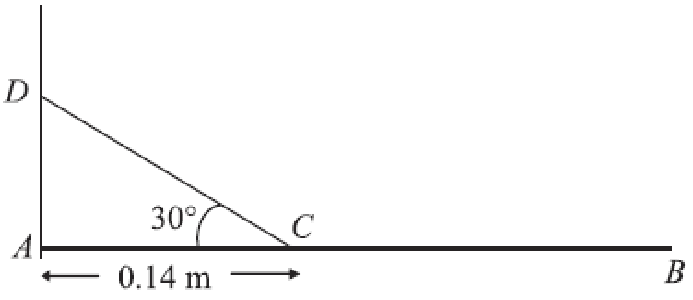
Hinges

1. **Figure 1**



A uniform beam *AB* of mass 2 kg is freely hinged at one end *A* to a vertical wall. The beam is held in equilibrium in a horizontal position by a rope which is attached to a point *C* on the beam, where *AC* = 0.14 m. The rope is attached to the point *D* on the wall vertically above *A*, where ∠*ACD* = 30°, as shown in Figure 1. The beam is modelled as a uniform rod and the rope as a light inextensible string. The tension in the rope is 63 N.

Find

(*a*) the length of *AB*, **(4)**

(*b*) the magnitude of the resultant reaction of the hinge on the beam at *A*. **(5)**

2.

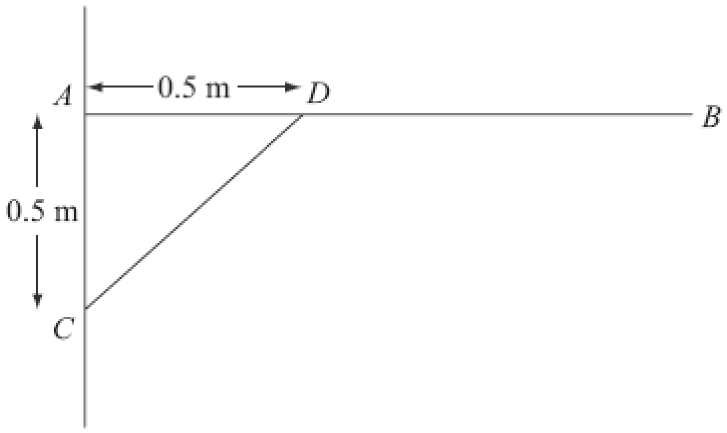


Figure 2

A uniform rod *AB*, of length 1.5 m and mass 3 kg, is smoothly hinged to a vertical wall at *A*. The rod is held in equilibrium in a horizontal position by a light strut *CD* as shown in Figure 2. The rod and the strut lie in the same vertical plane, which is perpendicular to the wall. The end *C* of the strut is freely jointed to the wall at a point 0.5 m vertically below *A*. The end *D* is freely joined to the rod so that *AD* is 0.5 m.

(*a*) Find the thrust in *CD*. **(4)**

(*b*) Find the magnitude and direction of the force exerted on the rod *AB* at *A*. **(7)**

3.

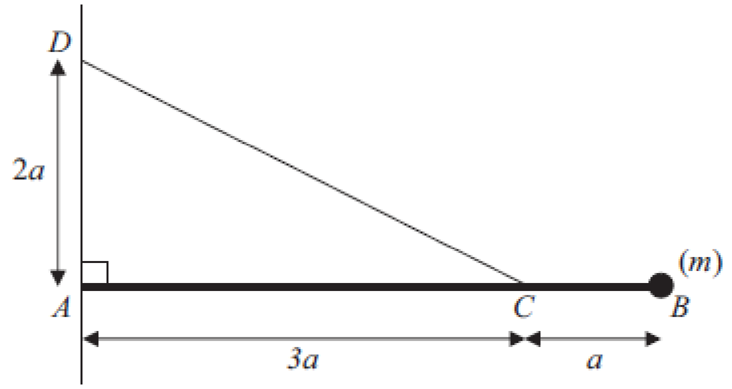


Figure 3

Figure 3 shows a uniform rod AB of mass m and length $4a$. The end A of the rod is freely hinged to a point on a vertical wall. A particle of mass m is attached to the rod at B. One end of a light inextensible string is attached to the rod at C, where $AC = 3a$. The other end of the string is attached to the wall at D, where $AD = 2a$ and D is vertically above A. The rod rests horizontally in equilibrium in a vertical plane perpendicular to the wall and the tension in the string is T.

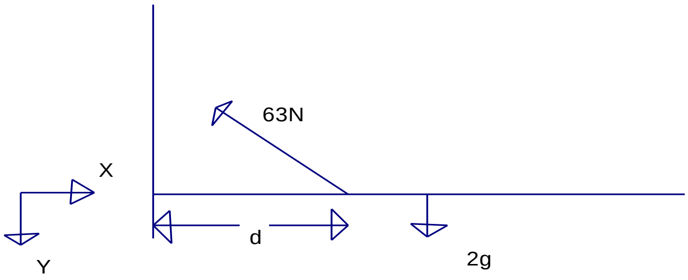
(a) Show that $T = mg\sqrt{13}$. **(5)**

The particle of mass m at B is removed from the rod and replaced by a particle of mass M which is attached to the rod at B. The string breaks if the tension exceeds $2mg\sqrt{13}$. Given that the string does not break,

(b) show that $M \leq \frac{5}{2}m$. **(3)**

Answers 1a) 45 cm b) 56 N 2a) 62.4 N b) 46.5 N, 161.6° below AB

1.

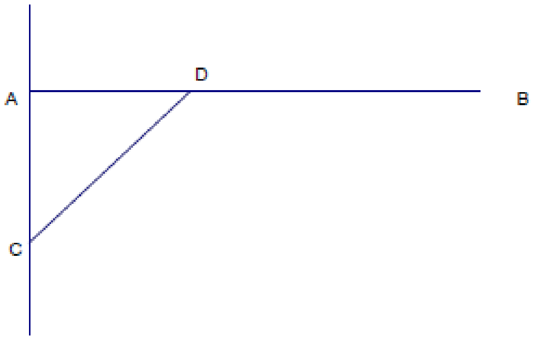


M(A) 63 sin 30 . 14 = $2g$. d
Solve: d = 0.225m
Hence AB = <u>45 cm</u>

R(→) $X = 63 \cos 30$ ($\approx$ 54.56)

R(↑) $Y = 63 \sin 30 - 2g$ ($\approx$ 11.9)

$R = \sqrt{(X^2 + Y^2)} \approx$ <u>55.8, 55.9 or 56 N</u>



Taking moments about A:

$$3g \times 0.75 = \frac{T}{\sqrt{2}} \times 0.5$$

$$T = 3\sqrt{2}g \times \frac{7.5}{5} = \frac{9\sqrt{2}g}{2} (= 62.4N)$$

$$\leftarrow \ \pm H = \frac{T}{\sqrt{2}} (= \frac{9g}{2} \approx 44.1N)$$

$$\uparrow \ \pm V + \frac{T}{\sqrt{2}} = 3g \quad (\square V = 3g - \frac{9g}{2} = \frac{-3g}{2} \approx -14.7 \square$$

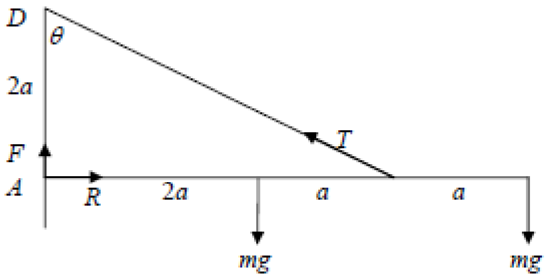
$$\Rightarrow |R| = \sqrt{81+9} \times \frac{g}{2} \approx 46.5(N)$$

at angle $\tan^{-1}\frac{1}{3} = 18.4^{\circ}$ (0.322 radians) below the line of BA

161.6° (2.82 radians) below the line of AB
(108.4° or 1.89 radians to upward vertical)

2.

3.



M(A) $3a \times T\cos\theta = 2amg + 4amg$

$$\cos\theta = \left(\frac{2}{\sqrt{9+4}} = \right)\frac{2}{\sqrt{13}}$$

$$\frac{6}{\sqrt{13}}T = 6mg$$

$$T = mg\sqrt{13} \quad *$$

$3a \times T \times \cos\theta = 2amg + 4aMg$

$$T = \frac{(2mg + 4Mg)}{6}\sqrt{13} \le 2mg\sqrt{13}$$

$mg + 2Mg \le 6mg$

$$M \le \frac{5}{2}m \quad *$$