

Exercise 3F

- 1 For each of the following binomial random variables, X :
- state, with reasons, whether X can be approximated by a normal distribution.
 - if appropriate, write down the normal approximation to X in the form $N(\mu, \sigma^2)$, giving the values of μ and σ .
- a $X \sim B(120, 0.6)$ b $X \sim B(20, 0.5)$ c $X \sim B(250, 0.52)$
d $X \sim B(300, 0.85)$ e $X \sim B(400, 0.48)$ f $X \sim B(1000, 0.58)$
- 2 The random variable $X \sim B(150, 0.45)$. Use a suitable approximation to estimate:
- $P(X \leq 60)$
 - $P(X > 75)$
 - $P(65 \leq X \leq 80)$
- 3 The random variable $X \sim B(200, 0.53)$. Use a suitable approximation to estimate:
- $P(X < 90)$
 - $P(100 \leq X < 110)$
 - $P(X = 105)$
- 4 The random variable $X \sim B(100, 0.6)$. Use a suitable approximation to estimate:
- $P(X > 58)$
 - $P(60 < X \leq 72)$
 - $P(X = 70)$
- 5 A fair coin is tossed 70 times. Use a suitable approximation to estimate the probability of obtaining more than 45 heads.
- 6 The probability of a roulette ball landing on red when the wheel is spun is $\frac{50}{101}$.
On one day in a casino, the wheel is spun 1200 times.
Estimate the probability that the ball lands on red in at least half of these spins.
- 7 a Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution. **(2 marks)**
A company sells orchids of which 45% produce pink flowers.
A random sample of 20 orchids is taken and X produce pink flowers.
b Find $P(X = 10)$. **(1 mark)**
A second random sample of 240 orchids is taken.
c Using a suitable approximation, find the probability that fewer than 110 orchids produce pink flowers. **(3 marks)**
d The probability that at least q orchids produce pink flowers is 0.2. Find q . **(3 marks)**
- 8 A drill bit manufacturer claims that 52% of its bits last longer than 40 hours.
A random sample of 30 bits is taken and X last longer than 40 hours.
a Find $P(X < 17)$. **(1 mark)**
A second random sample of 600 drill bits is taken.
b Using a suitable approximation, find the probability that between 300 and 350 bits last longer than 40 hours. **(3 marks)**
- 9 A particular breakfast cereal has prizes in 56% of the boxes. A random sample of 100 boxes is taken.
a Find the exact value of the probability that exactly 55 boxes contain a prize. **(1 mark)**
b Find the percentage error when using a normal approximation to calculate the probability that exactly 55 boxes contain prizes. **(4 marks)**