express $3 \sin x + 2 \cos x$ in the form $R \sin (x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(4)**

(*b*) Hence find the greatest value of $(3 \sin x + 2 \cos x)^4$. **(2)**

(*c*) Solve, for $0 < x < 2\pi$, the equation $3 \sin x + 2 \cos x = 1$, giving your answers to 3 decimal places. **(5)**

4. $f(x) = 5 \cos x + 12 \sin x$.

Given that $f(x) = R \cos (x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

(*a*) find the value of R and the value of α to 3 decimal places. **(4)**

(*b*) Hence solve the equation $5 \cos x + 12 \sin x = 6$ for $0 \leq x < 2\pi$. **(5)**

(*c*) (i) Write down the maximum value of $5 \cos x + 12 \sin x$. **(1)**

(ii) Find the smallest positive value of x for which this maximum value occurs. **(2)**

5. A curve C has equation $y = 3 \sin 2x + 4 \cos 2x$, $-\pi \leq x \leq \pi$.

The point $A(0, 4)$ lies on C.

(*a*) Find an equation of the normal to the curve C at A. **(5)**

(*b*) Express y in the form $R \sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the value of α to 3 significant figures. **(4)**

(*c*) Find the coordinates of the points of intersection of the curve C with the x-axis. Give your answers to 2 decimal places. **(4)**

6. (*a*) Using the identity $\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that $\cos 2A \equiv 1 - 2 \sin^2 A$. **(2)**

(*b*) Show that $2 \sin 2\theta - 3 \cos 2\theta - 3 \sin \theta + 3 \equiv \sin \theta(4 \cos \theta + 6 \sin \theta - 3)$. **(4)**

(*c*) Express $4 \cos \theta + 6 \sin \theta$ in the form $R \sin (\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. **(4)**

(*d*) Hence, for $0 \leq \theta < \pi$, solve $2 \sin 2\theta = 3(\cos 2\theta + \sin \theta - 1)$, giving your answers in radians to 3 significant figures, where appropriate. **(5)**