# Ladders

**1.** A ladder $AB$, of mass $m$ and length $4a$, has one end $A$ resting on rough horizontal ground. The other end $B$ rests against a smooth vertical wall. A load of mass $3m$ is fixed on the ladder at the point $C$, where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium making an angle of 30° with the wall, as shown in Figure 1.

Find the coefficient of friction between the ladder and the ground. **(10)**

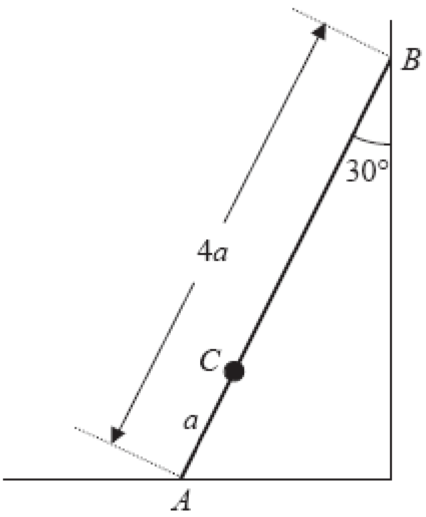



**Figure 1**

**2.** Figure 2 shows a ladder $AB$, of mass 25 kg and length 4 m, resting in equilibrium with one end $A$ on rough horizontal ground and the other end $B$ against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the ladder and the ground is $\frac{11}{25}$. The ladder makes an angle $\beta$ with the ground. When Reece, who has mass 75 kg, stands at the point $C$ on the ladder, where $AC$ = 2.8 m, the ladder is on the point of slipping. The ladder is modelled as a uniform rod and Reece is modelled as a particle.

(*a*) Find the magnitude of the frictional force of the ground on the ladder. **(3)**

(*b*) Find, to the nearest degree, the value of $\beta$. **(6)**

(*c*) State how you have used the modelling assumption that Reece is a particle. **(1)**

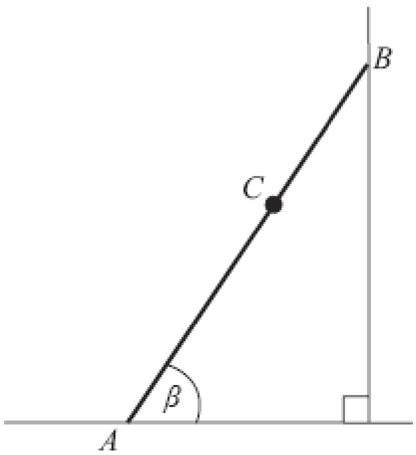



**Figure 2**

**3.** A plank rests in equilibrium against a fixed horizontal pole. The plank is modelled as a uniform rod *AB* and the pole as a smooth horizontal peg perpendicular to the vertical plane containing *AB*. The rod has length $3a$ and weight $W$ and rests on the peg at *C*, where $AC = 2a$. The end *A* of the rod rests on rough horizontal ground and *AB* makes an angle $\alpha$ with the ground, as shown in Figure 3.

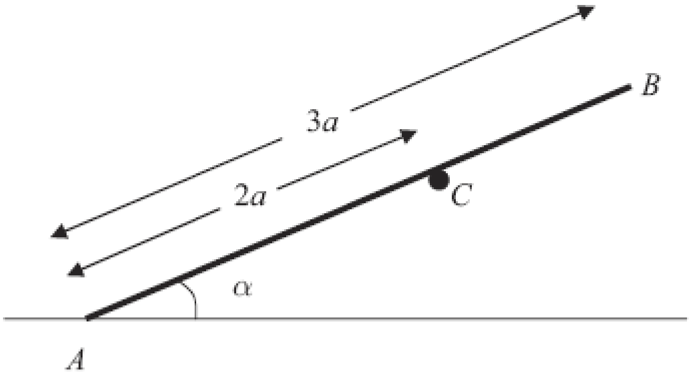



**Figure 3**

(*a*) Show that the normal reaction on the rod at *A* is $\frac{1}{4}(4 - 3\cos^2\alpha)W$. **(6)**

Given that the rod is in limiting equilibrium and that $\cos\alpha = \frac{2}{3}$,

(*b*) find the coefficient of friction between the rod and the ground. **(5)**

**4.**

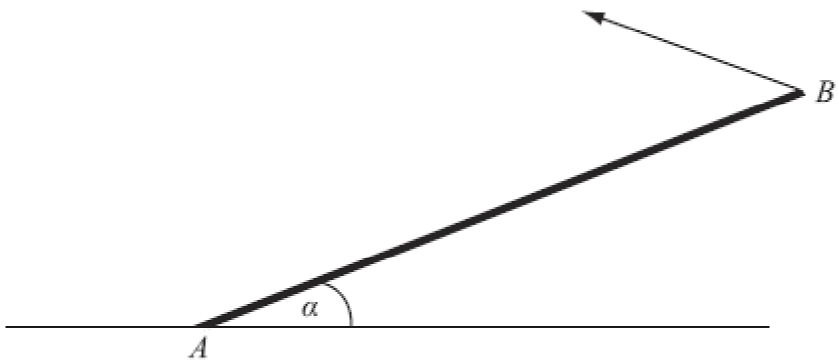



**Figure 4**

A uniform rod *AB*, of mass 20 kg and length 4 m, rests with one end *A* on rough horizontal ground. The rod is held in limiting equilibrium at an angle $\alpha$ to the horizontal, where $\tan\alpha = \frac{3}{4}$, by a force acting at *B*, as shown in Figure 4. The line of action of this force lies in the vertical plane which contains the rod. The coefficient of friction between the ground and the rod is 0.5.

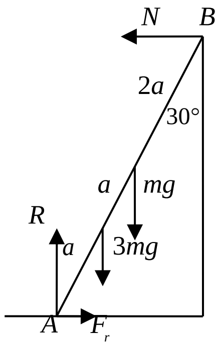
Find the magnitude of the normal reaction of the ground on the rod at *A*. **(7)**

**Answers**

**1** $\frac{5}{16\sqrt{3}}$ **2a) 431 N b) 56° 3b)** $\frac{\sqrt{5}}{4}$ **4. 157 N**

1.

(a)




M(A) $N \times 4a\cos 30° = 3mg \times a\sin 30° + mg \times 2a\sin 30°$

$N = \frac{5}{4} mg \tan 30°$ $\left( = \frac{5}{4\sqrt{3}} mg \right.$ $= 7.07\ldots \mathrm{m})$

$\rightarrow$ $F_r = N$ , $\uparrow$ $R = 4mg$

Using $F_r = \mu R$

$\frac{5}{4\sqrt{3}} mg = \mu R$ for their $R$

$\mu = \frac{5}{16\sqrt{3}}$ awrt 0.18

Alternative method:

M(B): $mg \times 2a\sin 30 + 3mg \times 3a\sin 30 + F \times 4a\cos 30 = R \times 4a\sin 30$

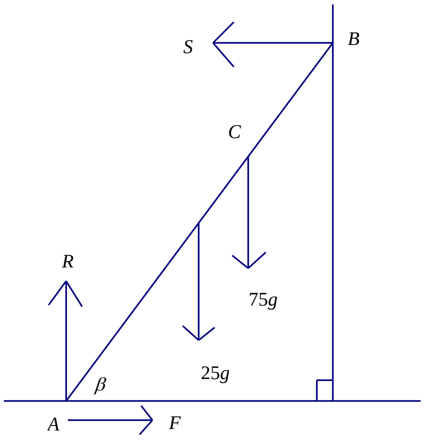
$11mga\sin 30 + F \times 4a\cos 30 = R \times 4a\sin 30$

$\frac{11mg}{2} + F\frac{4\sqrt{3}}{2} = 2R$

$\uparrow$ $R = 4mg$ ,

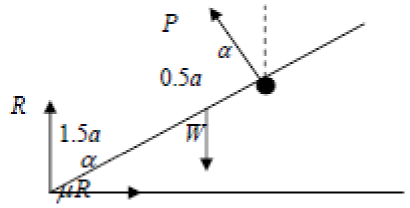
Using $F_r = \mu R$

$8\mu\sqrt{3} = \frac{5}{2}$, $\mu = \frac{5}{16\sqrt{3}}$

| | | | |
|---|---|---|---|
| (a | $R(\uparrow): R = 25g + 75g (= 100g)$ | B1 | |
| | $F = \mu R \Rightarrow F = \frac{11}{25} \times 100g$ | M1 | |
| | $= 44g\ (=431)$ | A1 | |
| | | | (3) |
| (b) |  | | |
| | M($A$): $25g \times 2\cos\beta + 75g \times 2.8\cos\beta = S \times 4\sin\beta$ | M1 A2,1,0 | |
| | $R(\leftrightarrow): F = S$ | | |
| | $176g\sin\beta = 260g\cos\beta$ | M1A1 | |
| | $\beta = 56(°)$ | A1 | |
| | | | (6) |
| (c) | So that Reece's weight acts directly at the point *C*. | B1 | |
| | | | **[10]** |

3.

(a)




$R(\uparrow) \quad R + P\cos\alpha = W$

$M(A) \quad P \times 2a = W \times 1.5a\cos\alpha$

$\left(P = \frac{3}{4}W\cos\alpha\right)$

$R = W - P\cos\alpha = W - \frac{3}{4}W\cos^2\alpha$

$= \frac{1}{4}\left(4 - 3\cos^2\alpha\right)W \quad *$ cso

(b) Using $\cos\alpha = \frac{2}{3}$, $R = \frac{2}{3}W$

$R(\rightarrow) \quad \mu R = P\sin\alpha$

Leading to $\mu = \frac{3}{4}\sin\alpha$

$\left(\sin\alpha = \sqrt{\left(1 - \frac{4}{9}\right)} = \frac{\sqrt{5}}{3}\right)$

$\mu = \frac{\sqrt{5}}{4}$ awrt 0.56

4. $m(B): R \times 4\cos\alpha = F \times 4\sin\alpha + 20g \times 2\cos\alpha$

Use of $F = \frac{1}{2}R$

Use of correct trig ratios

R = 160N or 157N